

DR. ALVIN'S PUBLICATIONS

DESIGN OF EXPERIMENTS WITH MINITAB

DR. ALVIN ANG



1 | PAGE

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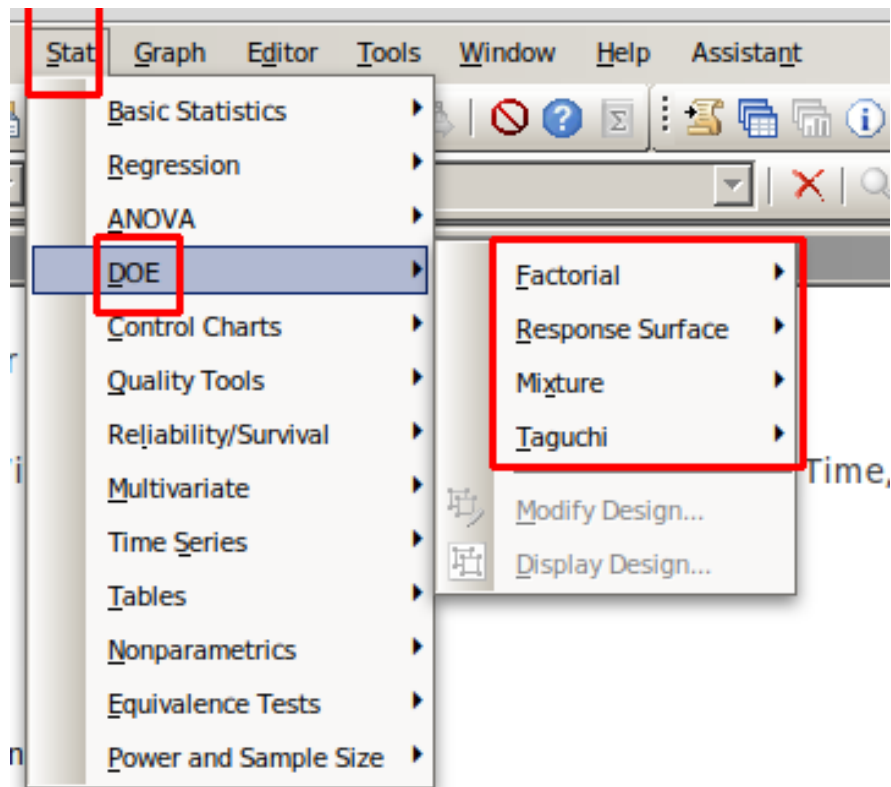
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I. INTRODUCTION



There are 4 types of DOEs in Minitab.

- Factorial (2 Levels)
 - Full Factorial
 - Fractional Factorial

- Response Surface Method (RSM) (>2 levels)
 - Central Composite Design (CCD)
 - Box-Behnken Design

- Mixture
 - Simplex Centroid
 - Simplex Lattice
 - Extreme Vertices

- Taguchi
 - Larger is Better
 - Nominal is Best
 - Nominal is Best (Default)
 - Smaller is Better

II. SIMPLE FULL FACTORIAL EXPERIMENT (2 LEVELS)

Single Cup Catapult



Practice Design of Experiments (DOE)

Go to <https://sigmazone.com/catapult-grid/>



Homepage Products Training Consulting About Us Articles Support

There are 5 Factors affecting the Distance (Response)

- Release Angle
- Firing Angle
- Cup Elevation
- Pin Elevation
- Bungee Positioning

Grid Interface

Run all rows

Number of rows 7

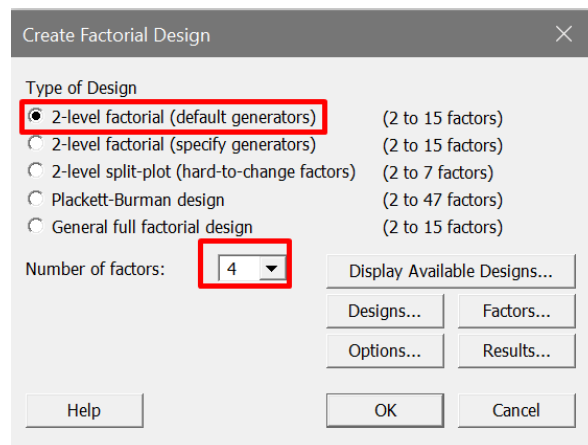
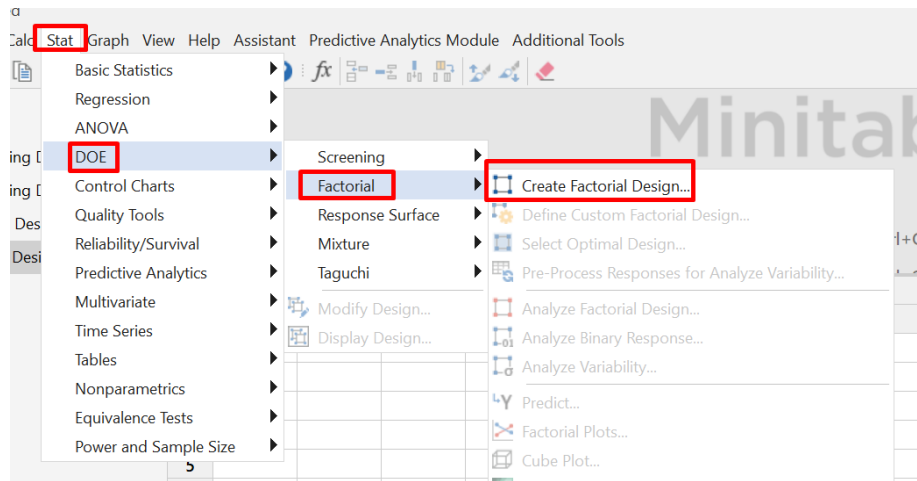
Update

| Release Angle | Firing Angle | Cup Elevation | Pin Elevation | Bungee Position | Distance |
|---------------|--------------|---------------|---------------|-----------------|------------|
| 100 | 100 | 300 | 200 | 200 | |
| 100 | 100 | 300 | 200 | 200 | (Response) |
| 100 | 100 | 300 | 200 | 200 | |
| 100 | 100 | 300 | 200 | 200 | |
| 100 | 100 | 300 | 200 | 200 | |
| 100 | 100 | 300 | 200 | 200 | |
| 100 | 100 | 300 | 200 | 200 | |

- We will test the following:
 - Release Angle: 140 / 180
 - Firing Angle: 110 (since this is constant, we will ignore it)
 - Cup Elevation: 220 / 280
 - Pin Elevation: 120 / 180
 - Bungee Position: 120 / 180

- 4 Factors 2 Levels, this is known as a 2^4 Factorial Design DOE.
- Total number of runs = $2^4 = 16$ runs
- Now go to Minitab

A. CREATION OF EXPERIMENT



1. DISPLAY AVAILABLE DESIGNS

Available Factorial Designs (with Resolution)

| Run | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|-----|------|------|-----|-----|-----|----|-----|-----|-----|-----|-----|-----|-----|-----|
| 4 | Full | III | | | | | | | | | | | | |
| 8 | Full | IV | III | III | III | | | | | | | | | |
| 16 | Full | V | IV | IV | IV | IV | III | III | III | III | III | III | III | III |
| 32 | Full | VI | IV | IV | IV | IV | IV | IV | IV | IV | IV | IV | IV | IV |
| 64 | Full | VII | V | IV | IV | IV | IV | IV | IV | IV | IV | IV | IV | IV |
| 128 | Full | VIII | VI | V | V | V | V | V | V | V | V | V | V | V |

Available Resolution III Plackett-Burman Designs

| Factors | Runs | Factors | Runs | Factors | Runs |
|---------|--------------------|---------|--------------------|---------|----------|
| 2-7 | 12,20,24,28,...,48 | 20-23 | 24,28,32,36,...,48 | 36-39 | 40,44,48 |
| 8-11 | 12,20,24,28,...,48 | 24-27 | 28,32,36,40,44,48 | 40-43 | 44,48 |
| 12-15 | 20,24,28,36,...,48 | 28-31 | 32,36,40,44,48 | 44-47 | 48 |
| 16-19 | 20,24,28,32,...,48 | 32-35 | 36,40,44,48 | | |

2. CREATE FACTORIAL DESIGNS

Designs

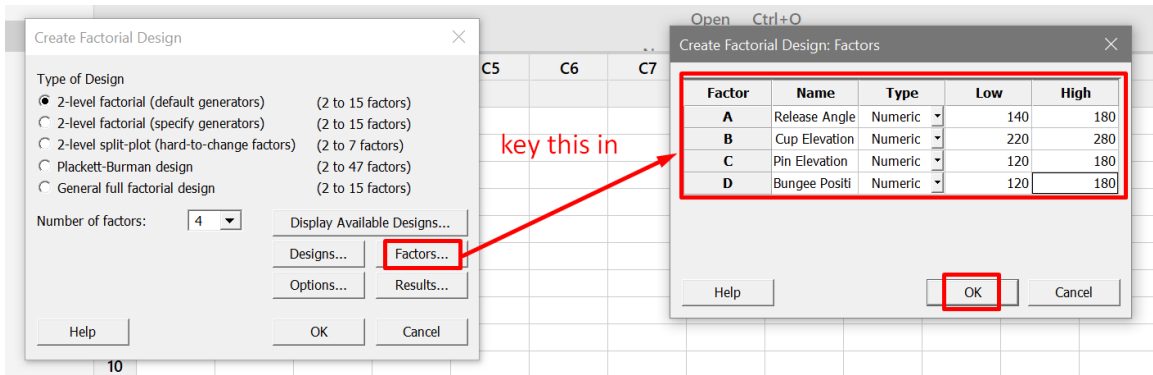
| Designs | Runs | Resolution | 2 ^{^(k-p)} |
|----------------|------|------------|---------------------|
| 1/2 fraction | 8 | IV | 2 ^{^(4-1)} |
| Full factorial | 16 | Full | 2 ^{^4} |

Number of center points per block: 0

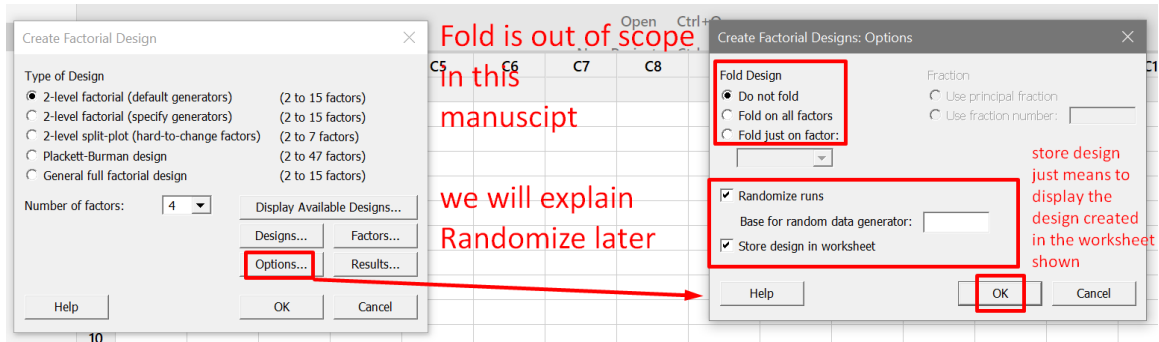
Number of replicates for corner points: 1

Number of blocks: 1

3. FACTORS



4. OPTIONS



view Help Assistant Predictive Analytics Module Additional Tools

we will explain these 4 columns later...

Open Ctrl+O

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
|----|----------|----------|----------|--------|---------------|---------------|---------------|-----------------|
| | StdOrder | RunOrder | CenterPt | Blocks | Release Angle | Cup Elevation | Pin Elevation | Bungee Position |
| 1 | 10 | 1 | 1 | 1 | 180 | 220 | 120 | 180 |
| 2 | 5 | 2 | 1 | 1 | 140 | 220 | 180 | 120 |
| 3 | 1 | 3 | 1 | 1 | 140 | 220 | 120 | 120 |
| 4 | 4 | 4 | 1 | 1 | 180 | 280 | 120 | 120 |
| 5 | 2 | 5 | 1 | 1 | 180 | 220 | 120 | 120 |
| 6 | 7 | 6 | 1 | 1 | 140 | 280 | 180 | 120 |
| 7 | 11 | 7 | 1 | 1 | 140 | 280 | 120 | 180 |
| 8 | 16 | 8 | 1 | 1 | 180 | 280 | 180 | 180 |
| 9 | 6 | 9 | 1 | 1 | 180 | 220 | 180 | 120 |
| 10 | 14 | 10 | 1 | 1 | 180 | 220 | 180 | 180 |
| 11 | 8 | 11 | 1 | 1 | 180 | 280 | 180 | 120 |
| 12 | 12 | 12 | 1 | 1 | 180 | 280 | 120 | 180 |
| 13 | 15 | 13 | 1 | 1 | 140 | 280 | 180 | 180 |
| 14 | 9 | 14 | 1 | 1 | 140 | 220 | 120 | 180 |
| 15 | 13 | 15 | 1 | 1 | 140 | 220 | 180 | 180 |
| 16 | 3 | 16 | 1 | 1 | 140 | 280 | 120 | 120 |

Create Factorial Design

Type of Design

- 2-level factorial (default generators) (2 to 15 factors)
- 2-level factorial (specify generators) (2 to 15 factors)
- 2-level split-plot (hard-to-change factors) (2 to 7 factors)
- Plackett-Burman design (2 to 47 factors)
- General full factorial design (2 to 15 factors)

Number of factors: 4

Display Available Designs...

Designs... Factors... Options... Results...

Help OK Cancel

click ok and these 4 columns will be generated...

a total of 16 runs with different combinations...

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|---------------|---------------|---------------|-----------------|----|----|----|----|----|
| | Release Angle | Cup Elevation | Pin Elevation | Bungee Position | | | | | |
| 1 | 140 | 220 | 120 | 120 | | | | | |
| 2 | 180 | 220 | 120 | 120 | | | | | |
| 3 | 140 | 280 | 120 | 120 | | | | | |
| 4 | 180 | 280 | 120 | 120 | | | | | |
| 5 | 140 | 220 | 180 | 120 | | | | | |
| 6 | 180 | 220 | 180 | 120 | | | | | |
| 7 | 140 | 280 | 180 | 120 | | | | | |
| 8 | 180 | 280 | 180 | 120 | | | | | |
| 9 | 140 | 220 | 120 | 180 | | | | | |
| 10 | 180 | 220 | 120 | 180 | | | | | |
| 11 | 140 | 280 | 120 | 180 | | | | | |
| 12 | 180 | 280 | 120 | 180 | | | | | |
| 13 | 140 | 220 | 180 | 180 | | | | | |
| 14 | 180 | 220 | 180 | 180 | | | | | |
| 15 | 140 | 280 | 180 | 180 | | | | | |
| 16 | 180 | 280 | 180 | 180 | | | | | |

copy and paste this 4 columns....

back into the <https://sigmazone.com/catapult-grid/> website ... to generate the Distance (response)

B. RUNNING THE EXPERIMENTS & GETTING THE RESULTS (RESPONSE)

- Go to <https://sigmazone.com/catapult-grid/>

Grid Interface

Recall this column is fixed at 110....is not a Factor we consider...

Run all rows Number of rows: 16 Update

when u click on Run all rows, the Response (Distance) appears...

| Release Angle | Firing Angle | Cup Elevation | Pin Elevation | Bungee Position | Distance |
|---------------|--------------|---------------|---------------|-----------------|----------|
| 140 | 110 | 220 | 120 | 120 | 90.03 |
| 180 | 110 | 220 | 120 | 120 | 175.27 |
| 140 | 110 | 280 | 120 | 120 | 127.44 |
| 180 | 110 | 280 | 120 | 120 | 243.28 |
| 140 | 110 | 220 | 180 | 120 | 121.25 |
| 180 | 110 | 220 | 180 | 120 | 239.62 |
| 140 | 110 | 280 | 180 | 120 | 169.23 |
| 180 | 110 | 280 | 180 | 120 | 338.75 |
| 140 | 110 | 220 | 120 | 180 | 118.35 |
| 180 | 110 | 220 | 120 | 180 | 232.99 |
| 140 | 110 | 280 | 120 | 180 | 173.89 |
| 180 | 110 | 280 | 120 | 180 | 321.57 |
| 140 | 110 | 220 | 180 | 180 | 159.22 |
| 180 | 110 | 220 | 180 | 180 | 320.12 |
| 140 | 110 | 280 | 180 | 180 | 225.81 |
| 180 | 110 | 280 | 180 | 180 | 450.74 |

Mode

remember to copy paste the blue boxes correctly... column for column...

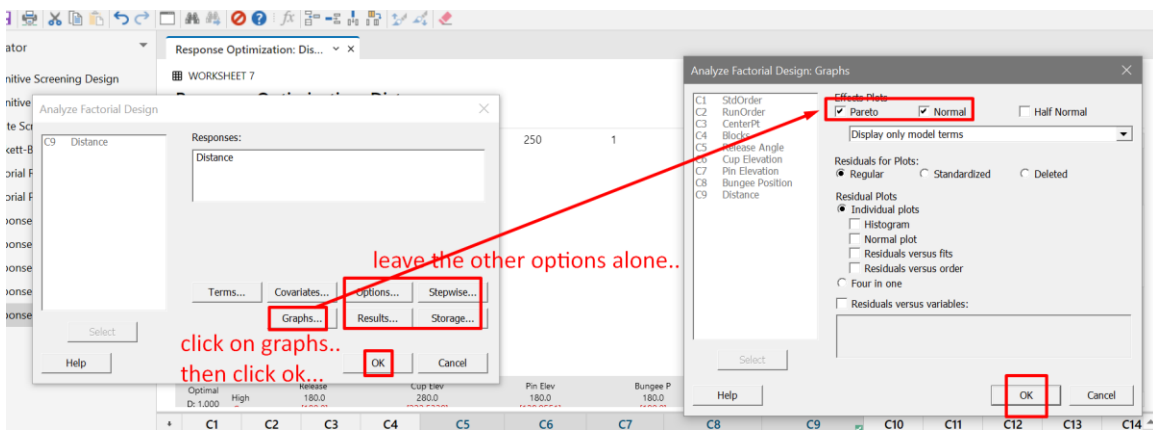
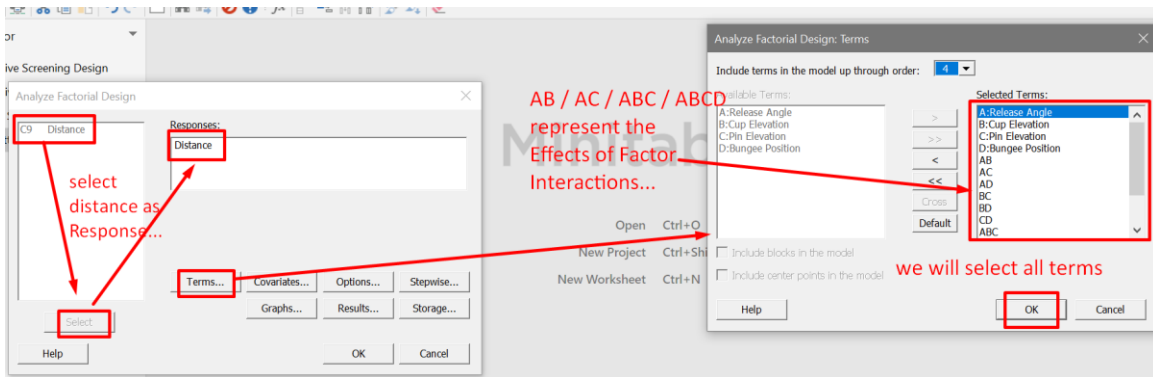
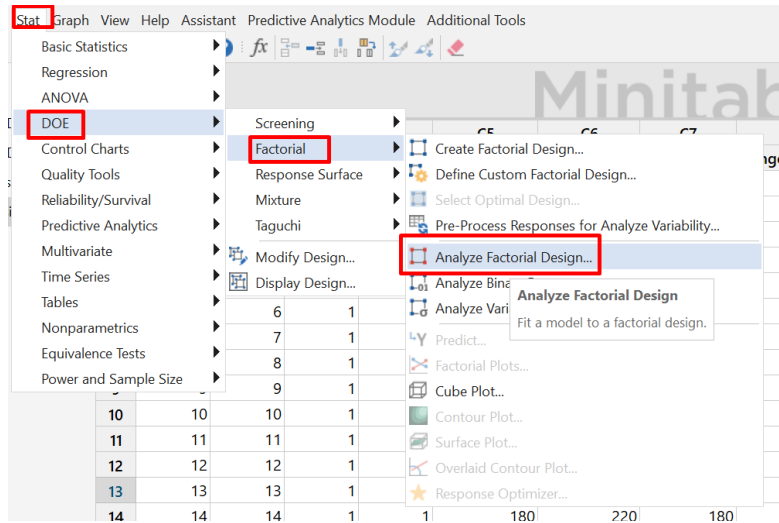
| | Distance |
|----|----------|
| 20 | 90.03 |
| 20 | 175.27 |
| 20 | 127.44 |
| 20 | 243.28 |
| 20 | 121.25 |
| 20 | 239.62 |
| 20 | 169.23 |
| 20 | 338.75 |
| 80 | 118.35 |
| 80 | 232.99 |
| 80 | 173.89 |
| 80 | 321.57 |
| 80 | 159.22 |
| 80 | 320.12 |
| 80 | 225.81 |
| 80 | 450.74 |

copy
paste this
Distance
Column...

