Defining the Manufacturing Process (MP) for a Single Server, Multiple Product Kanban Controlled System (SS/MP/KCS)

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ABSTRACT
This paper is a subset of a series of working papers on Multiple Product Kanban Controlled System (MP/KCS) (Ang, 2012, 2013a, 2013b; Ang & Piplani, 2010a, 2010b). Throughout the Kanban research literature, no researcher has formally defined the Manufacturing Process (MP) for a Single Server, Multiple Product Kanban Controlled System (SS/MP/KCS). Hence this paper tries to cover this gap. The work done here is important because it is a stepping stone towards comparing the performance of MP/KCS systems. The results for this performance comparison will then be presented in future papers. In other words, by properly defining and giving appropriate practical assumptions to the internal routings of the MP, simulations of SS/MP/KCS can then be possible. And this will result in a performance comparison for future managers of MP/KCS to decide which system is best to adopt.

Keywords: Kanban, Kanban Control System (KCS), Manufacturing Process (MP), M/M/1, Priority Queues

1. INTRODUCTION

This paper is a subset of a series of working papers on Multiple Product Kanban Controlled System (MP/KCS) (Ang, 2012, 2013a, 2013b; Ang & Piplani, 2010a, 2010b). Throughout the Kanban research literature, no researcher has formally defined the Manufacturing Process (MP) for a Single Server, Multiple Product Kanban Controlled System (SS/MP/KCS). Hence this paper tries to cover this gap.

Baynet et al. (2002) were the first researchers to draw detailed schematics of how Multiple Product Kanban Controlled Systems (MP/KCS) looked like. Specifically, they presented the Single Stage, Multiple Product Base Stock (SS/MP/BS), Dedicated Traditional Kanban Control System (SS/MP/De-TKCS)[1], Shared Extended Kanban Control System (SS/MP/Sh-EKCS), Dedicated Extended Kanban Control System (SS/MP/De-EKCS). They will not be presented here due to the lack of space. But reference can be made to their paper at (Baynat, Buzacott, & Dallery, 2002).

The problem with their schematics was that they did not make any assumptions about either workstations of a stage (which could be a production line, a job shop or any flexible manufacturing system), or on the routing of parts. Their discussion on MP/KCS focused on external routing and not on internal processing of MPs. In other words, if you located the MPs in their paper, you would not be able to find any description on what goes on inside it. It's like a 'black box'. However, in the following it is important to discuss the internal mechanism of MP/KCS so that they can be compared using simulation.

[1] Baynet et al. (2002) proposed two kinds of MP/TKCS. They are dedicated and shared TKCS. However, they have been proven to be equivalent and it is not worthwhile to consider the shared case. Hence, in this research, only the SS/MP/De-TKCS is considered. In further discussion 'De' is dropped and the system is simply termed as SS/MP/TKCS.

2. MANUFACTURING PROCESS (MP) FOR SS/MP/KCS – M/M/1 WITH PRIORITY QUEUES

Since Baynet et al. (2002) did not describe the internal mechanism of the MP for MP/KCS, an attempt is made here. Figure 1 shows what goes on inside the Manufacturing Process (MP) of an MP/KCS.

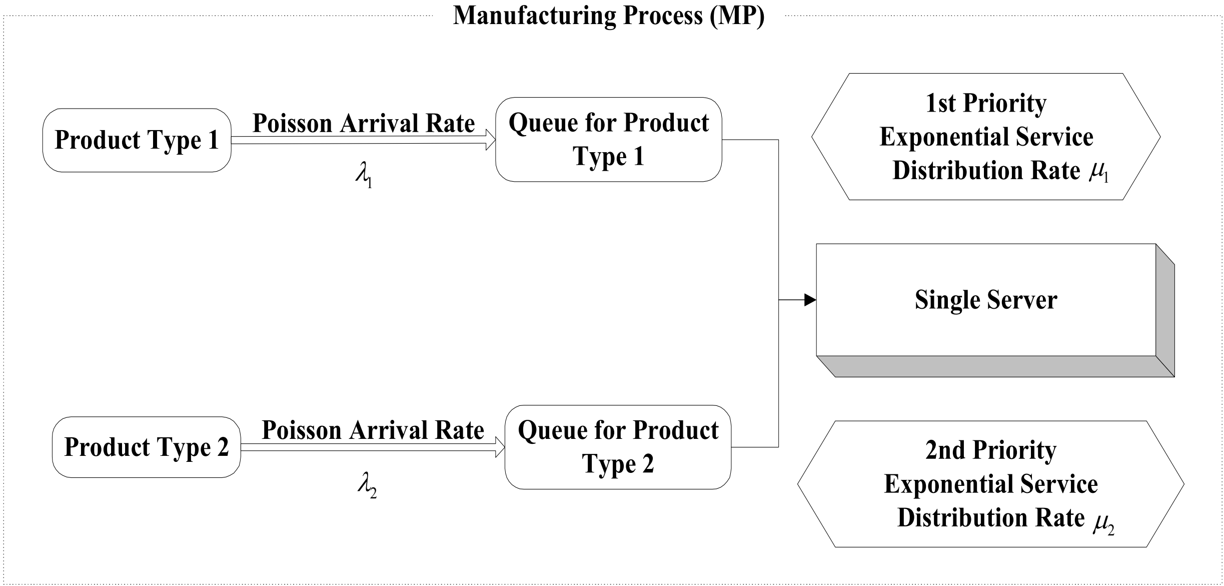

Figure 1: Manufacturing Process (MP) as an M/M/1 queue with two priorities

The MP operates as follows:

- Type 1 and 2 demands have Poisson arrival rates of λ_1 and λ_2, respectively.
- Type 1 demand arrivals have higher priority.
- There is no pre-emption of service.
- Type 1 and 2 demands have exponential service distribution rates of μ_1 μ_2, respectively.
- $\lambda_1 < \lambda_2$: Demand arrival rate of type 2 is greater than type 1; else type 2 will never get processed.
- $\lambda_1 < \mu_1$: Standard queuing theory assumption, arrival rate of type 1 customers cannot be greater than its processing rate.
- $\lambda_2 < \mu_2$: Standard queuing theory assumption, arrival rate of Type 2 customers cannot be greater than its processing rate.
- $\lambda_2 < \mu_1$: Demand arrival rate for type 2 is smaller than processing rate for type 1. This assumption is needed to prevent system from getting choked with type 2 demands. Consider for example the reverse, $\mu_1 < \lambda_2$; each time a type 1 product enters, it has priority for processing. But it has a slower processing rate compared to type 2 arrivals, so that once it completes, the system would have a long queue of type 2 demands waiting and will be flooded with type 2 demands.
- $\lambda_1 < \mu_2$: Demand arrival rate for type 1 is smaller than MP rate for type 2. This assumption is similar to the one above and is needed to prevent the system from getting flooded with type 1 demands.
- Since the law of queuing theory states that average demand arrival rate, λ_A, be lower than average Manufacturing Process (MP) rate, μ_A, that is $\lambda_A < \mu_A$; two possibilities exist:
  i. $\lambda_1 < \lambda_2 < \mu_1 < \mu_2$
  ii. $\lambda_1 < \lambda_2 < \mu_2 < \mu_1$

3. AVERAGE MANUFACTURING PROCESS (MP) RATE

Now that the MP for MP/KCS has been properly defined, the next important parameter that needs to be formulated is the Average MP Processing Rate, μ_A. The average MP rate is used in optimization models for MP/KCS. Thus it is important because only after these MP/KCS have been optimized can a fair performance comparison take place. Figure 1 showed an M/M/1 system with priority queues of two product types. (Ross, 2007)used a General service distribution time to compute the average arrival rate and distribution of arrival as follows:

$$\lambda_A = \lambda_1 + \lambda_2 \quad (1)$$

$$G(x) = \frac{\lambda_1}{\lambda} G_1(x) + \frac{\lambda_2}{\lambda} G_2(x) \quad (2)$$

Equation (1) shows the average arrival rate of two Poisson arrival rates while Equation (2) shows average arrival distribution of two independent arrival distributions. This could either be the 1st or 2nd moment of Service Distribution i.e. $E[S_i]$ or $E[S_i^2]$ (where $E[S_i]$ is expected service time for customer i. Equations (1) and (2) hold since the combination of two independent Poisson Processes is itself a Poisson Process, whose rate is the sum of rates of the component processes. Also, the 1st and 2nd moments of the Exponential distribution are

$$E[S_i] = \frac{1}{\mu_i} \quad (3)$$

$$E\left[S_i^2\right] = \frac{2}{\mu_i^2} \quad (4)$$

Substituting Equations (3) and (4) into (2):

$$E[S_A] = \frac{\lambda_1}{\lambda} E[S_1] + \frac{\lambda_2}{\lambda} E[S_2] \quad (5)$$

Also

$$\frac{1}{\mu_A} = \frac{\lambda_1}{\lambda}\left[\frac{1}{\mu_1}\right] + \frac{\lambda_2}{\lambda}\left[\frac{1}{\mu_2}\right] \quad (6)$$

The validity of Equation (6) can be tested by a system with no type 2 customer arrivals, that is, $\lambda_2 = 0$. From Equation (1), $\lambda = \lambda_1$. Substituting this into Equation (6)

$$\frac{1}{\mu_A} = \frac{1}{\mu_1}$$

$$E[S_A] = E[S_1] \quad (7)$$

This shows that without type 2 arrivals, the system behaves as a a single product system, validating Equations (1) and (2). Continuing to obtain average MP rate:

$$\frac{1}{\mu_A} = \frac{\lambda_1}{\lambda\mu_1} + \frac{\lambda_2}{\lambda\mu_2} = \frac{\lambda_1\mu_2 + \lambda_2\mu_1}{\lambda\mu_1\mu_2} \quad (8)$$

And

$$\mu_A = \frac{\lambda\mu_1\mu_2}{\lambda_1\mu_2 + \lambda_2\mu_1} \quad (9)$$

4. SUMMARY

In this short paper, the Manufacturing Process (MP) for a Single Server, Multiple Product Kanban Controlled System (SS/MP/KCS) is properly defined. In addition, the Average MP Processing Rate, μ_A, has also been formulated. The μ_A formulated here is useful for optimizing MP/KCS–and important because only after these MP/KCS have been optimized can a fair performance comparison take place.

The overall work done here is important because it is a stepping stone towards comparing the performance of MP/KCS systems. The results for this performance comparison will then be presented in future papers. In other words, by properly defining and giving appropriate practical assumptions to the internal routings of the MP, simulations of SS/MP/KCS can then be possible. And this will result in a performance comparison for future managers of MP/KCS to decide which system is best to adopt.

5. REFERENCES

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