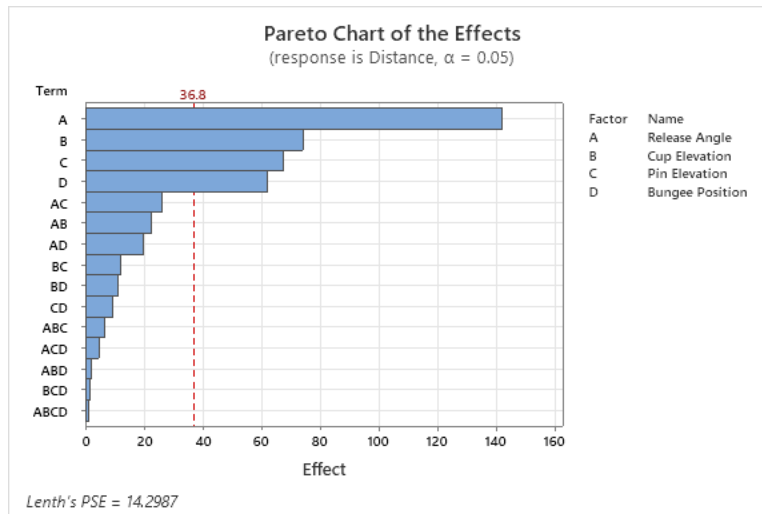
| C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 |
|---------------|---------------|---------------|-----------------|----------|-----|-----|-----|-----|-----|
| Release Angle | Cup Elevation | Pin Elevation | Bungee Position | Distance | | | | | |
| 140 | 220 | 120 | 120 | 90.03 | | | | | |
| 180 | 220 | 120 | 120 | 175.27 | | | | | |
| 140 | 280 | 120 | 120 | 127.44 | | | | | |
| 180 | 280 | 120 | 120 | 243.28 | | | | | |
| 140 | 220 | 180 | 120 | 121.25 | | | | | |
| 180 | 220 | 180 | 120 | 239.62 | | | | | |
| 140 | 280 | 180 | 120 | 169.23 | | | | | |
| 180 | 280 | 180 | 120 | 338.75 | | | | | |
| 140 | 220 | 120 | 180 | 118.35 | | | | | |
| 180 | 220 | 120 | 180 | 232.99 | | | | | |
| 140 | 280 | 120 | 180 | 173.89 | | | | | |
| 180 | 280 | 120 | 180 | 321.57 | | | | | |
| 140 | 220 | 180 | 180 | 159.22 | | | | | |
| 180 | 220 | 180 | 180 | 320.12 | | | | | |
| 140 | 280 | 180 | 180 | 225.81 | | | | | |
| 180 | 280 | 180 | 180 | 450.74 | | | | | |

Back here
into Minitab...

C. ANALYZING THE RESULTS

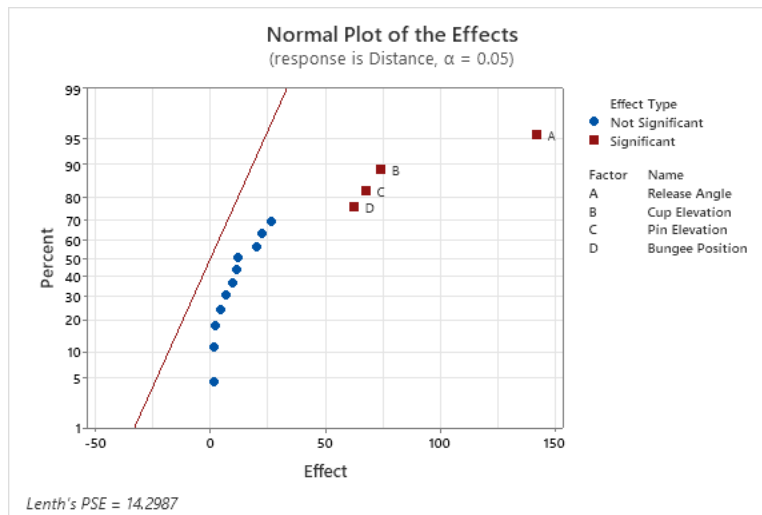


1. PARETO CHART



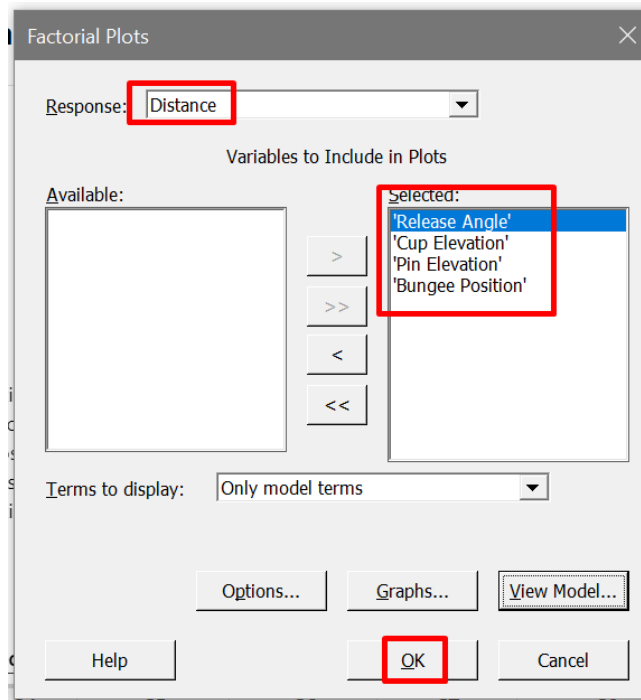
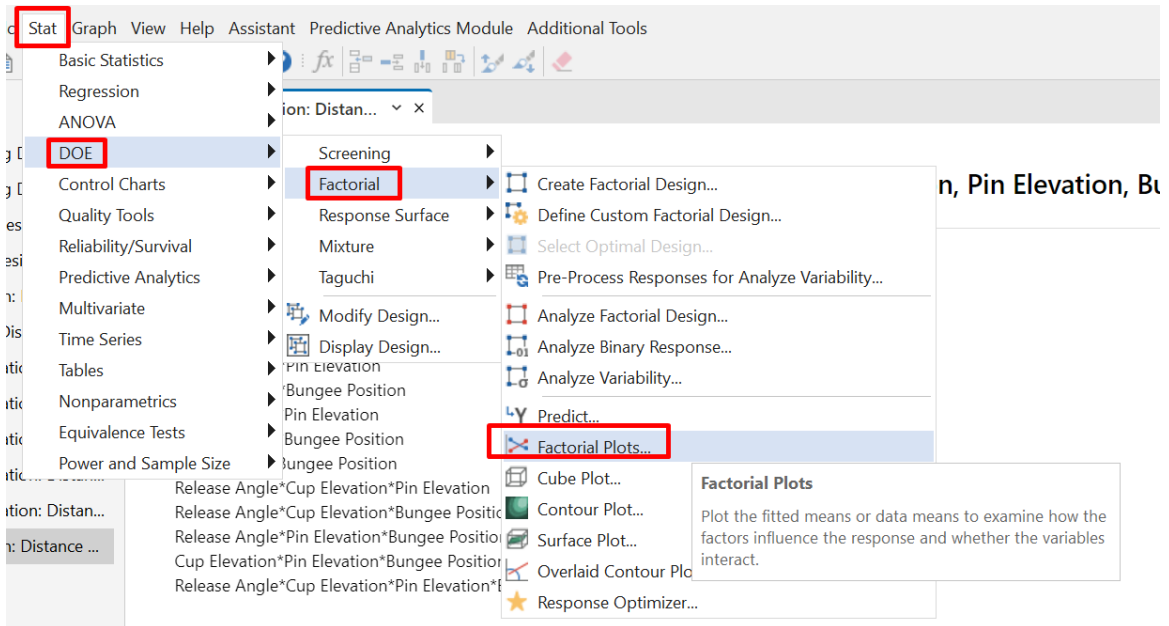
- We see that the Main Effects are: A, B, C and D.
- Anything that surpasses 36.8 is an important effect.
- We see that all other interaction effects fall below 36.8 (thus unimportant)

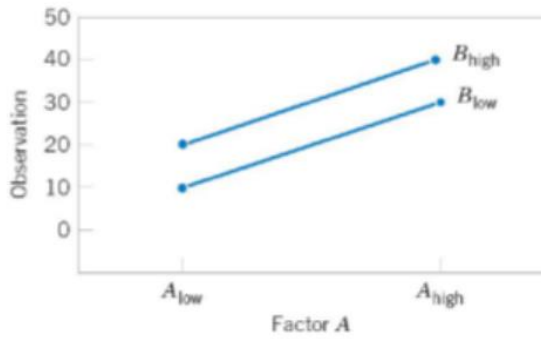
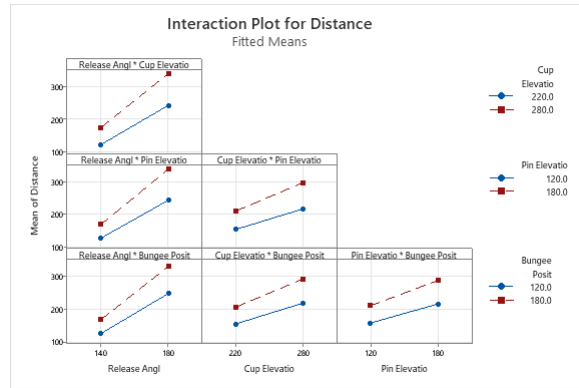
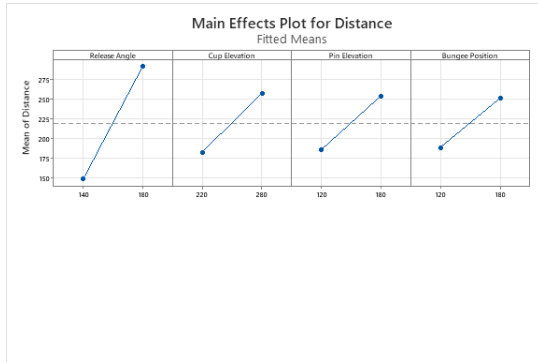
2. NORMAL PLOT



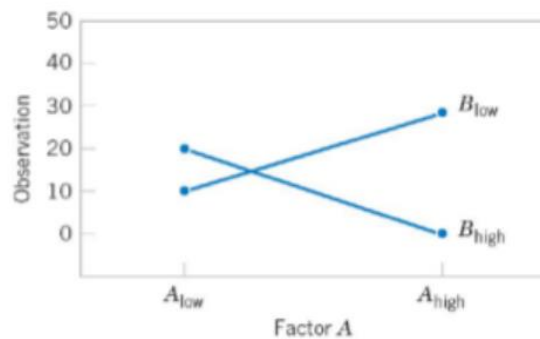
- Normal plot shows us the same thing.

3. FACTORIAL PLOT





No Interaction

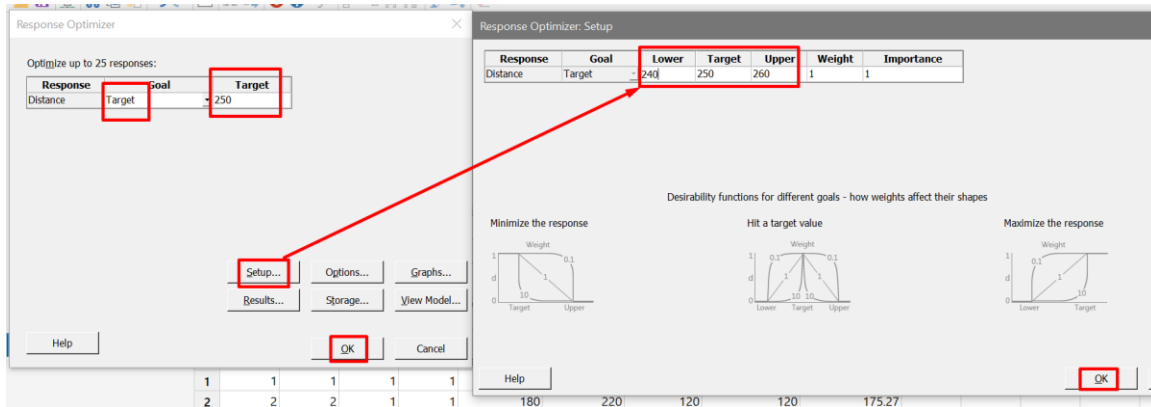
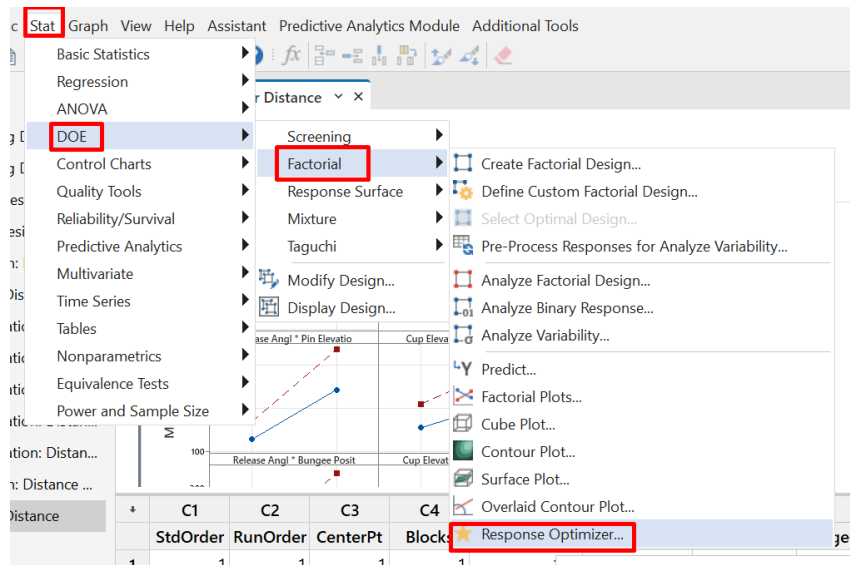


With Interaction

If there is a overlap / cross of the lines, there is interaction between the Factors.

However, we see that all 4 Factors have no relationship with each other.

D. PREDICTING RESULTS



Multiple Response Prediction

| Variable | Setting |
|-----------------|---------|
| Release Angle | 180 |
| Cup Elevation | 222.534 |
| Pin Elevation | 128.955 |
| Bungee Position | 180 |

we need to set these Factor at these settings should we want to hit 250 as Distance

| Response | Fit | SE Fit | 95% CI | 95% PI |
|----------|-------|--------|---------|---------|
| Distance | 250.0 | * | (* , *) | (* , *) |

III. DEFINITIONS OF DOE

A. FACTORS / LEVELS / RESPONSES

ce **FACTORS**

| | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 |
|---|---------------|---------------|---------------|-----------------|----------|----------|-----|-----|-----|
| | Release Angle | Cup Elevation | Pin Elevation | Bungee Position | Distance | | | | |
| 1 | 140 | 220 | 120 | 120 | 90.03 | | | | |
| 1 | 180 | 220 | 120 | 120 | 175.27 | | | | |
| 1 | 140 | 280 | 120 | 120 | 127.44 | | | | |
| 1 | 180 | 280 | 120 | 120 | 243.28 | | | | |
| 1 | 140 | 220 | 180 | 120 | 121.25 | | | | |
| 1 | 180 | 220 | 180 | 120 | 239.62 | RESPONSE | | | |
| 1 | 140 | 280 | 180 | 120 | 169.23 | | | | |
| 1 | 180 | 280 | 180 | 120 | 338.75 | | | | |
| 1 | 140 | 220 | 120 | 180 | 118.35 | | | | |
| 1 | 180 | 220 | 120 | 180 | 232.99 | | | | |
| 1 | 140 | 280 | 120 | 180 | 173.89 | | | | |
| 1 | 180 | 280 | 120 | 180 | 321.57 | | | | |
| 1 | 140 | 220 | 180 | 180 | 159.22 | | | | |
| 1 | 180 | 220 | 180 | 180 | 320.12 | | | | |
| 1 | 140 | 280 | 180 | 180 | 225.81 | | | | |
| 1 | 180 | 280 | 180 | 180 | 450.74 | | | | |

LEVELS

- Earlier, we did this test:
 - Release Angle: 140 / 180 → LOW / HI
 - Firing Angle: 110 (since this is constant, we will ignore it)
 - Cup Elevation: 220 / 280 → LOW / HI
 - Pin Elevation: 120 / 180 → LOW / HI
 - Bungee Position: 120 / 180 → LOW / HI
- 4 Factors 2 Levels, this is known as a 2^4 Factorial Design DOE.
- Total number of runs = $2^4 = 16$ runs

B. RUN / RANDOMIZATION

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
|----|----------|----------|----------|--------|----|----|----|----|
| | StdOrder | RunOrder | CenterPt | Blocks | | | | |
| 1 | 1 | 1 | 1 | 1 | | | | |
| 2 | 2 | 2 | 1 | 1 | | | | |
| 3 | 3 | 3 | 1 | 1 | | | | |
| 4 | 4 | 4 | 1 | 1 | | | | |
| 5 | 5 | 5 | 1 | 1 | | | | |
| 6 | 6 | 6 | 1 | 1 | | | | |
| 7 | 7 | 7 | 1 | 1 | | | | |
| 8 | 8 | 8 | 1 | 1 | | | | |
| 9 | 9 | 9 | 1 | 1 | | | | |
| 10 | 10 | 10 | 1 | 1 | | | | |
| 11 | 11 | 11 | 1 | 1 | | | | |
| 12 | 12 | 12 | 1 | 1 | | | | |
| 13 | 13 | 13 | 1 | 1 | | | | |
| 14 | 14 | 14 | 1 | 1 | | | | |
| 15 | 15 | 15 | 1 | 1 | | | | |
| 16 | 16 | 16 | 1 | 1 | | | | |
| 17 | | | | | | | | |
| 18 | | | | | | | | |
| 19 | | | | | | | | |
| 20 | | | | | | | | |

16 runs

No Randomization

The screenshot shows the Minitab interface with a worksheet titled 'Response Optimization: Distance'. The worksheet data is as follows:

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|----------|----------|----------|--------|---------------|---------------|---------------|-----------------|----|
| | StdOrder | RunOrder | CenterPt | Blocks | Release Angle | Cup Elevation | Pin Elevation | Bungee Position | |
| 1 | 15 | 1 | 1 | 1 | 140 | 280 | 180 | 180 | |
| 2 | 5 | 2 | 1 | 1 | 140 | 220 | 180 | 120 | |
| 3 | 7 | 3 | 1 | 1 | 140 | 280 | 180 | 120 | |
| 4 | 12 | 4 | 1 | 1 | 180 | 280 | 120 | 180 | |
| 5 | 3 | 5 | 1 | 1 | 140 | 280 | 120 | 120 | |
| 6 | 6 | 6 | 1 | 1 | 180 | 220 | 180 | 120 | |
| 7 | 9 | 7 | 1 | 1 | 140 | 220 | 120 | 180 | |
| 8 | 13 | 8 | 1 | 1 | 140 | 220 | 180 | 180 | |
| 9 | 8 | 9 | 1 | 1 | 180 | 280 | 180 | 120 | |
| 10 | 11 | 10 | 1 | 1 | 140 | 280 | 120 | 180 | |
| 11 | 16 | 11 | 1 | 1 | 180 | 280 | 180 | 180 | |
| 12 | 14 | 12 | 1 | 1 | 180 | 220 | 180 | 180 | |
| 13 | 10 | 13 | 1 | 1 | 180 | 220 | 120 | 180 | |
| 14 | 2 | 14 | 1 | 1 | 180 | 220 | 120 | 120 | |
| 15 | 1 | 15 | 1 | 1 | 140 | 220 | 120 | 120 | |
| 16 | 4 | 16 | 1 | 1 | 180 | 280 | 120 | 120 | |

Annotations on the screenshot:

- A red box highlights the 'StdOrder' column, with the text "the runs are now randomized" below it.
- A red box highlights the 'RunOrder' column, with the text "same order but random" below it.
- The 'Create Factorial Design' dialog box is open, showing '2-level factorial (default generators)' selected.
- The 'Create Factorial Designs: Options' dialog box is open, with 'Randomize runs' checked. A red box highlights the 'Base for random data generator' field, which contains the value '9'. Red text next to it says: "you only key this in if you want your random runs order to be fixed everytime e.g. if you put 9.. and the random runs is 15...5...7...12... next time you put 9 again it will come out as 15...5...7...12".

- Randomization is important to protect against uncontrolled and/or unknown influences of variables that are not part of the experiment.
- In order to minimize this risk of unknown influence, experimenters randomly assign the order of testing to improve the chances of averaging out this bias or distortion of the responses related to the factor(s) under study.
- If you are unable to randomize due to physical or cost constraints, we will need to do Blocking.

C. BLOCKING

- If you cannot randomize due to lack of resources, example, not enough raw materials to do so many runs, one may block.
- 1 Block = 1 Batch.
- Possibly, 1 Block = 1 Day. Which means, running an entire experiment in 1 day = 1 block.
- Running the experiment again the next day will be 2 blocks → Block 2.
- Blocking minimizes the risk of the nuisance-factor batches creating excessive estimates of the inherent variation.
- For example, you want to test the quality of a new printing press.
- However, press arrangement takes several hours and can only be done four times a day.
- Because the design of the experiment requires at least eight runs, you need at least two days to test the press.
- To distinguish between any block effect (incidental differences between days) and effects because of the experimental factors (temperature, humidity, and press operator), you must include the block (day) in the designed experiment. You should randomize run order within blocks.

View Help Assistant Predictive Analytics Module Additional Tools

Response Optimization: Dis... x

WORKSHEET 7

Response Optimization: Distance

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|----------|----------|----------|--------|---------------|---------------|---------------|-----------------|----|
| | StdOrder | RunOrder | CenterPt | Blocks | Release Angle | Cup Elevation | Pin Elevation | Bungee Position | |
| 1 | 15 | 1 | 1 | 1 | 140 | 280 | 180 | 180 | |
| 2 | 5 | 2 | 1 | 1 | 140 | 220 | 180 | 120 | |
| 3 | 7 | 3 | 1 | 1 | 140 | 280 | 180 | 120 | |
| 4 | 12 | 4 | 1 | 1 | 180 | 280 | 120 | 180 | |
| 5 | 3 | 5 | 1 | 1 | 140 | 280 | 120 | 120 | |
| 6 | 6 | 6 | 1 | 1 | 180 | 220 | 180 | 120 | |
| 7 | 9 | 7 | 1 | 1 | 140 | 220 | 120 | 180 | |
| 8 | 13 | 8 | 1 | 1 | 140 | 220 | 180 | 180 | |
| 9 | 8 | 9 | 1 | 1 | 180 | 280 | 180 | 120 | |
| 10 | 11 | 10 | 1 | 1 | 140 | 280 | 120 | 180 | |
| 11 | 16 | 11 | 1 | 1 | 180 | 280 | 180 | 180 | |
| 12 | 14 | 12 | 1 | 1 | 180 | 220 | 180 | 180 | |
| 13 | 10 | 13 | 1 | 1 | 180 | 220 | 120 | 180 | |
| 14 | 2 | 14 | 1 | 1 | 180 | 220 | 120 | 120 | |
| 15 | 1 | 15 | 1 | 1 | 140 | 220 | 120 | 120 | |
| 16 | 4 | 16 | 1 | 1 | 180 | 280 | 120 | 120 | |
| 17 | | | | | | | | | |
| 18 | | | | | | | | | |
| 19 | | | | | | | | | |
| 20 | | | | | | | | | |

Create Factorial Design

Type of Design

- 2-level factorial (default generators) (2 to 15 factors)
- 2-level factorial (specify generators) (2 to 15 factors)
- 2-level split-plot (hard-to-change factors) (2 to 7 factors)
- Plackett-Burman design (2 to 47 factors)
- General full factorial design (2 to 15 factors)

Number of factors: 4

Display Available Designs...
 Designs... Factors...
 Options... Results...

Help OK Cancel

Create Factorial Design: Designs

| Designs | Runs | Resolution | 2^(k-p) |
|----------------|------|------------|---------|
| 1/2 fraction | 8 | IV | 2^(4-1) |
| Full factorial | 16 | Full | 2^4 |

Number of center points per block: 0

Number of replicates for corner points: 1

Number of blocks: 1

Help OK Cancel

For this manuscript, we only assume 1 Block throughout (take it as just 1 Day, or 1 Batch)
 Meaning, we run the entire experiment in 1 day, not more.

D. REPLICATE VS REPEAT

we will discuss center points later

one corner point = 1 run
we will show what does Corner Point mean later

| Designs | Runs | Resolution | $2^{(k-p)}$ |
|----------------|------|------------|-------------|
| 1/2 fraction | 8 | IV | $2^{(4-1)}$ |
| Full factorial | 16 | Full | 2^4 |

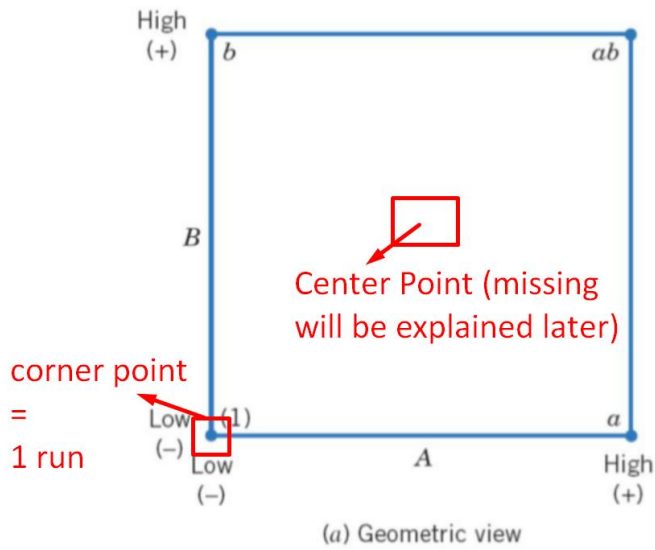
| | | | |
|-----|-----|-----|-----|
| 180 | 220 | 180 | 180 |
| 180 | 220 | 120 | 180 |
| 180 | 220 | 120 | 120 |
| 140 | 220 | 120 | 120 |
| 180 | 280 | 120 | 120 |

- Replication = Performing more than one trial of each run
- (Completely NEW SETUP each trial).
- A Replicate is an Independent and Random application of the run, including the setup.
- Repeat = a Repetition of a run WITHOUT going through a NEW SETUP.

E. WHAT IF MANY OF THE EFFECTS ARE SIGNIFICANT?

- Check → Was the setup for each trial really randomly replicated?
- If not, the responses are repeats, not replicates.
- Replication assumes that each trial is an independent and random performance of the process, specifically including any process setup.
- If not, the experimenter has repeats that may be neither independent nor random.
- The estimate of s (standard deviation), using repeats will be much smaller than the actual inherent variation of the process.
- This will make most effects appear significant, when in reality they have not been randomly replicated to properly estimate the inherent variation.

F. CORNER POINT / CENTER POINT

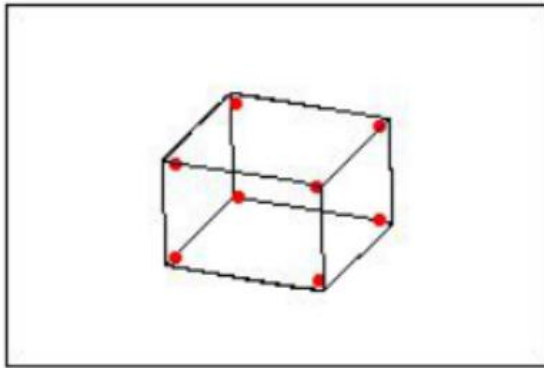


| Run | Factor | | Label |
|-----|--------|---|-------|
| | A | B | |
| 1 | - | - | (1) |
| 2 | + | - | a |
| 3 | - | + | b |
| 4 | + | + | ab |

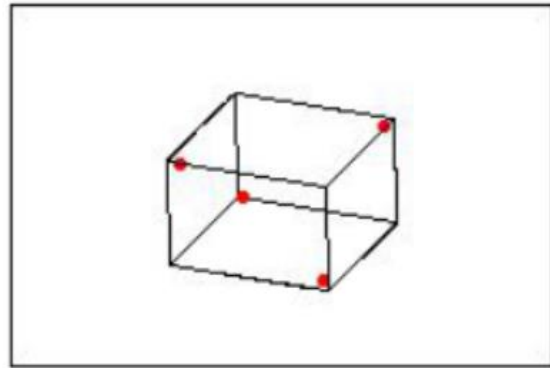
(b) Design or test matrix for the 2² factorial design

- There are 4 Corner Points:
 - (-1, -1) / (-1, 1) / (1, -1) / (1, 1) → Each is 1 Run
- The Centre Point is (0,0).
- Currently, we don't use it.
- We will explore Centre Point in the next section on Response Surface Design (RSM): Central Composite Design (CCD).

G. FULL VS FRACTIONAL FACTORIALS



Full factorial design



1/2 Fraction factorial design

Create Factorial Design

Type of Design

- 2-level factorial (default generators) (2 to 15 factors)
- 2-level factorial (specify generators) (2 to 15 factors)
- 2-level split-plot (hard-to-change factors) (2 to 7 factors)
- Plackett-Burman design (2 to 47 factors)
- General full factorial design (2 to 15 factors)

Number of factors: 4

Display Available Designs...

Designs... Factors... Options... Results... OK Cancel Help

Create Factorial Design: Display Available Designs

Available Factorial Designs (with Resolution)

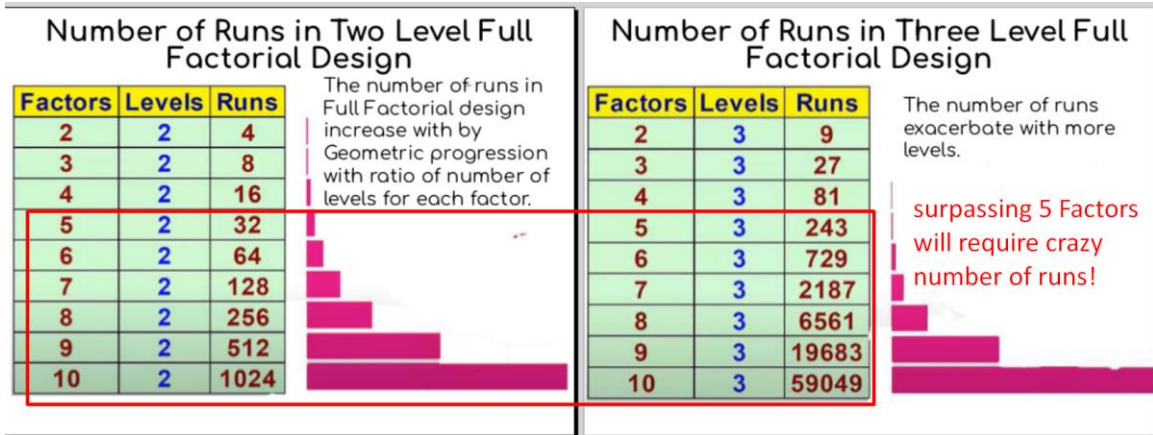
| Run | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|-----|------|-----|------|------|------|------|-----|-----|-----|-----|-----|-----|-----|-----|
| 4 | Full | III | | | | | | | | | | | | |
| 8 | Full | IV | III | III | III | | | | | | | | | |
| 16 | Full | V | IV | IV | IV | IV | III | III | III | III | III | III | III | III |
| 32 | | | Full | VI | IV | IV | IV | IV | IV | IV | IV | IV | IV | IV |
| 64 | | | | Full | VII | V | IV | IV | IV | IV | IV | IV | IV | IV |
| 128 | | | | | Full | VIII | VI | V | V | IV | IV | IV | IV | IV |

Available Resolution III Plackett-Burman Designs

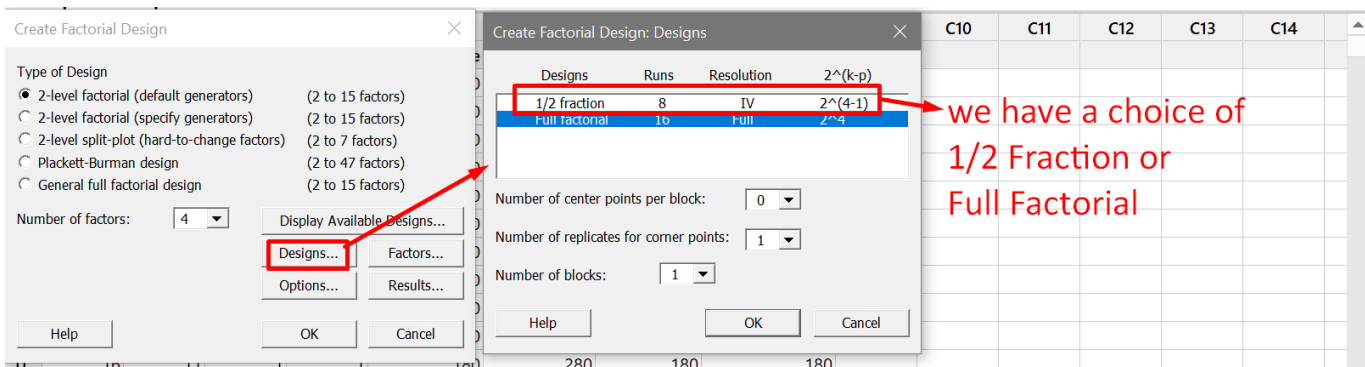
| Factors | Runs | Factors | Runs | Factors | Runs |
|---------|--------------------|---------|--------------------|---------|----------|
| 2-7 | 12,20,24,28,...,48 | 20-23 | 24,28,32,36,...,48 | 36-39 | 40,44,48 |
| 8-11 | 12,20,24,28,...,48 | 24-27 | 28,32,36,40,44,48 | 40-43 | 44,48 |
| 12-15 | 20,24,28,36,...,48 | 28-31 | 32,36,40,44,48 | 44-47 | 48 |
| 16-19 | 20,24,28,32,...,48 | 32-35 | 36,40,44,48 | | |

OK Help

- Full Factorial means All Runs
- For example, $2^3 \rightarrow 2$ Levels 3 Factors = Total 8 Runs
- But if you lack resources, are you able to do it in just 4 Runs?
- Yes, but its labelled as Resolution III Design (see picture above, in Red \rightarrow 4 Run, 3 Factors).



- You can see that the number of Runs increase dramatically as number of Factors increase!
- Thus we need Fractional Factorial to cut down the number of Runs (we can do Screening to Screen out the unimportant Runs as shown later).



- Similarly, in the previous experiment (Create Factorial Designs) we did a 2^4 (2 Levels 4 Factors) which we have the option of running the Full Experiment (=16 runs... in which we did...), or we could just do 8 runs.

Create Factorial Design: Display Available Designs

Available Factorial Designs (with Resolution)

| Run | Factors | | | | | | | | | | | | | |
|-----|---------|------|------|------|------|------|------|-----|-----|-----|-----|-----|-----|-----|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 4 | Full | III | | | | | | | | | | | | |
| 8 | | Full | IV | III | III | III | | | | | | | | |
| 16 | | | Full | V | IV | IV | IV | III | III | III | III | III | III | III |
| 32 | | | | Full | VI | IV | IV | IV | IV | IV | IV | IV | IV | IV |
| 64 | | | | | Full | VII | V | IV | IV | IV | IV | IV | IV | IV |
| 128 | | | | | | Full | VIII | VI | V | V | IV | IV | IV | IV |

Available Resolution III Plackett-Burman Designs

| Factors | Runs | Factors | Runs | Factors | Runs |
|---------|--------------------|---------|--------------------|---------|----------|
| 2-7 | 12,20,24,28,...,48 | 20-23 | 24,28,32,36,...,48 | 36-39 | 40,44,48 |
| 8-11 | 12,20,24,28,...,48 | 24-27 | 28,32,36,40,44,48 | 40-43 | 44,48 |
| 12-15 | 20,24,28,36,...,48 | 28-31 | 32,36,40,44,48 | 44-47 | 48 |
| 16-19 | 20,24,28,32,...,48 | 32-35 | 36,40,44,48 | | |

Help OK

- If we had chosen 8 Runs, this would be a resolution IV (see picture above, in Yellow)

H. RESOLUTION AND CONFOUNDING

- So long as you do not Run Full Factorials (which means, Run all possible combinations i.e. perform ALL experiments), you will experience Confounding.

$2^3 = \text{total } 8 \text{ runs}$

| A | B | C | AB | BC | AC | ABC |
|----|----|----|----|----|----|-----|
| -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 | 1 | -1 | 1 |
| -1 | 1 | -1 | -1 | -1 | 1 | 1 |
| 1 | 1 | -1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 | 1 | -1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Full Factorial

Fractional Factorial

Only 4 Runs chosen

- Presume we have the above: 8 runs cut down to 4 runs....

| A | B | C | AB | BC | AC | ABC |
|----|----|----|----|----|----|-----|
| -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 | 1 | -1 | 1 |
| -1 | 1 | -1 | -1 | -1 | 1 | 1 |
| 1 | 1 | -1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 | 1 | -1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

- Notice that C and AB have the same effects! They are Confounded!

| A | B | C | AB | BC | AC | ABC |
|----|----|----|----|----|----|-----|
| -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 | 1 | -1 | 1 |
| -1 | 1 | -1 | -1 | -1 | 1 | 1 |
| 1 | 1 | -1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 | 1 | -1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

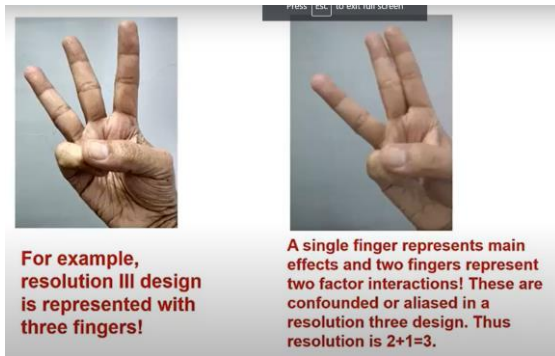
- Notice that B and AC have the same effects! They are Confounded!

| A | B | C | AB | BC | AC | ABC |
|----|----|----|----|----|----|-----|
| -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 | 1 | -1 | 1 |
| -1 | 1 | -1 | -1 | -1 | 1 | 1 |
| 1 | 1 | -1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 | 1 | -1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

- Notice that A and BC have the same effects! They are Confounded!

- Since they have the same effects, Confounding means that you cannot tell whether the Response is due to
 - A or BC?
 - B or AC?
 - C or AB?

1. RESOLUTION III



A + BC

B + AC

C + AB

Resolution III

2. RESOLUTION IV



A + BCD

B + ACD

C + ABD

D + ABC

AB + CD

AC + BD

AD + BC

Resolution IV

3. RESOLUTION V

Resolution V design is represented with five fingers!

One finger represents main effects and four fingers represents four-factor interactions and one finger represents main effects! Thus main effects are aliased with four-factor interactions! Resolution of the design is therefore $4+1=5$.

Three fingers represents three-factor interactions and two fingers represents two-factor interactions! Thus some two-factor interactions are aliased with some three factor interactions!

| | |
|----------|---------------------|
| A + BCDE | AE + BCD |
| B + ACDE | BC + ADE |
| C + ABDE | BD + ACE |
| D + ABCE | BE + ACD |
| E + ABCD | CD + ABE |
| AB + CDE | CE + ABD |
| AC + BDE | DE + ABC |
| AD + BCE | Resolution V |

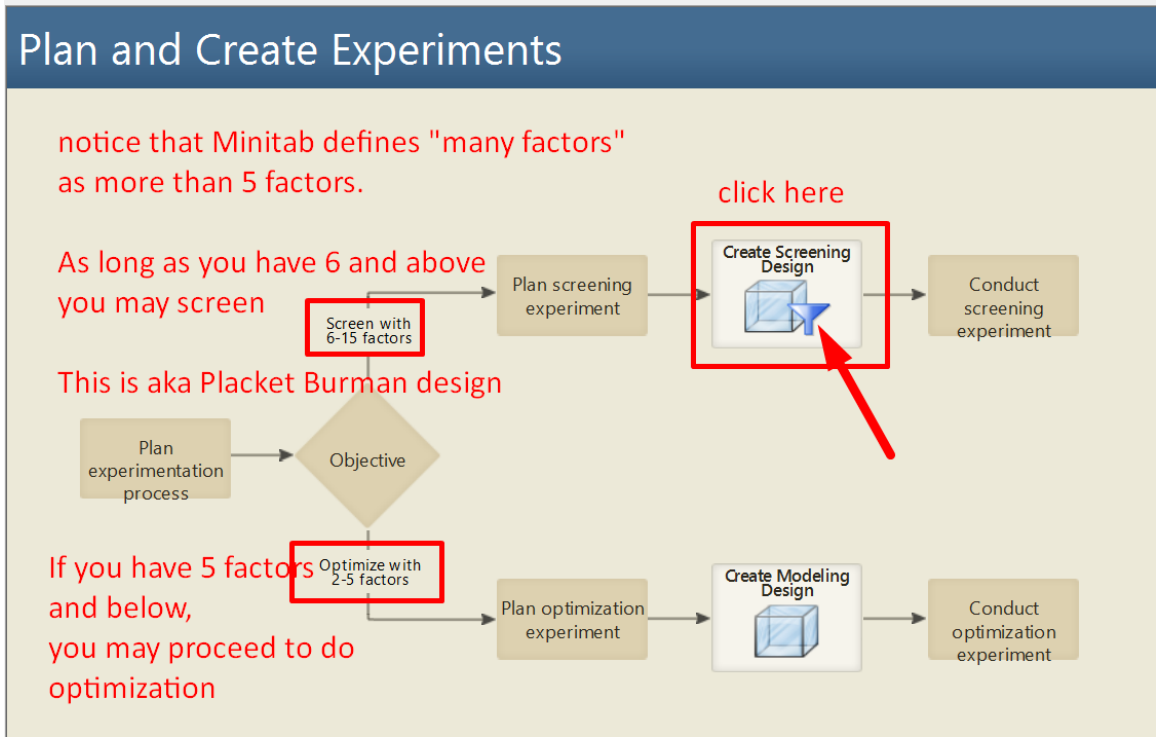
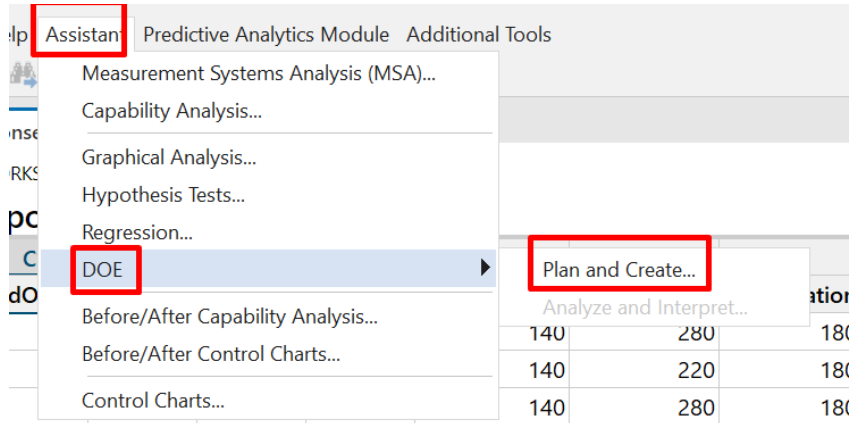
- Confounding should be Avoided because we cannot differentiate which Factor is affecting the Response.
- However, due to limitations of Resources, we need to Screen out the unimportant Runs.

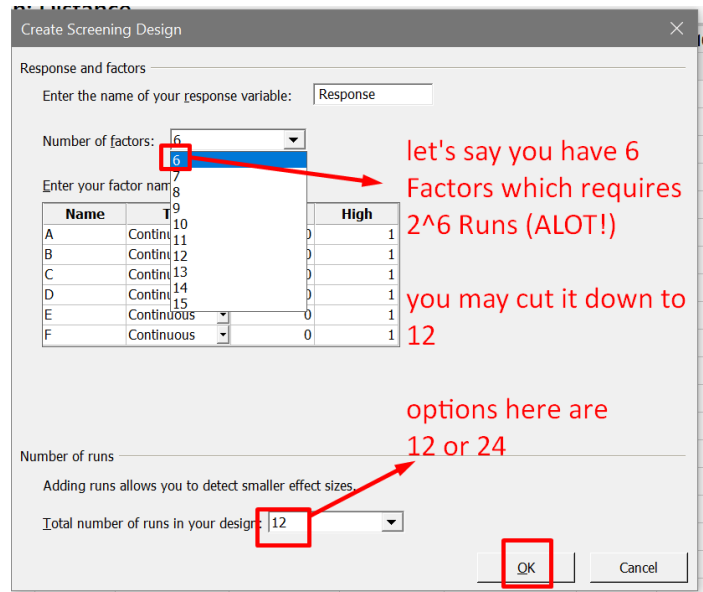
I. SCREENING (PLACKETT BURMAN DESIGN)

- We already know (from Section: Full vs Fractional Factorials) that that the number of Runs increase dramatically as number of Factors increase (>5 Factors)!
- Thus we need Fractional Factorial to cut down the number of Runs
- Screening helps to Screen out unimportant Runs and cut it down to a Fractional Factorial.
- In this manuscript, we consider 2 ways of Screening:
 - Using Assistant
 - Screening while Creating the Factorial Design
- Note that in this Manuscript we only consider Plackett Burman Screening Design.
- And if we use Assistant option in Minitab, we are actually doing Plackett Burman Screening.
- Plackett Burman designs are only Resolution III experiments AND 6 or more factors.
- It only identifies Main effects and ignore Interaction effects
- In other words, Minitab only performs Screening when we have >6 Factors.
- Else, Minitab will auto-generate the Fractional Factorial Options for us to choose from (while we create the Factorial Design).

1. USING ASSISTANT

Note that Assistant Screening = Plackett Burman Screening





Create Screening Design

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 | C16 |
|----|----------|----------|--------|--------|----|----|----|----|----|-----|----------|-----|-----|-----|-----|-----|
| | StdOrder | RunOrder | PtType | Blocks | A | B | C | D | E | F | Response | | | | | |
| 1 | 4 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | | | | | | |
| 2 | 7 | 2 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | | | | | | |
| 3 | 10 | 3 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | | | | | | |
| 4 | 3 | 4 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | | | | | | |
| 5 | 12 | 5 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | |
| 6 | 2 | 6 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | | | | | | |
| 7 | 9 | 7 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | | | | | | |
| 8 | 8 | 8 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | | | | | | |
| 9 | 1 | 9 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | | | | | | |
| 10 | 5 | 10 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | | | | | | |
| 11 | 6 | 11 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | | | | | | |
| 12 | 11 | 12 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | | | | | | |
| 13 | | | | | | | | | | | | | | | | |

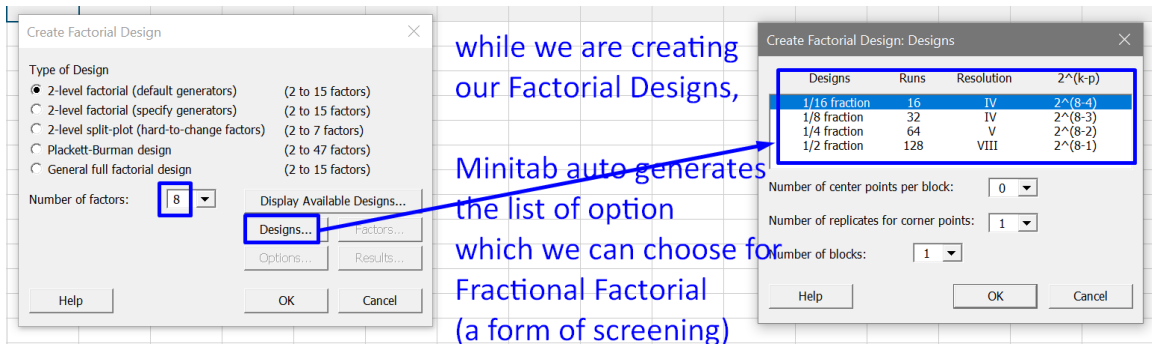
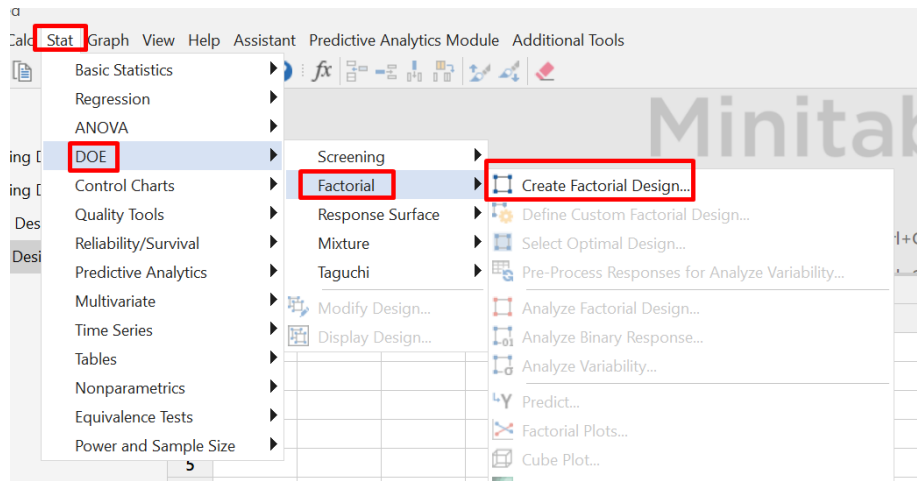
Minitab automatically generates a Recipe for you to Run

6 Factors with 2 Levels using only 12 Runs

Just follow the Recipe and input to get the Response

- C3 (CenterPt or PtType) stores the point type.
- If you create a 2-level design, Minitab names this column CenterPt.
- If you create a Plackett-Burman or general full factorial design, Minitab names this column PtType.
- The codes are: 0 is a Center Point run and 1 is a Corner Point. (we will explain Centre Point and Corner Point later in RSM – CCD, next section).

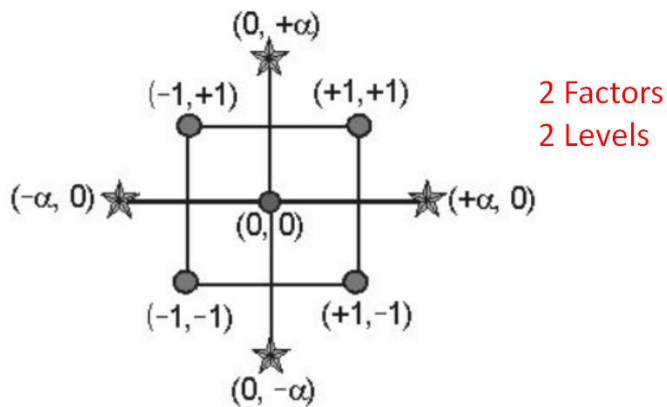
2. SCREENING WHILE CREATING FACTORIAL DESIGN



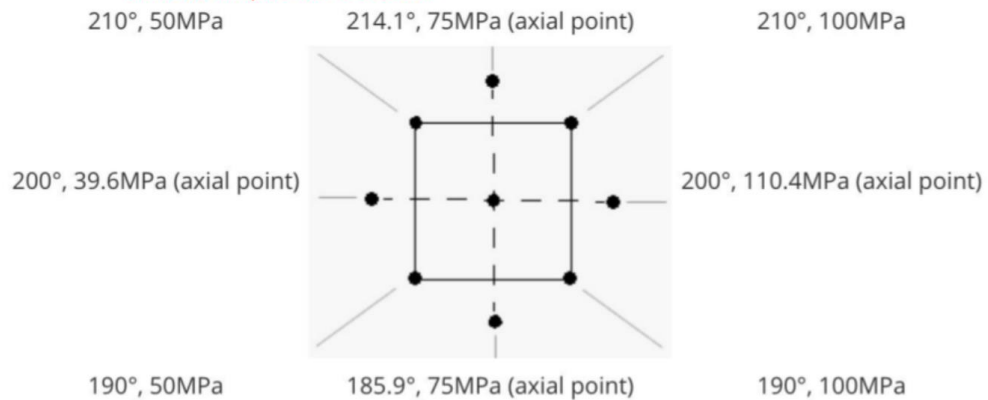
IV. RESPONSE SURFACE METHODS (>2 LEVELS)

- There are two types of Response Surface Methods (RSM)
 - Central Composite Design (CCD)
 - Box-Behnken

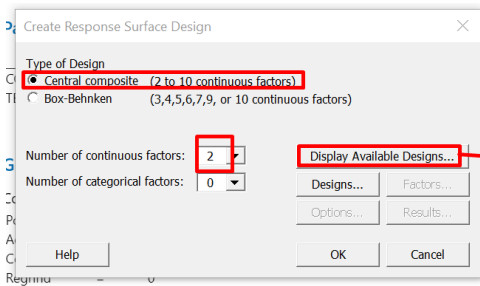
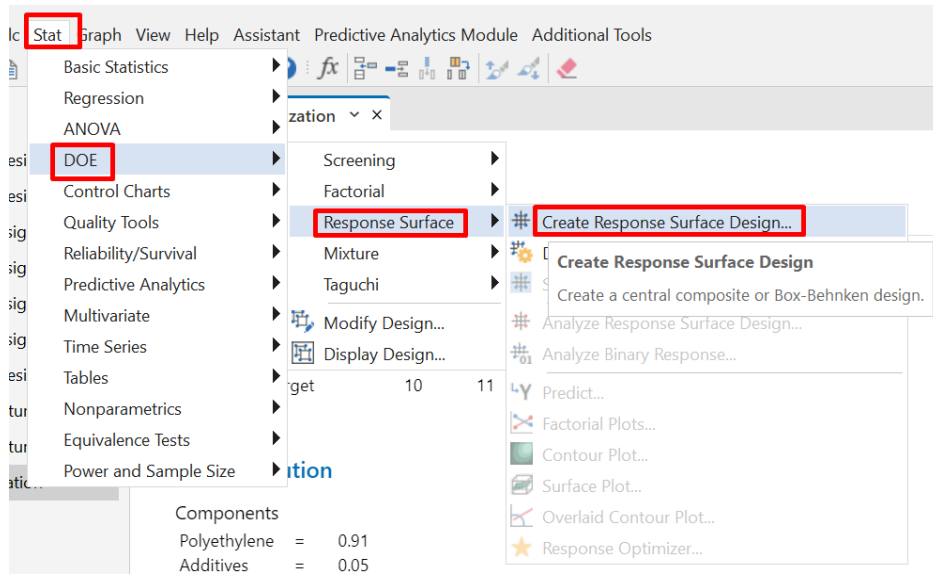
A. CENTRAL COMPOSITE DESIGN (CCD)



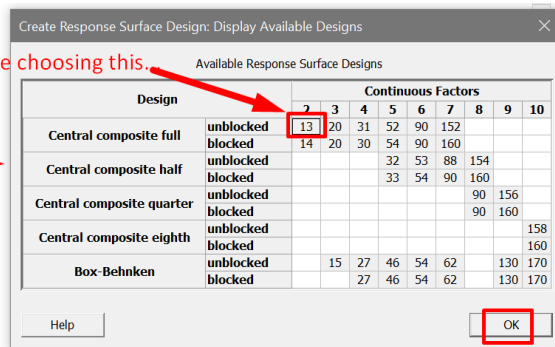
an example of CCD....



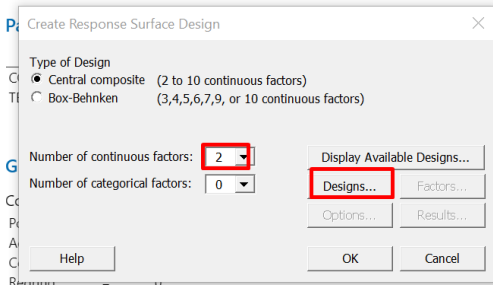
1. CREATING THE EXPERIMENT



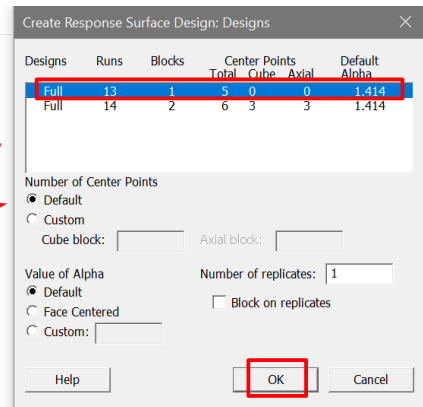
we shall be choosing this.

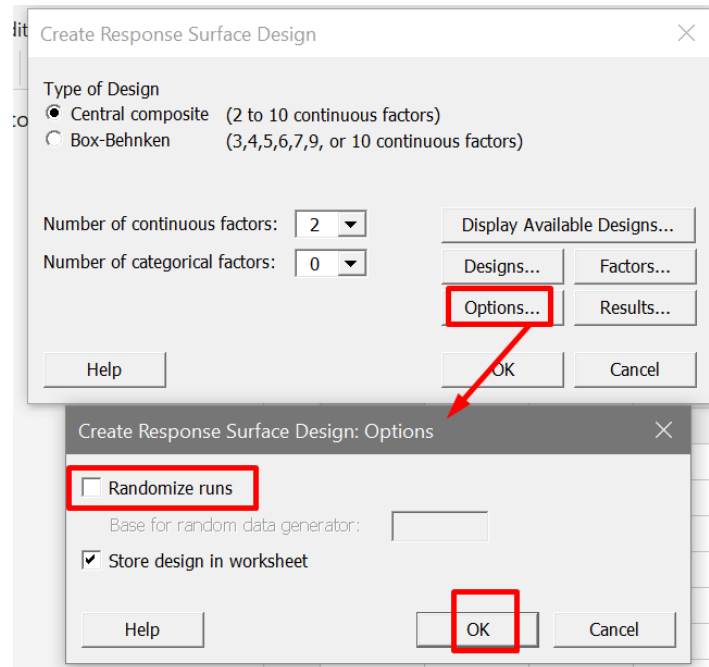
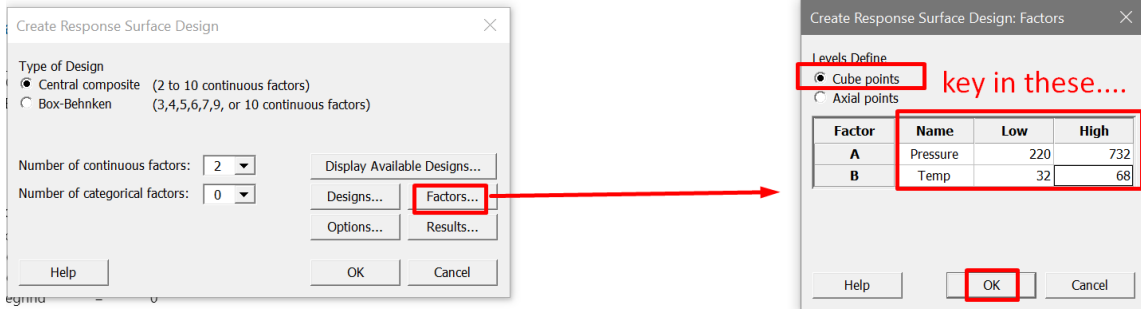


Predicted Responses



we shall choose this...





As you can see from the above, we have many CCDs....

- Full / Half / Quarter / Eighth....
- We will not go through them
- More explanation here: <https://support.minitab.com/en-us/minitab/21/help-and-how-to/statistical-modeling/doe/supporting-topics/response-surface-designs/summary-of-central-composite-designs/>

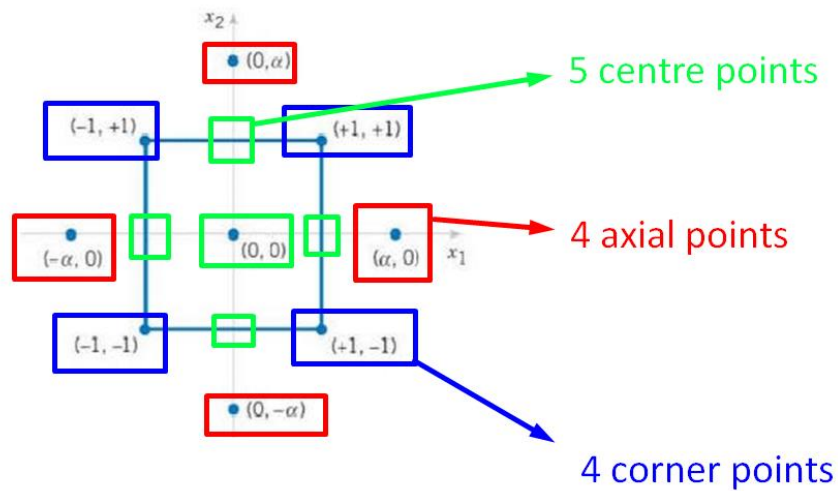
2. EXPLAINING THE PT TYPE

| | + | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|-----------------|----|----------|----------|--------|--------|----------|---------|----------------|----|----|
| | | StdOrder | RunOrder | PtType | Blocks | Pressire | Temp | Retention Time | | |
| 4 corner points | 1 | 1 | 1 | 1 | 1 | 220.000 | 32.0000 | 72 | | |
| | 2 | 2 | 2 | 1 | 1 | 732.000 | 32.0000 | 79 | | |
| | 3 | 3 | 3 | 1 | 1 | 220.000 | 68.0000 | 60 | | |
| | 4 | 4 | 4 | 1 | 1 | 732.000 | 68.0000 | 68 | | |
| 4 axial points | 5 | 5 | 5 | -1 | 1 | 113.961 | 50.0000 | 55 | | |
| | 6 | 6 | 6 | -1 | 1 | 838.039 | 50.0000 | 85 | | |
| | 7 | 7 | 7 | -1 | 1 | 476.000 | 24.5442 | 104 | | |
| | 8 | 8 | 8 | -1 | 1 | 476.000 | 75.4558 | 135 | | |
| 5 center points | 9 | 9 | 9 | 0 | 1 | 476.000 | 50.0000 | 118 | | |
| | 10 | 10 | 10 | 0 | 1 | 476.000 | 50.0000 | 119 | | |
| | 11 | 11 | 11 | 0 | 1 | 476.000 | 50.0000 | 123 | | |
| | 12 | 12 | 12 | 0 | 1 | 476.000 | 50.0000 | 123 | | |
| | 13 | 13 | 13 | 0 | 1 | 476.000 | 50.0000 | 127 | | |
| | 14 | | | | | | | | | |
| | 15 | | | | | | | | | |

13 runs produced

key the responses in

How did we manage to achieve 13 runs?



- 5 Centre Points + 4 Axial Points + 4 Corner Points = 13 Runs in total

a) Blocking within CCD

- Previously, we saw that 5 Centre Points + 4 Axial Points + 4 Corner Points = 13 Runs in total
- But they were all run in 1 Block. (unblocked)

we did this previously
13 runs

| Design | | Continuous Factors | | | | | | | | | |
|---------------------------|-----------|--------------------|----|----|----|----|-----|-----|-----|-----|--|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| Central composite full | unblocked | 13 | 20 | 31 | 52 | 90 | 152 | | | | |
| | blocked | 14 | 20 | 30 | 54 | 90 | 160 | | | | |
| Central composite half | unblocked | | | | 32 | 53 | 88 | 154 | | | |
| | blocked | | | | 33 | 54 | 90 | 160 | | | |
| Central composite quarter | unblocked | | | | | | | 90 | 156 | | |
| | blocked | | | | | | | 90 | 160 | | |
| Central composite eighth | unblocked | | | | | | | | | 158 | |
| | blocked | | | | | | | | | 160 | |
| Box-Behnken | unblocked | | 15 | 27 | 46 | 54 | 62 | | 130 | 170 | |
| | blocked | | | 27 | 46 | 54 | 62 | | 130 | 170 | |

- Presume now there was Blocking (we wanted to Run the experiment in 2 days or 2 blocks).

| Design | | Continuous Factors | | | | | | | | | |
|---------------------------|-----------|--------------------|----|----|----|----|-----|-----|-----|-----|--|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| Central composite full | unblocked | 13 | 20 | 31 | 52 | 90 | 152 | | | | |
| | blocked | 14 | 20 | 30 | 54 | 90 | 160 | | | | |
| Central composite half | unblocked | | | | 32 | 53 | 88 | 154 | | | |
| | blocked | | | | 33 | 54 | 90 | 160 | | | |
| Central composite quarter | unblocked | | | | | | | 90 | 156 | | |
| | blocked | | | | | | | 90 | 160 | | |
| Central composite eighth | unblocked | | | | | | | | | 158 | |
| | blocked | | | | | | | | | 160 | |
| Box-Behnken | unblocked | | 15 | 27 | 46 | 54 | 62 | | 130 | 170 | |
| | blocked | | | 27 | 46 | 54 | 62 | | 130 | 170 | |

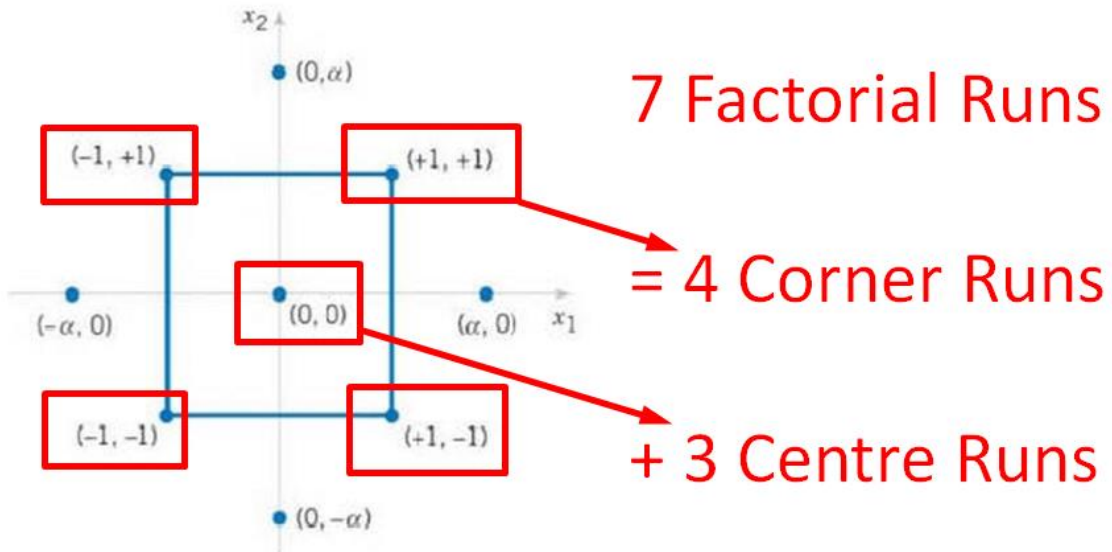
why 14 runs?

- Why do we need 14 runs?

| Pattern | Block | X_1 | X_2 | Comment |
|---------|-------|-----------|-----------|-----------------------|
| -- | 1 | -1 | -1 | Full Factorial |
| -+ | 1 | -1 | +1 | Full Factorial |
| + - | 1 | +1 | -1 | Full Factorial |
| ++ | 1 | +1 | +1 | Full Factorial |
| 00 | 1 | 0 | 0 | Center-Full Factorial |
| 00 | 1 | 0 | 0 | Center-Full Factorial |
| 00 | 1 | 0 | 0 | Center-Full Factorial |
| -0 | 2 | -1.414214 | 0 | Axial |
| +0 | 2 | +1.414214 | 0 | Axial |
| 0- | 2 | 0 | -1.414214 | Axial |
| 0+ | 2 | 0 | +1.414214 | Axial |
| 00 | 2 | 0 | 0 | Center-Axial |
| 00 | 2 | 0 | 0 | Center-Axial |
| 00 | 2 | 0 | 0 | Center-Axial |

(this picture shows 14 runs = 7 Corner (Factorial) Runs and 7 Axial Runs)

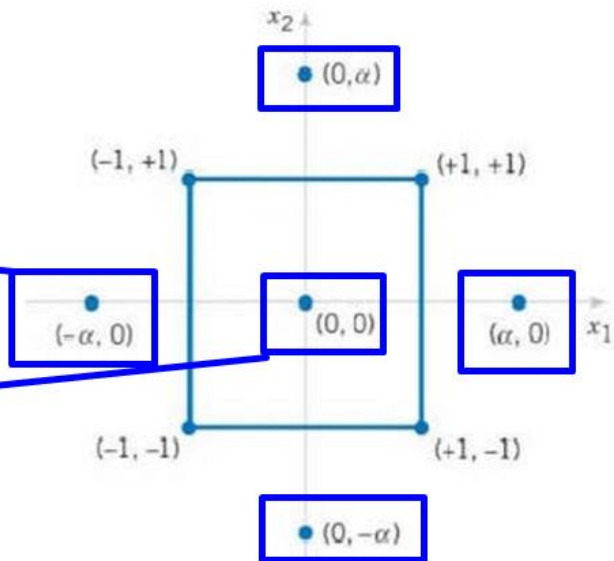
- 14 Runs is because, when two blocks are required there should be a
 - Factorial Block (Block 1 = 7 Factorial Runs) and
 - An Axial Block (Block 2 = 7 Axial Runs).



Block 2:
 7 Axial Runs

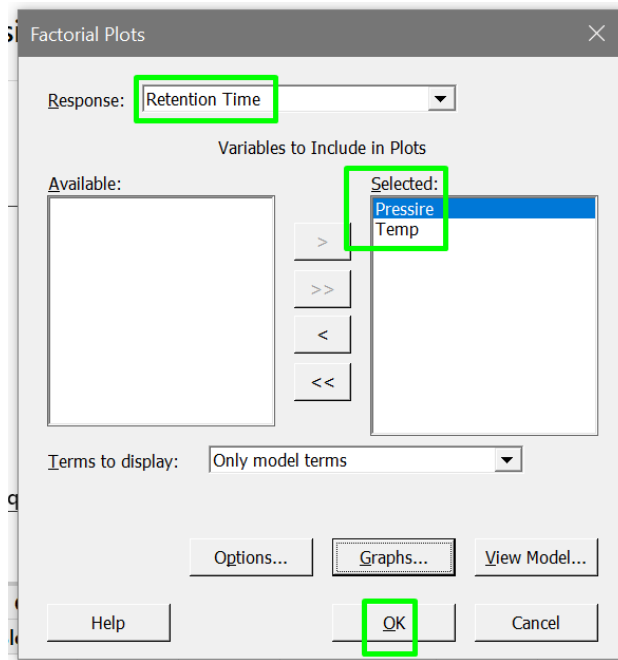
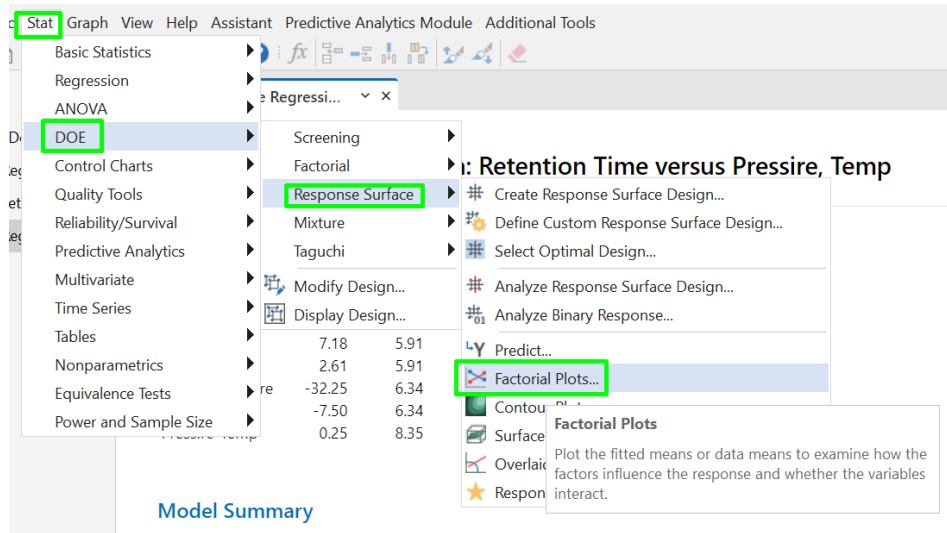
= 4 Axial Runs

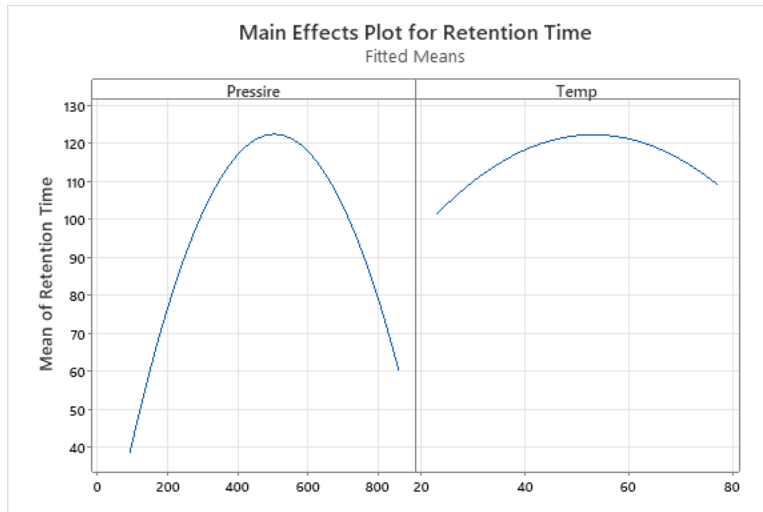
+ 3 Centre Runs



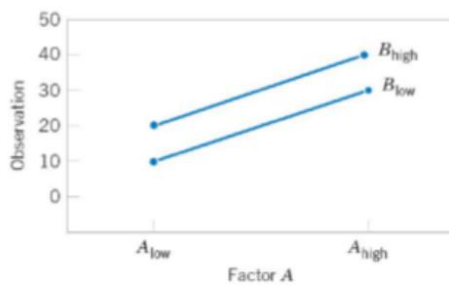
3. ANALYZING THE CCD EXPERIMENT

a) Factorial Plot

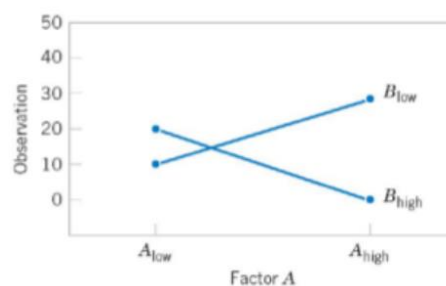




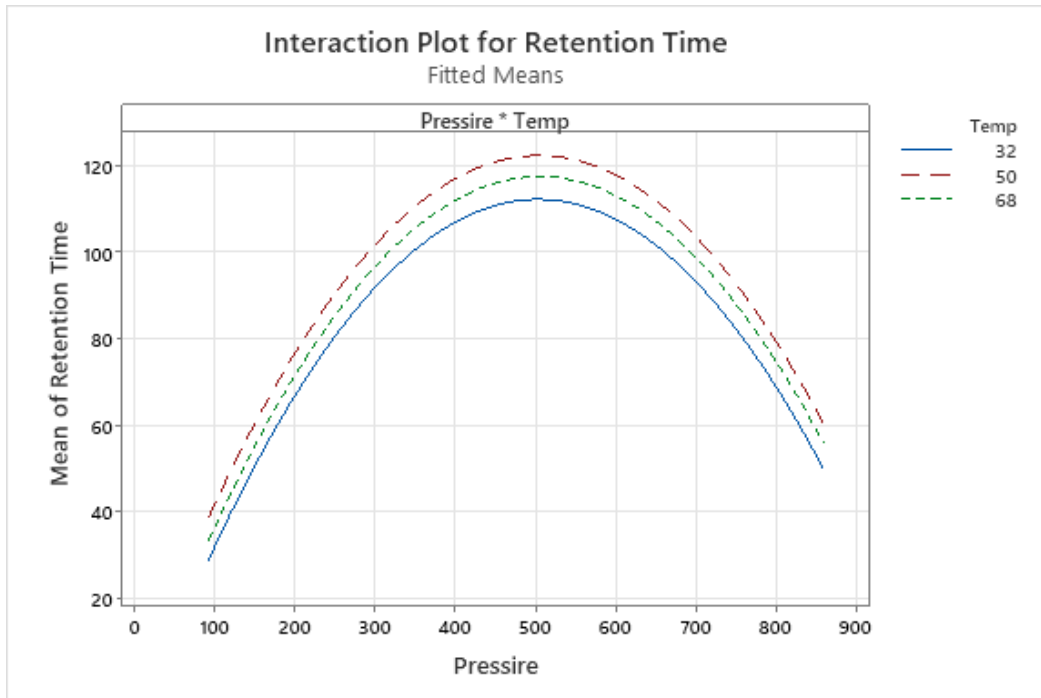
- Notice the difference between the chart above vs the one in Section: Factorial Plot (ctrl click the link).
- The Factorial Plot of a CCD is curved because Minitab generated many points to test per Factor. E.g. Pressure = 220 / 732 / 114 / 838 / 476...
- If you compare this to a Factorial Plot of a Simple 2 points experiment (shown below).... You see that its impossible to generate a curve with just 2 points....(high and low)
- Thus, CCD (RSM) is better than a Simple Factorial Experiment because it can detect curvature (Non-Linear)



No Interaction

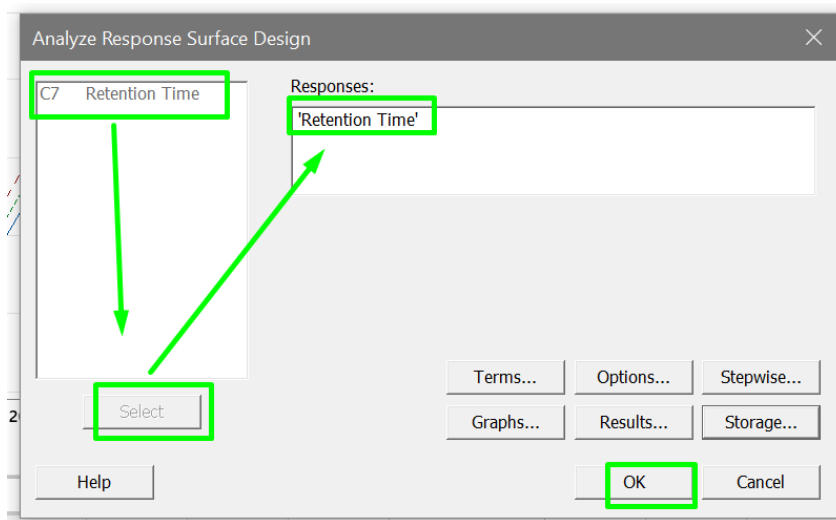
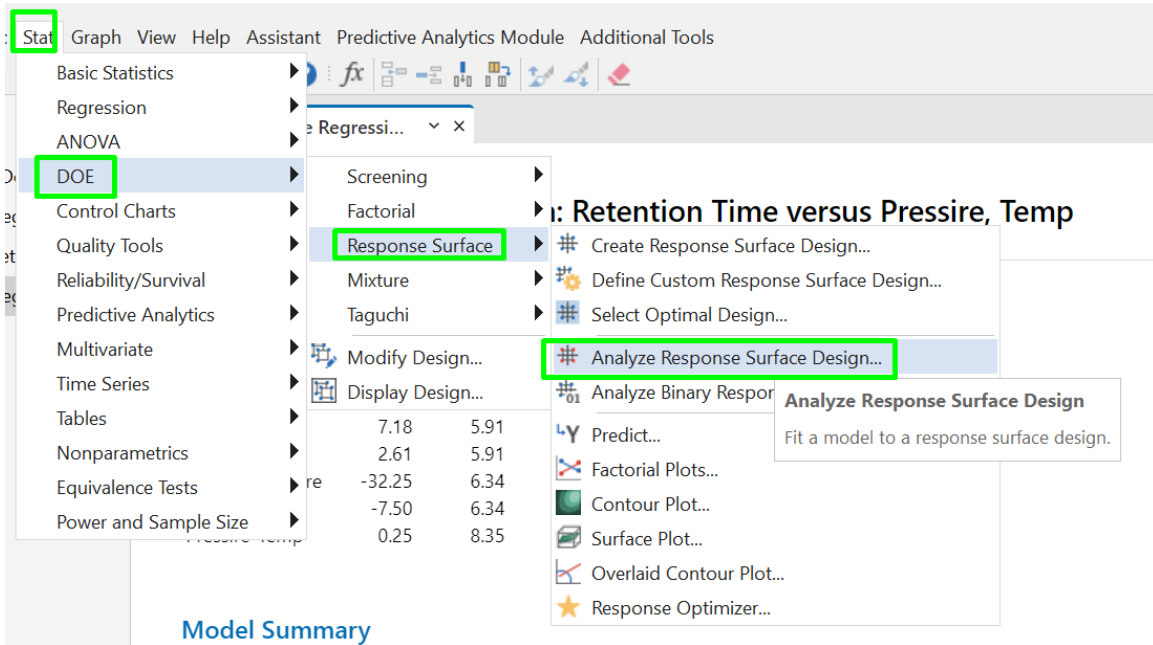


With Interaction



We note that there is a maximum Retention Time when the Pressure is around 500 and the Temperature around 50.

b) Pareto Chart and Model Equation



Regression Equation in Uncoded Units

$$\text{Retention Time} = -66.7 + 0.494 \text{ Pressire} + 2.43 \text{ Temp} - 0.000492 \text{ Pressire} * \text{Pressire} - 0.0231 \text{ Temp} * \text{Temp} + 0.00005 \text{ Pressire} * \text{Temp}$$

An equation to model the relationship is created...
Note that its non-linear...

Analysis of Variance

| Source | DF | Adj SS | Adj MS | F-Value | P-Value |
|-------------------|----|---------|---------|---------|---------|
| Model | 5 | 7778.75 | 1555.75 | 5.57 | 0.022 |
| Linear | 2 | 466.52 | 233.26 | 0.84 | 0.473 |
| Pressire | 1 | 412.22 | 412.22 | 1.48 | 0.264 |
| Temp | 1 | 54.29 | 54.29 | 0.19 | 0.673 |
| Square | 2 | 7311.98 | 3655.99 | 13.09 | 0.004 |
| Pressire*Pressire | 1 | 7235.22 | 7235.22 | 25.91 | 0.001 |
| Temp*Temp | 1 | 391.30 | 391.30 | 1.40 | 0.275 |
| 2-Way Interaction | 1 | 0.25 | 0.25 | 0.00 | 0.977 |
| Pressire*Temp | 1 | 0.25 | 0.25 | 0.00 | 0.977 |
| Error | 7 | 1954.48 | 279.21 | | |
| Lack-of-Fit | 3 | 1902.48 | 634.16 | 48.78 | 0.001 |
| Pure Error | 4 | 52.00 | 13.00 | | |
| Total | 12 | 9733.23 | | | |

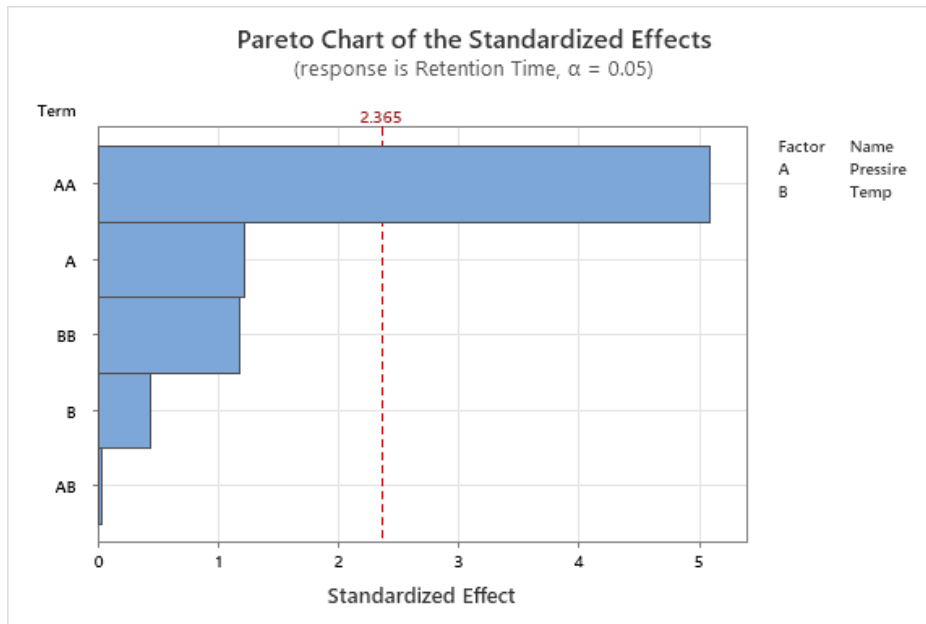
we ignore the linear model since curvature is significant

0.001 < 0.05 means that Pressure is significant

0.275 > 0.05 means that Temperature is insignificant

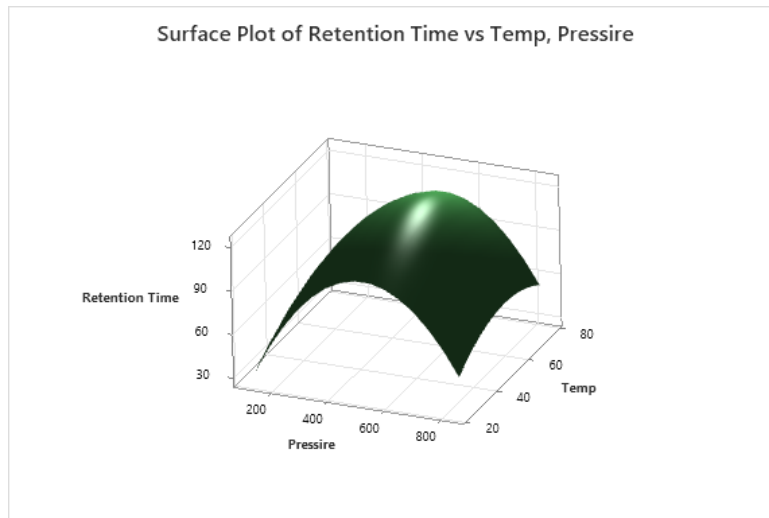
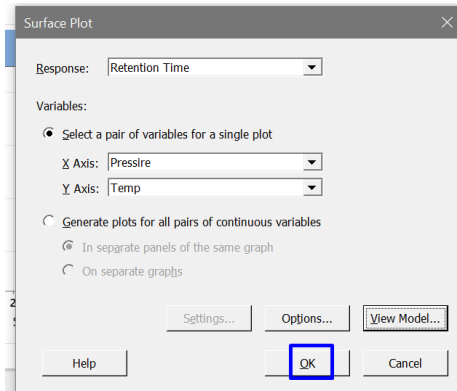
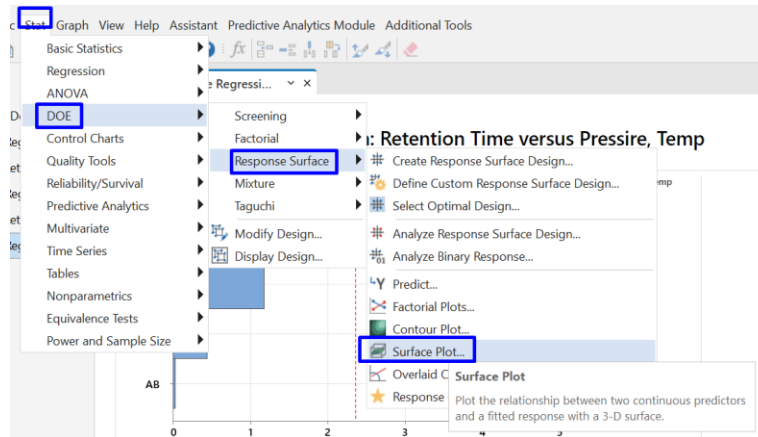
the interaction between Press and Temp is insignificant

0.001 < 0.05 means that the curvature is significant

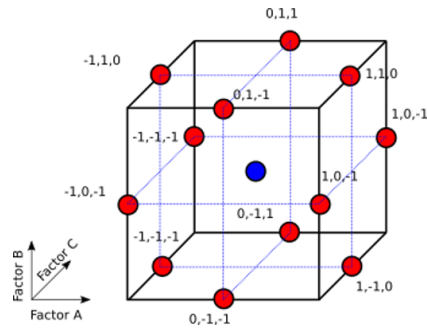


- We note that Pressure is the Key Factor affecting Response Time.
- Temperature is insignificant.

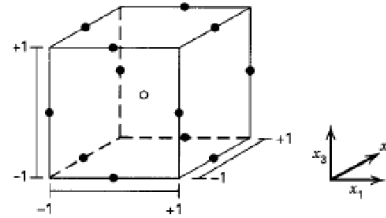
c) Surface Plot



B. BOX-BEHNKEN DESIGN



| Run | x_1 | x_2 | x_3 |
|-----|-------|-------|-------|
| 1 | -1 | -1 | 0 |
| 2 | -1 | 1 | 0 |
| 3 | 1 | -1 | 0 |
| 4 | 1 | 1 | 0 |
| 5 | -1 | 0 | -1 |
| 6 | -1 | 0 | 1 |
| 7 | 1 | 0 | -1 |
| 8 | 1 | 0 | 1 |
| 9 | 0 | -1 | -1 |
| 10 | 0 | -1 | 1 |
| 11 | 0 | 1 | -1 |
| 12 | 0 | 1 | 1 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |



Create Response Surface Design

Type of Design

Central composite (2 to 10 continuous factors)

Box-Behnken (3,4,5,6,7,9, or 10 continuous factors)

Number of continuous factors:

Number of categorical factors:

Display Available Designs...

min runs are 15 runs

Min BB Design is 3 Factors
You can't go lower than 3 factors

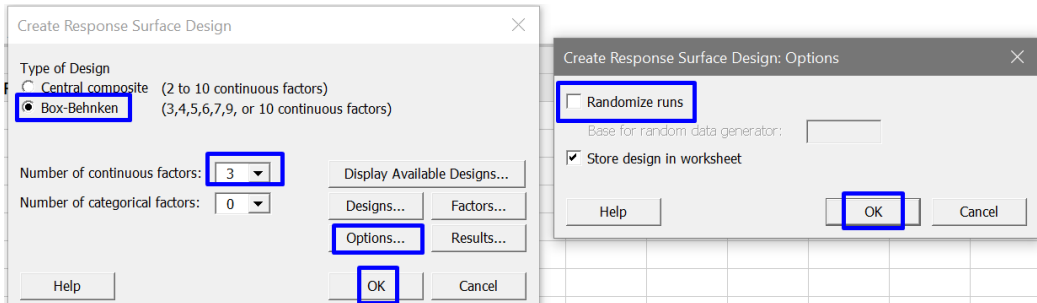
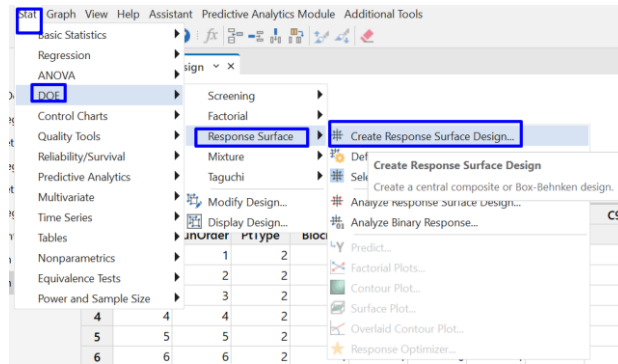
Create Response Surface Design: Display Available Designs

Available Response Surface Designs

| Design | | Continuous Factors | | | | | | | | | |
|---------------------------|-----------|--------------------|----|----|----|----|-----|-----|-----|-----|-----|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| Central composite full | unblocked | 13 | 20 | 31 | 52 | 90 | 152 | | | | |
| | blocked | 14 | 20 | 30 | 54 | 90 | 160 | | | | |
| Central composite half | unblocked | | | | 32 | 53 | 88 | 154 | | | |
| | blocked | | | | 33 | 54 | 90 | 160 | | | |
| Central composite quarter | unblocked | | | | | | | 90 | 156 | | |
| | blocked | | | | | | | 90 | 160 | | |
| Central composite eighth | unblocked | | | | | | | | | 158 | |
| | blocked | | | | | | | | | 160 | |
| Box-Behnken | unblocked | | 15 | 27 | 46 | 54 | 62 | | | 130 | 170 |
| | blocked | | 15 | 27 | 46 | 54 | 62 | | | 130 | 170 |

- Note that a 3 Factor 2 Level BB Design minimum is 15 Runs
- Note that BB Design are ALL Centre Point Runs!!!
- In other words, BB is a special subset of the CCD (you can imagine it to be a Fractional Factorial of a Full Factorial Design).

1. CREATING THE BB EXPERIMENT



Box-Behnken Design

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C |
|----|----------|----------|--------|--------|----|----|----|----|----|-----|-----|---|
| | StdOrder | RunOrder | PtType | Blocks | A | B | C | | | | | |
| 1 | 1 | 1 | 2 | 1 | -1 | -1 | 0 | | | | | |
| 2 | 2 | 2 | 2 | 1 | 1 | -1 | 0 | | | | | |
| 3 | 3 | 3 | 2 | 1 | -1 | 1 | 0 | | | | | |
| 4 | 4 | 4 | 2 | 1 | 1 | 1 | 0 | | | | | |
| 5 | 5 | 5 | 2 | 1 | -1 | 0 | -1 | | | | | |
| 6 | 6 | 6 | 2 | 1 | 1 | 0 | -1 | | | | | |
| 7 | 7 | 7 | 2 | 1 | -1 | 0 | 1 | | | | | |
| 8 | 8 | 8 | 2 | 1 | 1 | 0 | 1 | | | | | |
| 9 | 9 | 9 | 2 | 1 | 0 | -1 | -1 | | | | | |
| 10 | 10 | 10 | 2 | 1 | 0 | 1 | -1 | | | | | |
| 11 | 11 | 11 | 2 | 1 | 0 | -1 | 1 | | | | | |
| 12 | 12 | 12 | 2 | 1 | 0 | 1 | 1 | | | | | |
| 13 | 13 | 13 | 0 | 1 | 0 | 0 | 0 | | | | | |
| 14 | 14 | 14 | 0 | 1 | 0 | 0 | 0 | | | | | |
| 15 | 15 | 15 | 0 | 1 | 0 | 0 | 0 | | | | | |

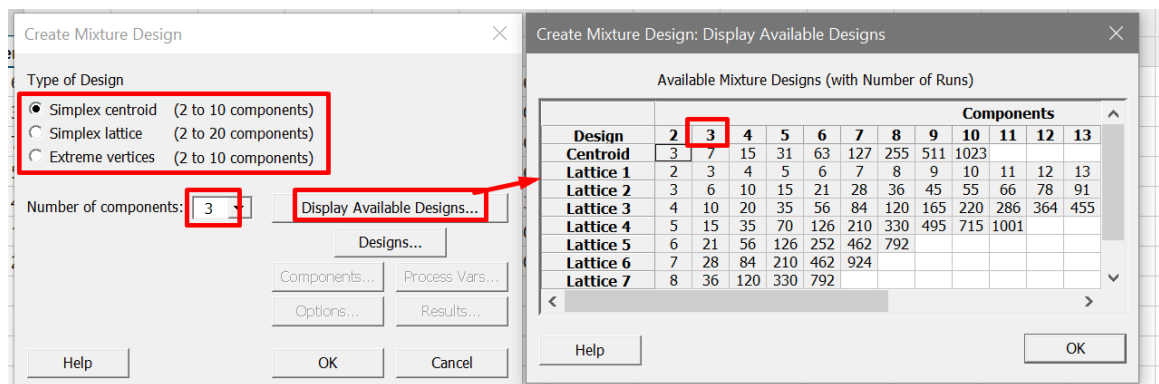
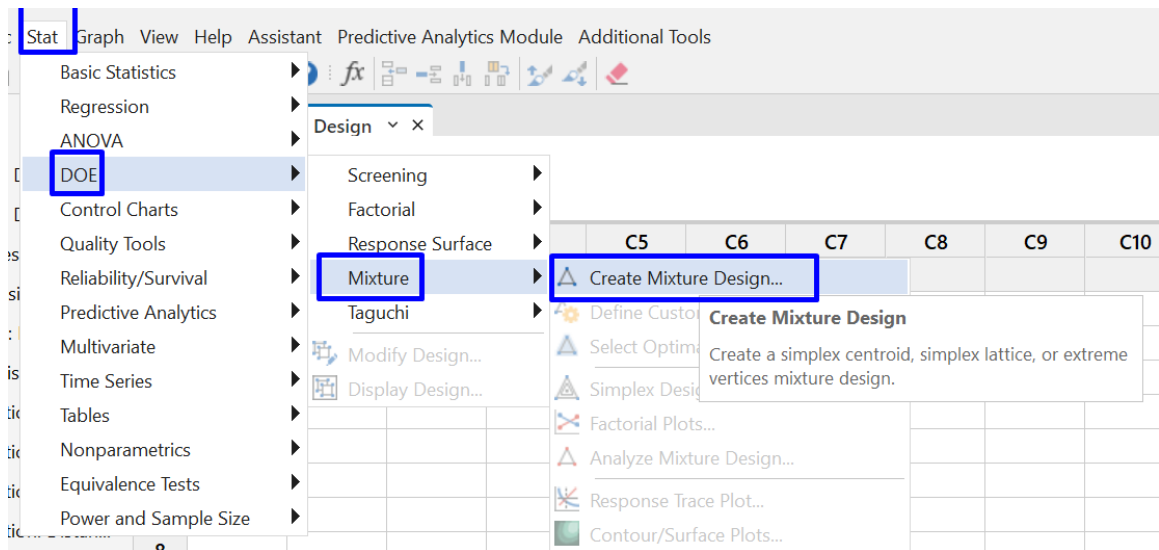
Every Run has a Factor that Must include 0

Which means they are ALL Centre Point Runs!

Note that the PtType indicates that all runs are only Centre Points!

- We shall not carry on with further experiments.
- We shall stop here at only creating the BB experimental design.
- This is because the rest of the steps are similar and can be repeated as above (see section: CCD: Creating the Experiment (ctrl click the link))

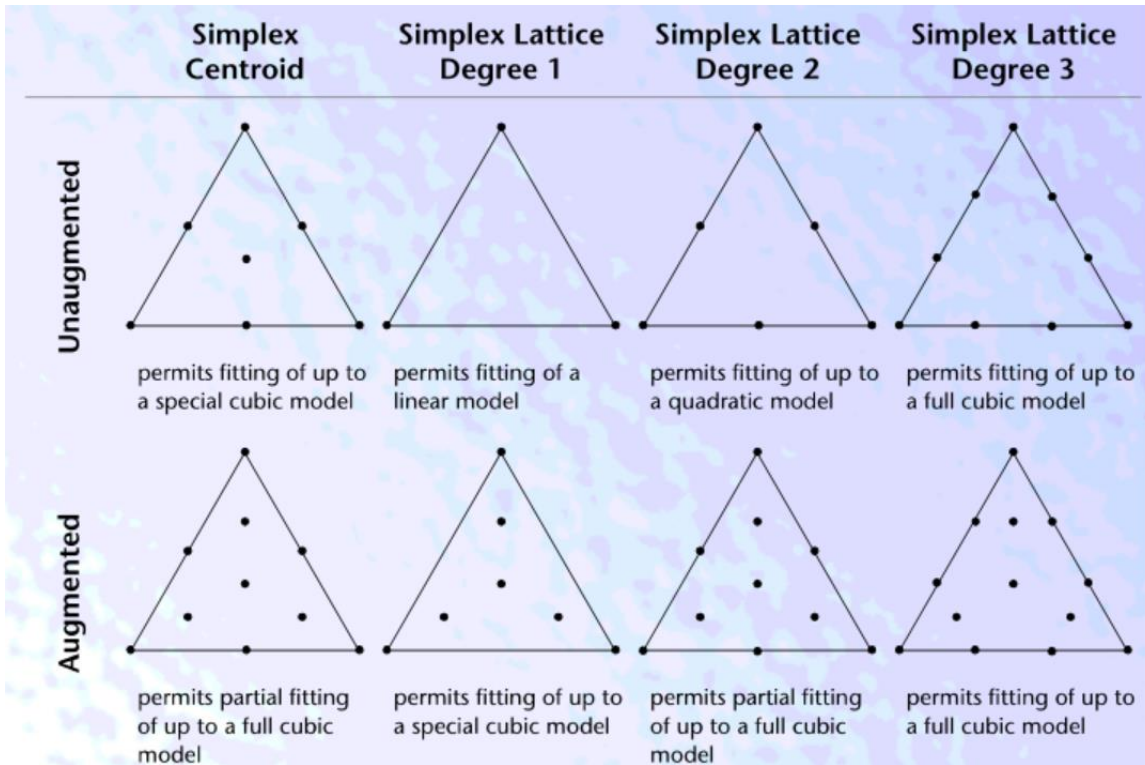
V. MIXTURE DESIGNS



There are 3 types of Mixture Designs:

- Simplex Centroid
- Simplex Lattice
- Extreme Vertices

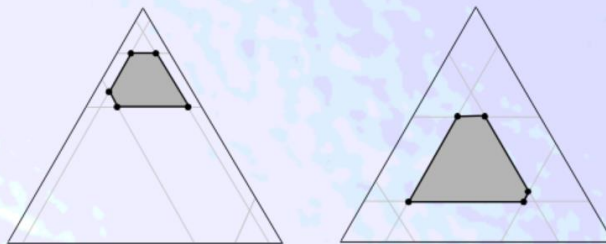
A. WHAT IS A MIXTURE DESIGN



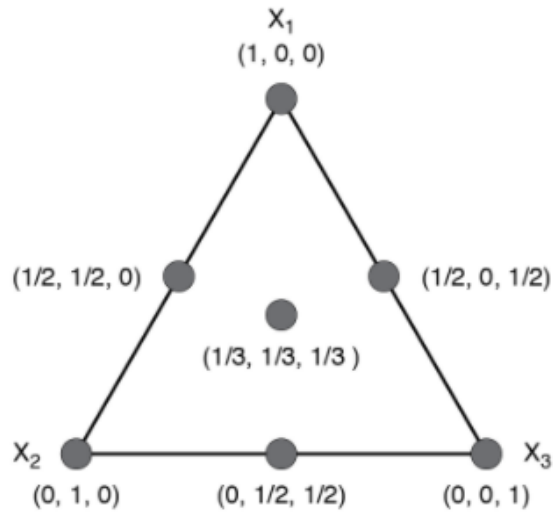
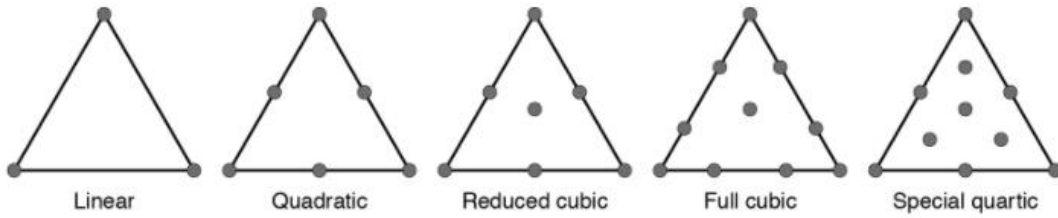
Extreme vertices designs

Very Complex
We will not
dwell further into this....

In extreme vertices designs, MINTAB employs an algorithm that generates extreme vertices and their blends up to the specified degree. These designs must be used when your chosen design space is not an L-simplex. The presence of both lower and upper bound constraints on the components often create this condition. The goal of an extreme vertices design is to choose design points that adequately cover the design space. The illustration below shows the extreme vertices for two three-component designs with both upper and lower constraints:



The light gray lines represent the lower and upper bound constraints on the components. The dark gray area represents the design space. The points are placed at the extreme vertices of design space.

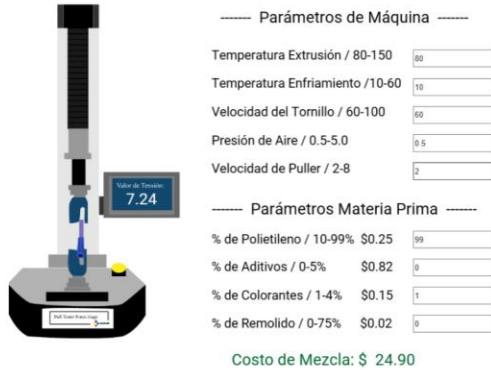


- Picture above shows a Simplex Centroid (Unaugmented) with 7 Points (7 Runs)
- There are 3 Components / Factors (X_1 / X_2 / X_3)
- $X_1 + X_2 + X_3$ Must == 100% → They are ingredients (mixture) which make up a total percentage of 100%.

B. MIXTURE EXPERIMENT EXAMPLE

- Go <https://cusum.mx/en/simuladores/>

Mixture and Gage R&R Destructive Simulator 2022



----- Parámetros de Máquina -----

Temperatura Extrusión / 80-150

Temperatura Enfriamiento / 10-60

Velocidad del Tornillo / 60-100

Presión de Aire / 0.5-5.0

Velocidad de Puller / 2-8

----- Parámetros Materia Prima -----

% de Polietileno / 10-99% \$0.25

% de Aditivos / 0-5% \$0.82

% de Colorantes / 1-4% \$0.15

% de Remolido / 0-75% \$0.02

Costo de Mezcla: \$ 24.90

When optimizing raw materials Engineers commonly use mixtures. This type of DOE is different from the previous ones and this simulator teaches students how to approach and optimize a mixture process. It also teaches them how to validate a destructive type Gage R&R.

--> [Download Excel Grid for Faster Analysis](#)

[click this to download the .xls for faster results...](#)

| | Machine Parameters | | | | | Raw Material Parameters | | | |
|---------|-----------------------|---------------------|-------------|--------------|--------------|-------------------------|------------|------------|----------|
| | Extrusion Temperature | Cooling Temperature | Screw Speed | Air Pressure | Puller Speed | %Polyethylene | %Additives | %Colorants | %Regrind |
| Minimum | 80 | 10 | 60 | 0.5 | 2 | 10% | 0% | 1% | 0% |
| Maximum | 150 | 60 | 100 | 5 | 8 | 99% | 5% | 4% | 75% |
| Run No. | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | | | | |
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | | | | | | | | | |
| 6 | | | | | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9 | | | | | | | | | |
| 10 | | | | | | | | | |
| 11 | | | | | | | | | |
| 12 | | | | | | | | | |
| 13 | | | | | | | | | |
| 14 | | | | | | | | | |
| 15 | | | | | | | | | |
| 16 | | | | | | | | | |
| 17 | | | | | | | | | |
| 18 | | | | | | | | | |
| 19 | | | | | | | | | |
| 20 | | | | | | | | | |
| 21 | | | | | | | | | |

key this in for Machine Parameters
Not that these are FIXED and are NOT our Factors / Mixture ingredients....

these are our Mixture Ingredients which we will use Minitab to generatewe will copy paste here later...

The Machine Parameters (Fixed Constants) are:

- Extrusion: 80
- Cooling: 10
- Screw: 60
- Air: 0.5
- Puller: 2

The Raw Material Parameters (Mixture Ingredients) are:

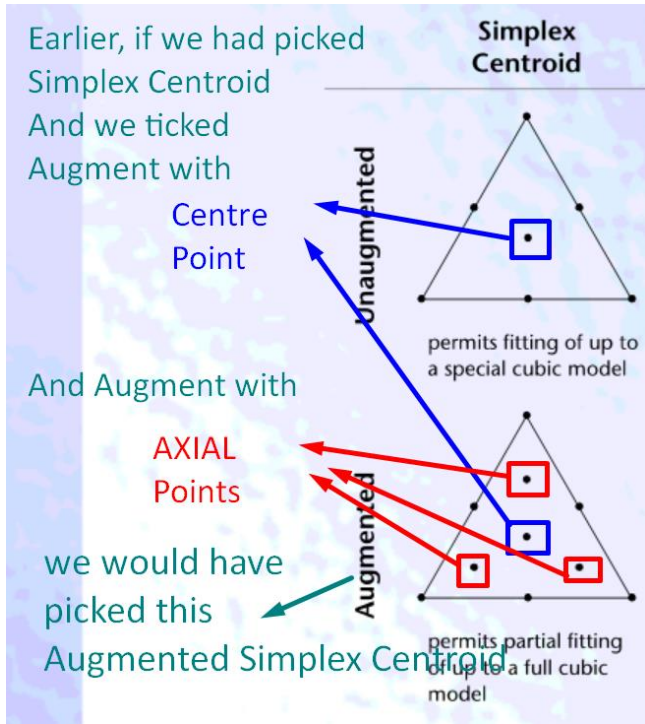
- Polyethylene: 10 ~ 99%
- Additives: 0 ~5%
- Colorants: 1 ~ 4%
- Regrind: 0~75%

1. CREATING THE MIXTURE EXPERIMENT

we simply choose Extreme Vertices because its the best one (won't be explaining why...)

we shall explain what is meant by Augment with Center and Axial Point

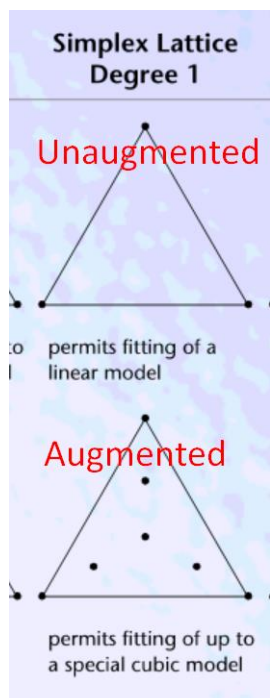
| Point Type | Description | Number |
|------------|--------------|--------|
| 1 | vertex | 1 |
| 2 | double blend | 1 |
| 3 | triple blend | 1 |
| 0 | center point | 1 |
| -1 | axial point | 1 |



An Augmented Simplex Centroid has 10 Points (see above picture).. thus it will produce a table of 10 runs...

similarly if you chose Simplex Lattice 1 Degree

Don't have Center Point Nor Axial Point....



| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 |
|---|----------|----------|--------|--------|----|----|----|----|----|-----|-----|-----|
| | StdOrder | RunOrder | PtType | Blocks | A | B | C | | | | | |
| 1 | 2 | 1 | 1 | 1 | 0 | 1 | 0 | | | | | |
| 2 | 1 | 2 | 1 | 1 | 1 | 0 | 0 | | | | | |
| 3 | 3 | 3 | 1 | 1 | 0 | 0 | 1 | | | | | |
| 4 | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | |

this creates a 3 run 3 component table because it has 3 points on the triangle....

Create Mixture Design

Type of Design

Simplex centroid (2 to 10 components)

Simplex lattice (2 to 20 components)

Extreme vertices (2 to 10 components)

Number of components: **4**

Components... Process Vars... Options... Results... Help OK Cancel

Create Mixture Design: Components

Total Mixture Amount

Single total: **1.0**

Multiple totals (up to 5):

Component Bounds Specified in Amount
(lower and upper are for the first total, if you specified more than one)

| Component | Name | Lower | Upper |
|-----------|--------------|-------|-------|
| A | Polyethylene | 0.1 | 0.99 |
| B | Additives | 0 | 0.05 |
| C | Colorants | 0.01 | 0.04 |
| D | Regrind | 0 | 0.75 |

Linear Constraints... Help OK Cancel

Mixture should add up to 100%

Key in the 4 component names as well as the Lower and Upper constrained percentages...

Create Mixture Design

Type of Design

Simplex centroid (2 to 10 components)

Simplex lattice (2 to 20 components)

Extreme vertices (2 to 10 components)

Number of components: **4**

Options... Results... Help OK Cancel

Create Mixture Design: Options

Randomize runs

Base for random data generator:

Store design in worksheet

Help OK Cancel

deselect randomize runs...

and leave the other options as defaults.. then click OK.....

Extreme Vertices Design this new table has been created....

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 |
|----|----------|----------|--------|--------|--------------|-----------|-----------|---------|----|-----|-----|-----|-----|
| | StdOrder | RunOrder | PtType | Blocks | Polyethylene | Additives | Colorants | Regrind | | | | | |
| 1 | 1 | 1 | 1 | 1 | 0.9900 | 0.0000 | 0.0100 | 0.0000 | | | | | |
| 2 | 2 | 2 | 1 | 1 | 0.2400 | 0.0000 | 0.0100 | 0.7500 | | | | | |
| 3 | 3 | 3 | 1 | 1 | 0.2100 | 0.0000 | 0.0400 | 0.7500 | | | | | |
| 4 | 4 | 4 | 1 | 1 | 0.1900 | 0.0500 | 0.0100 | 0.7500 | | | | | |
| 5 | 5 | 5 | 1 | 1 | 0.1600 | 0.0500 | 0.0400 | 0.7500 | | | | | |
| 6 | 6 | 6 | 1 | 1 | 0.9600 | 0.0000 | 0.0400 | 0.0000 | | | | | |
| 7 | 7 | 7 | 1 | 1 | 0.9400 | 0.0500 | 0.0100 | 0.0000 | | | | | |
| 8 | 8 | 8 | 1 | 1 | 0.9100 | 0.0500 | 0.0400 | 0.0000 | | | | | |
| 9 | 9 | 9 | 0 | 1 | 0.5750 | 0.0250 | 0.0250 | 0.3750 | | | | | |
| 10 | 10 | 10 | -1 | 1 | 0.7825 | 0.0125 | 0.0175 | 0.1875 | | | | | |
| 11 | 11 | 11 | -1 | 1 | 0.4075 | 0.0125 | 0.0175 | 0.5625 | | | | | |
| 12 | 12 | 12 | -1 | 1 | 0.3925 | 0.0125 | 0.0325 | 0.5625 | | | | | |
| 13 | 13 | 13 | -1 | 1 | 0.3825 | 0.0375 | 0.0175 | 0.5625 | | | | | |
| 14 | 14 | 14 | -1 | 1 | 0.3675 | 0.0375 | 0.0325 | 0.5625 | | | | | |
| 15 | 15 | 15 | -1 | 1 | 0.7675 | 0.0125 | 0.0325 | 0.1875 | | | | | |
| 16 | 16 | 16 | -1 | 1 | 0.7575 | 0.0375 | 0.0175 | 0.1875 | | | | | |
| 17 | 17 | 17 | -1 | 1 | 0.7425 | 0.0375 | 0.0325 | 0.1875 | | | | | |

copy and paste these 4 columns back into the Excel spreadsheet

to generate - Results - Costs as the Response....

- Recall: PtType stores the point type.
- The codes are:
 - 0 is a Centre Point run and
 - 1 is a Corner Point.
 - I think -1 is an Axial Point

| | Machine Parameters | | | | | Raw Material Parameters | | | | Results | | |
|---------|-----------------------|---------------------|-------------|--------------|--------------|-------------------------|------------|------------|----------|------------|--------------|-----------------------|
| | Extrusion Temperature | Cooling Temperature | Screw Speed | Air Pressure | Puller Speed | %Polyethylene | %Additives | %Colorants | %Regrind | Multiplier | Mixture Cost | Tension Results Lbs/f |
| Minimum | 80 | 10 | 60 | 0.5 | 2 | 10% | 0% | 1% | 0% | N/A | N/A | 10 |
| Maximum | 150 | 60 | 100 | 5 | 8 | 99% | 5% | 4% | 75% | N/A | N/A | 12 |
| Run No. | | | | | | | | | | | | |
| 1 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.99 | 0 | 0.01 | 0 | 0.993 | \$ 24.90 | 7.411 |
| 2 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.24 | 0 | 0.01 | 0.75 | 0.183 | \$ 7.65 | 1.354 |
| 3 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.21 | 0 | 0.04 | 0.75 | 0.162 | \$ 7.35 | 1.201 |
| 4 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.19 | 0.05 | 0.01 | 0.75 | 0.633 | \$ 10.50 | 4.648 |
| 5 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.16 | 0.05 | 0.04 | 0.75 | 0.612 | \$ 10.20 | 4.449 |
| 6 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.96 | 0 | 0.04 | 0 | 0.972 | \$ 24.60 | 7.294 |
| 7 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.94 | 0.05 | 0.01 | 0 | 1.443 | \$ 27.75 | 10.670 |
| 8 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.91 | 0.05 | 0.04 | 0 | 1.422 | \$ 27.45 | 10.591 |
| 9 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.575 | 0.025 | 0.025 | 0.375 | 0.8025 | \$ 17.55 | 5.992 |
| 10 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.7825 | 0.0125 | 0.0175 | 0.1875 | 0.89775 | \$ 21.23 | 6.695 |
| 11 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.4075 | 0.0125 | 0.0175 | 0.5625 | 0.49275 | \$ 12.60 | 3.588 |
| 12 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.3925 | 0.0125 | 0.0325 | 0.5625 | 0.48225 | \$ 12.45 | 3.618 |
| 13 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.3825 | 0.0375 | 0.0175 | 0.5625 | 0.71775 | \$ 14.03 | 5.267 |
| 14 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.3675 | 0.0375 | 0.0325 | 0.5625 | 0.70725 | \$ 13.88 | 5.155 |
| 15 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.7675 | 0.0125 | 0.0325 | 0.1875 | 0.88725 | \$ 21.08 | 6.606 |
| 16 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.7575 | 0.0375 | 0.0175 | 0.1875 | 1.12275 | \$ 22.65 | 8.326 |
| 17 | 80.00 | 10.00 | 60.00 | 0.50 | 2.00 | 0.7425 | 0.0375 | 0.0325 | 0.1875 | 1.11225 | \$ 22.50 | 8.085 |
| 18 | | | | | | | | | | | | |
| 19 | | | | | | | | | | | | |
| 20 | | | | | | | | | | | | |
| 21 | | | | | | | | | | | | |
| 22 | | | | | | | | | | | | |
| 23 | | | | | | | | | | | | |
| 24 | | | | | | | | | | | | |
| 25 | | | | | | | | | | | | |
| 26 | | | | | | | | | | | | |
| 27 | | | | | | | | | | | | |
| 28 | | | | | | | | | | | | |
| 29 | | | | | | | | | | | | |
| 30 | | | | | | | | | | | | |

these are the machine parameters that are left constant.....

these are my Mixture Ingredients which should add up to 100% per row.....

which i copied pasted from Minitab....

these are my 2 responses which i will copy paste into Minitab....

2. PREDICTING THE MIXTURE EXPERIMENT

then click here...

| C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 | C16 |
|---------|----------|---------|-----|-----|-----|-----|-----|-----|
| Regrind | COST | TENSION | | | | | | |
| 0.0000 | \$ 24.90 | 7.411 | | | | | | |
| 0.7500 | \$ 7.65 | 1.354 | | | | | | |
| 0.7500 | \$ 7.35 | 1.201 | | | | | | |
| 0.7500 | \$ 10.50 | 4.648 | | | | | | |
| 0.7500 | \$ 10.20 | 4.449 | | | | | | |
| 0.0000 | \$ 24.60 | 7.294 | | | | | | |
| 0.0000 | \$ 27.75 | 10.670 | | | | | | |
| 0.0000 | \$ 27.45 | 10.591 | | | | | | |

copy paste these 2 columns in from the Excel spreadsheet...as Responses...

the most important is to minimize cost... followed by targetting Tension...

| Response | Goal | Lower | Target | Upper | Weight | Importance |
|-------------|----------|-------|--------|-------|--------|------------|
| C9 COST | Minimize | | 15 | 30 | 1 | 2 |
| C10 TENSION | Target | 10 | 11 | 12 | 1 | 1 |

Response Optimization

Parameters

| | Goal | Lower | Target | Upper | Weight | Import |
|---------|---------|-------|--------|-------|--------|--------|
| COST | Minimum | 15 | 15 | 30 | 1 | 2 |
| TENSION | Target | 10 | 11 | 12 | 1 | 1 |

in order to achieve a minimum COST of \$15 and targetted TENSION of 11....

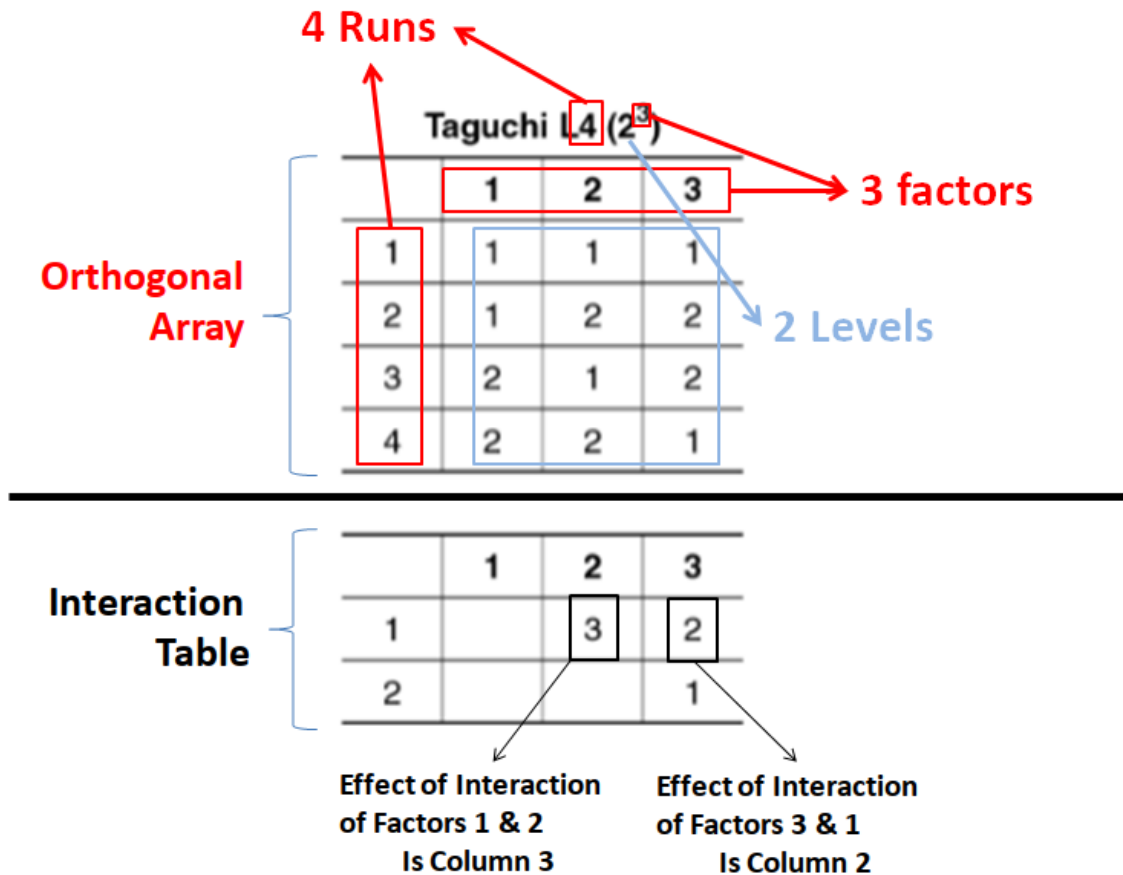
Global Solution

| Components | |
|--------------|--------|
| Polyethylene | = 0.91 |
| Additives | = 0.05 |
| Colorants | = 0.04 |
| Regrind | = 0 |

we should use these Mixture %

VI. TAGUCHI METHOD

A. CONCEPT



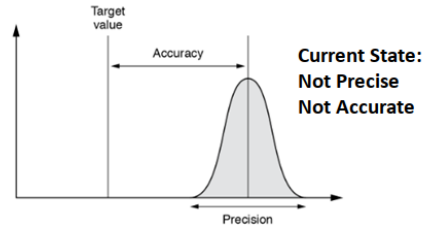
- By right, a Full Factorial for a 3 factor 2 level (2^3) = 8 runs required.
- Taguchi can reduce the number of runs to only 4.
- This is possible because Taguchi assumes the 3rd column (3rd factor) to be the interaction effect of factors 1 and 2, thus it can be ignored.

| | 1 | 2 | 3 |
|----------|----------|----------|----------|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 |
| 3 | 2 | 1 | 2 |
| 4 | 2 | 2 | 1 |

- Taguchi regards Factor Interaction Effects as Noise (thus they are insignificant and ignored).
- Taguchi only regards Main Factor Effects.

B. TAGUCHI'S OPTIMIZATION

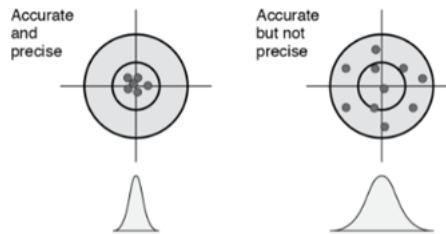
Optimization Step 2:
Try to Move Mean to
Target, thus
improving Accuracy



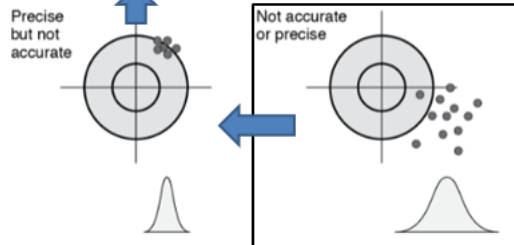
Optimization Step 1:
Try to Reduce
Variation, thus
improving Precision

- Precision relates to the Standard Deviation (SD)
- Target Value relates to the Mean

Optimization
Step 2: Try to
Move Mean to
Target, thus
improving
Accuracy



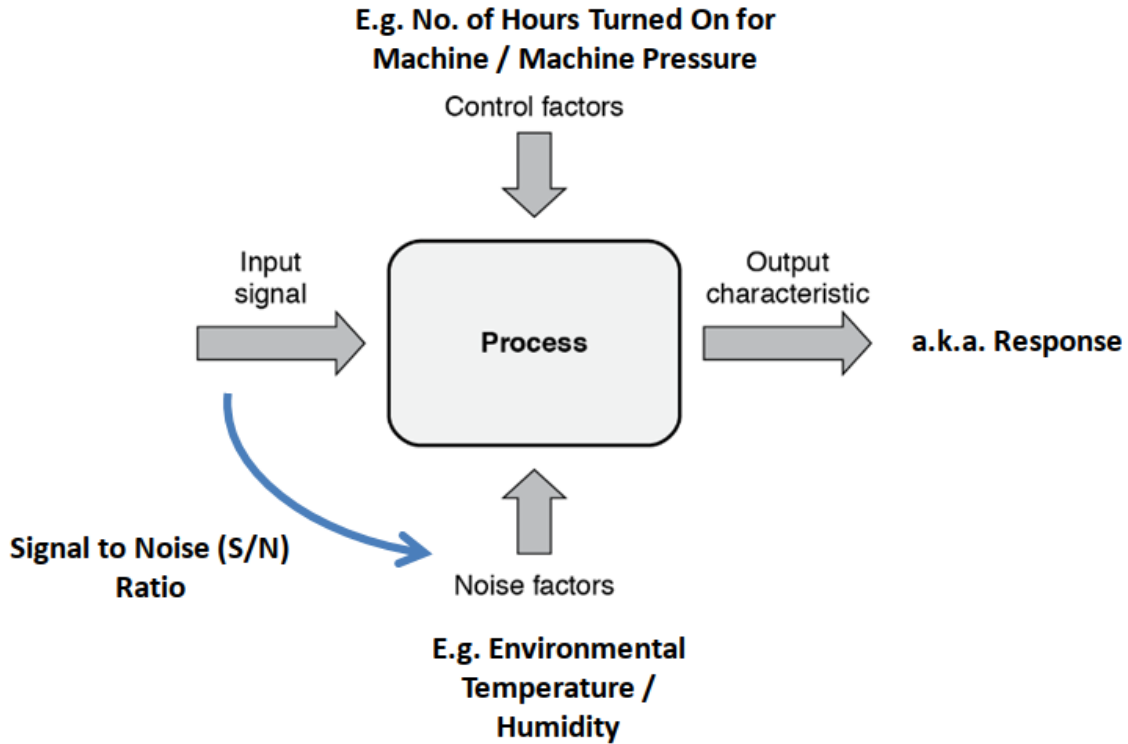
Optimization
Step 1: Try to
Reduce
Variation, thus
improving
Precision



Current State:

- Low Signal to Noise (S/N) Ratio
- High Variation
- Mean far from Target

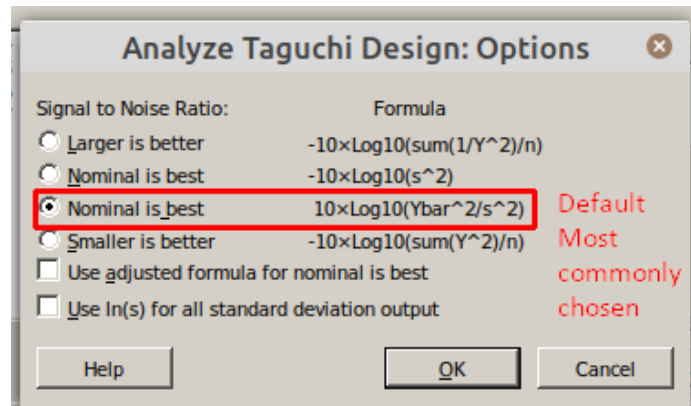
C. MEANING OF SIGNAL TO NOISE RATIO (S/N)



- S/N ratio combines the effect of the Mean and Standard Deviation **into one value** used to gage the process output.¹
- In other words, the larger the S/N ratio, the more robust the process is to Noise.
- As mentioned above,
 - Precision relates to the Standard Deviation (Step 1 for Optimization). We use the S/N ratio to identify those Control Factors that reduce variability.

¹ <https://www.amazon.com/Practical-Design-Experiments-DOE-Optimizing/dp/0873899245>

- Accuracy relates to the Target Value (Mean) (Step 2 for Optimization). We identify Control Factors that move the mean to target and have no effect on the S/N ratio.
- There are 4 types of S/N ratio in Minitab: (note that for all types, our objective is to Maximize the S/N ratio → increase Signal, lower Noise)



1. LARGER IS BETTER

- Objective is to Maximize the Response.
- Example: Identify the Factors that Increase the hardness of the steel alloy.

2. NOMINAL IS BEST

- Objective is to try to hit the Target Response using S/N ratio based on Standard Deviations.

3. NOMINAL IS BEST (DEFAULT)

- Objective is to try to hit the Target Response using S/N ratio based on Means and Standard Deviations.
- Example: Identify the Factors that allow the manufacture to find the nominal specification.

4. SMALLER IS BETTER

- Objective is to Minimize the Response.
- Example: Identify the Factors that Reduce the force necessary to open the sealed packaging.

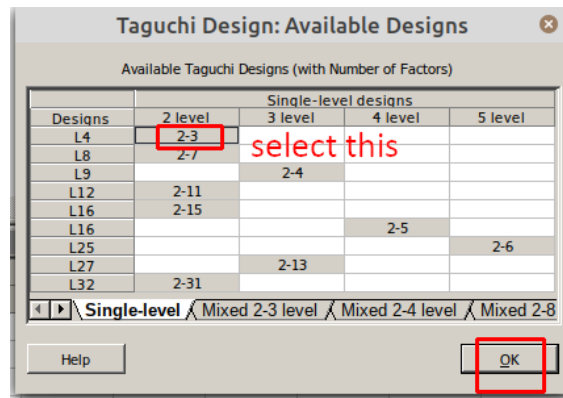
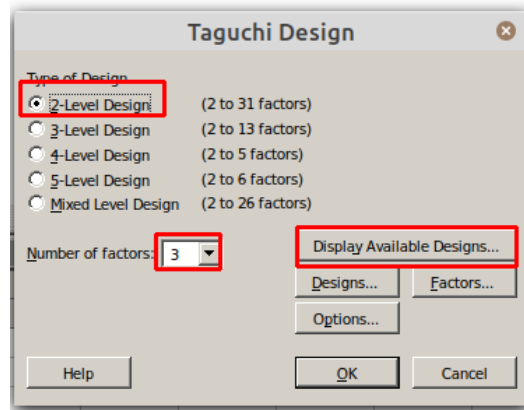
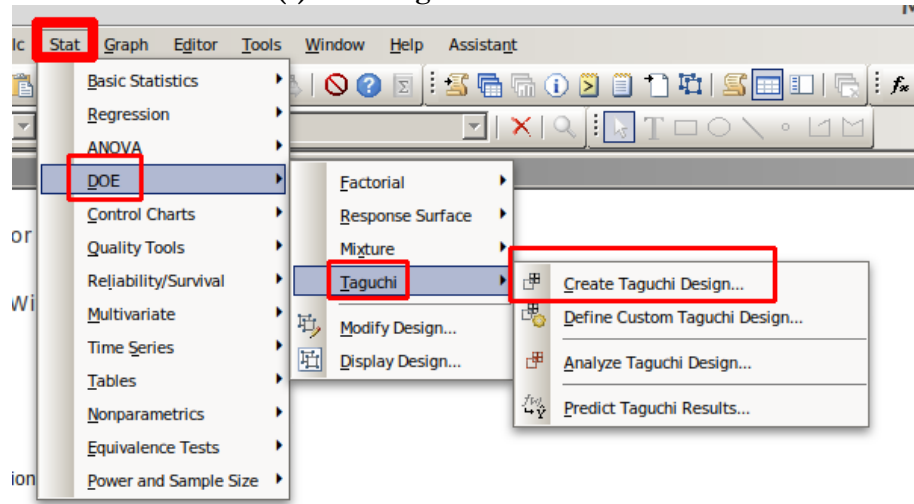
D. EXAMPLE: MINIMIZING THE RESPONSE (SMALLER IS BETTER)

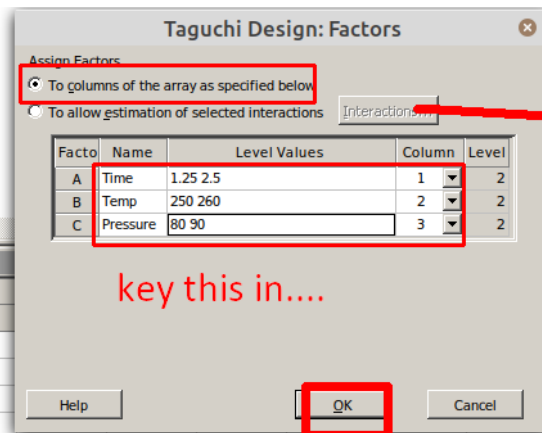
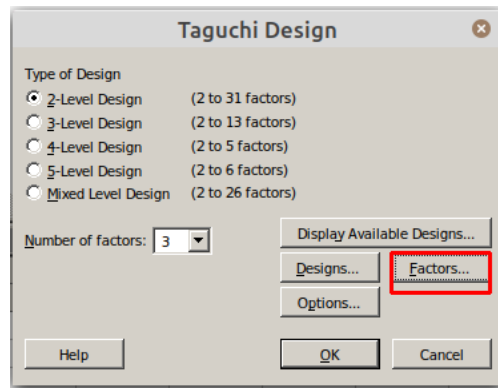
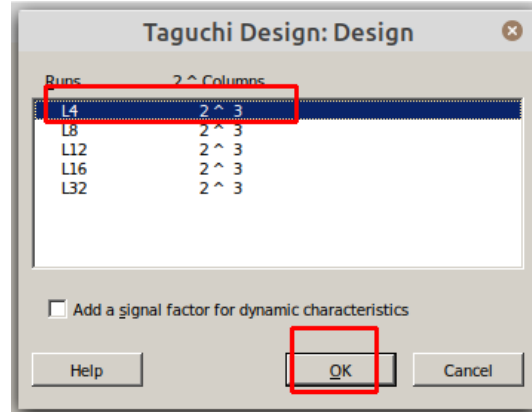
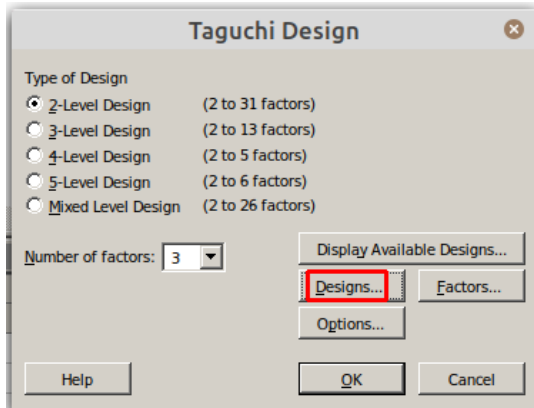
| Time | Temp | Pressure | With Noise 1 | With Noise 2 | With Noise 3 |
|------|------|----------|--------------|--------------|--------------|
| 1.25 | 250 | 80 | 2.9 | 3 | 2.8 |
| 1.25 | 260 | 90 | 2.4 | 2.2 | 2.3 |
| 2.5 | 250 | 90 | 3.5 | 3.6 | 3.7 |
| 2.5 | 260 | 80 | 2.6 | 2.5 | 2.7 |

- This example is taken from (Durivage, 2016)
- A sealing process requires large force to open the packaging.
- 3 Factors, 2 Levels
 - Time: 1.25 and 2.5 seconds
 - Temperature: 250 and 260 Degree Celsius
 - Pressure: 80 and 90 PSI
- By right, a 2^3 will take total 8 runs, but we will do a L4 (only 4 runs)
- Responses (Three outputs collected under three different noise conditions)
 - With Noise 1
 - With Noise 2
 - With Noise 3
- Objective:
 - What are the Factors that help to reduce the Force required to open the package?
 - Noise is irremovable, we can only try to Maximize the S/N ratio

a) Minitab Solution

(1) Creating the Worksheet

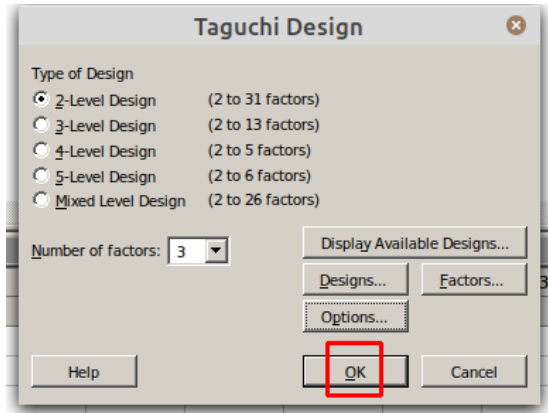




we do not select this option because we are not keen to study the Interaction Effects... i.e. AB / AC ...

key this in....

Taguchi feels that Interaction Effects are negligible and we should only study Main Effects... i.e. A / B / C



Worksheet 4 ***

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
|---|------|------|----------|----|----|----|----|----|
| | Time | Temp | Pressure | | | | | |
| 1 | 1.25 | 250 | 80 | | | | | |
| 2 | 1.25 | 260 | 90 | | | | | |
| 3 | 2.50 | 250 | 90 | | | | | |
| 4 | 2.50 | 260 | 80 | | | | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | | | | | | | | |
| 8 | | | | | | | | |

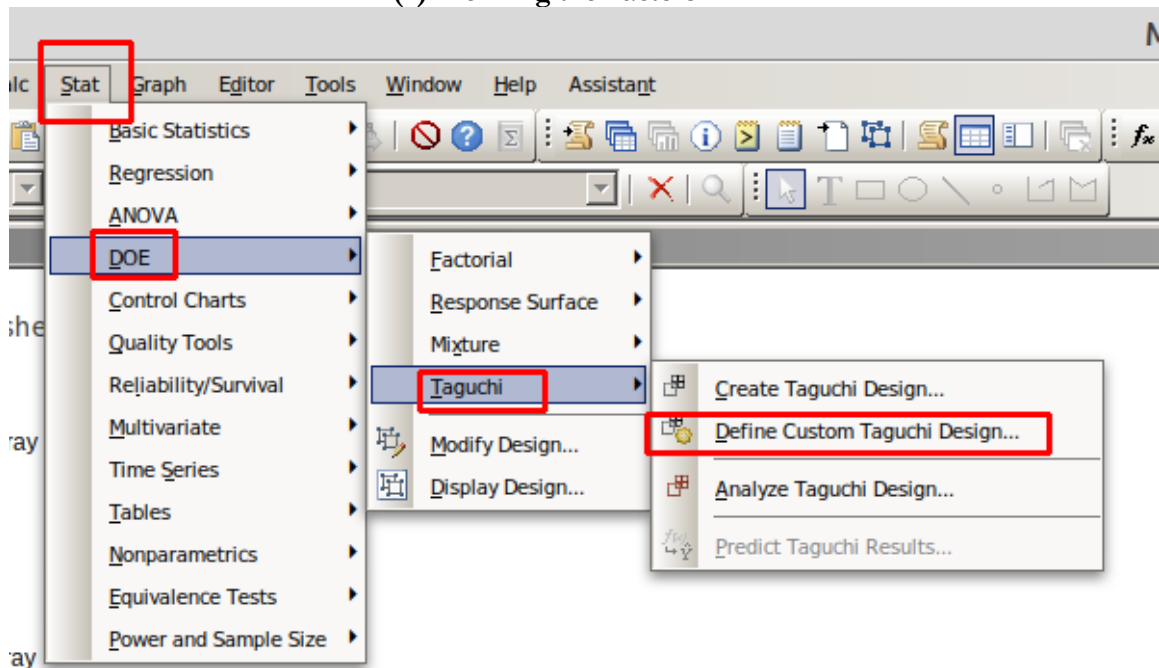
this table pops up and has been created..

Worksheet 4 ***

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 |
|----|------|------|----------|--------------|--------------|--------------|----|
| | Time | Temp | Pressure | With Noise 1 | With Noise 2 | With Noise 3 | |
| 1 | 1.25 | 250 | 80 | 2.9 | 3.0 | 2.8 | |
| 2 | 1.25 | 260 | 90 | 2.4 | 2.2 | 2.3 | |
| 3 | 2.50 | 250 | 90 | 3.5 | 3.6 | 3.7 | |
| 4 | 2.50 | 260 | 80 | 2.6 | 2.5 | 2.7 | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |
| 8 | | | | | | | |
| 9 | | | | | | | |
| 10 | | | | | | | |

key the three responses in.....

(2) Defining the Factors



Define Custom Taguchi Design

C1 Time
C2 Temp
C3 Pressure
C4 With Noise 1
C5 With Noise 2
C6 With Noise 3

Factors:
Time-Temperature

Signal Factor
 No signal factor
 Specify by column:

Select OK Cancel

we choose "no signal factor" because we are only testing static designs

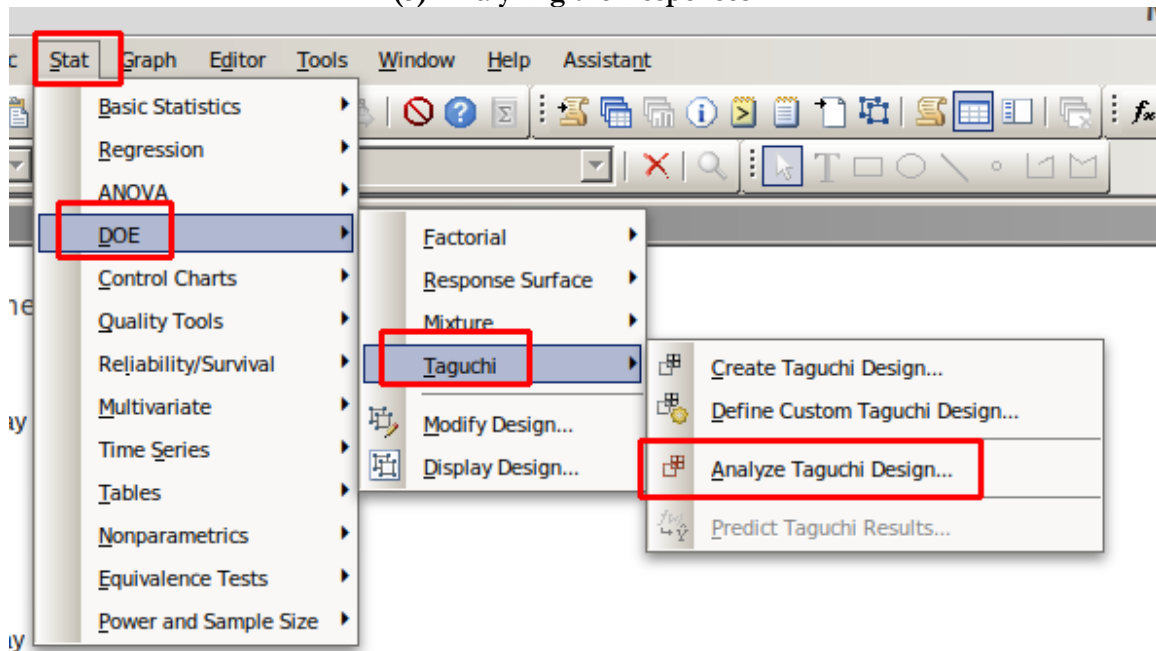
Signal Factor = Dynamic Design = instead of just 2 levels like 1 / 2 or 0 / 1 (like Pressure is set only to 80 or 90)... a Signal Factor can have a range of values like 80...81...82... 83... etc.....

select these three and it will appear as your factors

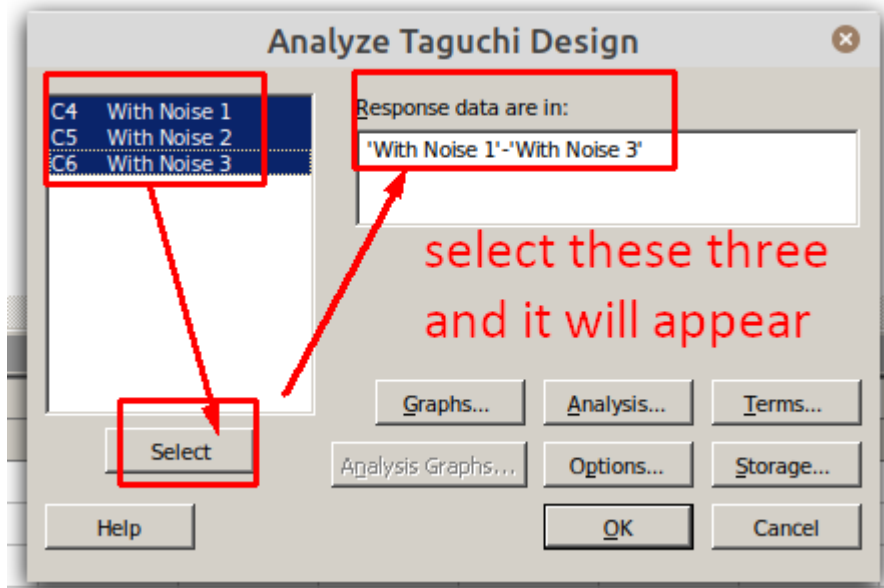
C15 C16 C17 C18 C19 C20 C21

The image shows the 'Define Custom Taguchi Design' dialog box in Minitab. The 'Factors' list on the left contains C1 Time, C2 Temp, and C3 Pressure. The 'Factors' field on the right contains 'Time-Temperature'. The 'Signal Factor' section has the 'No signal factor' radio button selected. Red arrows point from the text annotations to the 'Time-Temperature' field, the 'No signal factor' radio button, and the 'Select' button. The background shows a spreadsheet with columns C15 through C21.

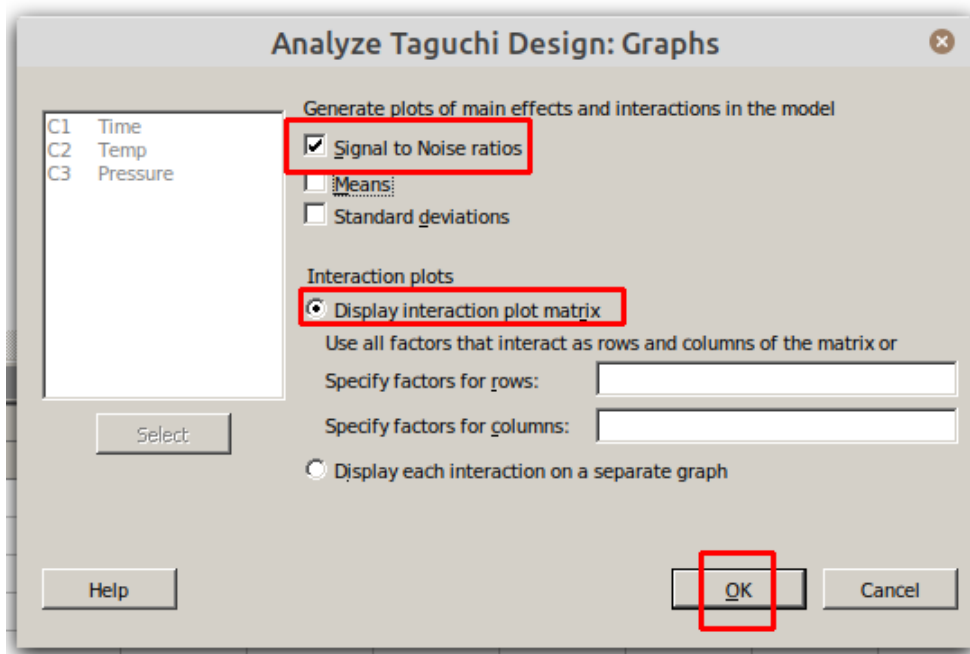
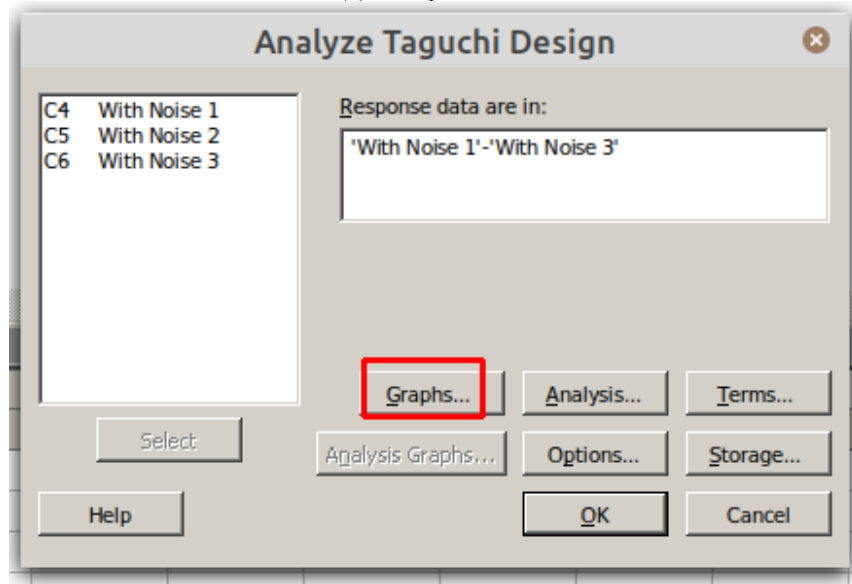
(3) Analyzing the Responses



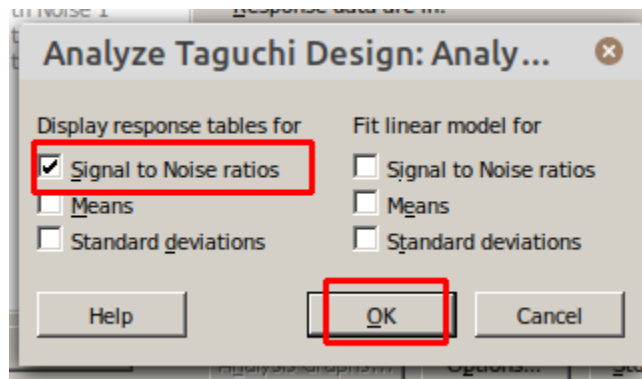
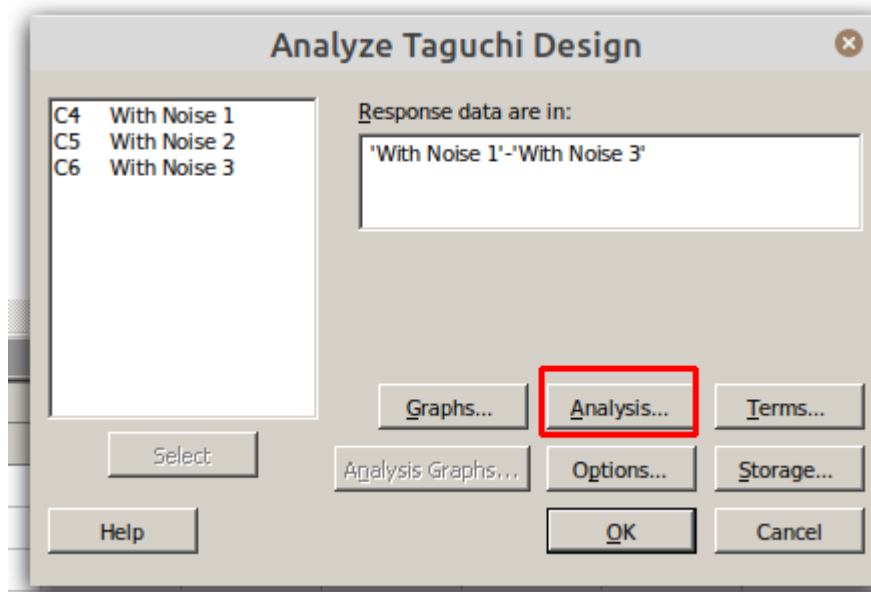
(a) Defining the Responses



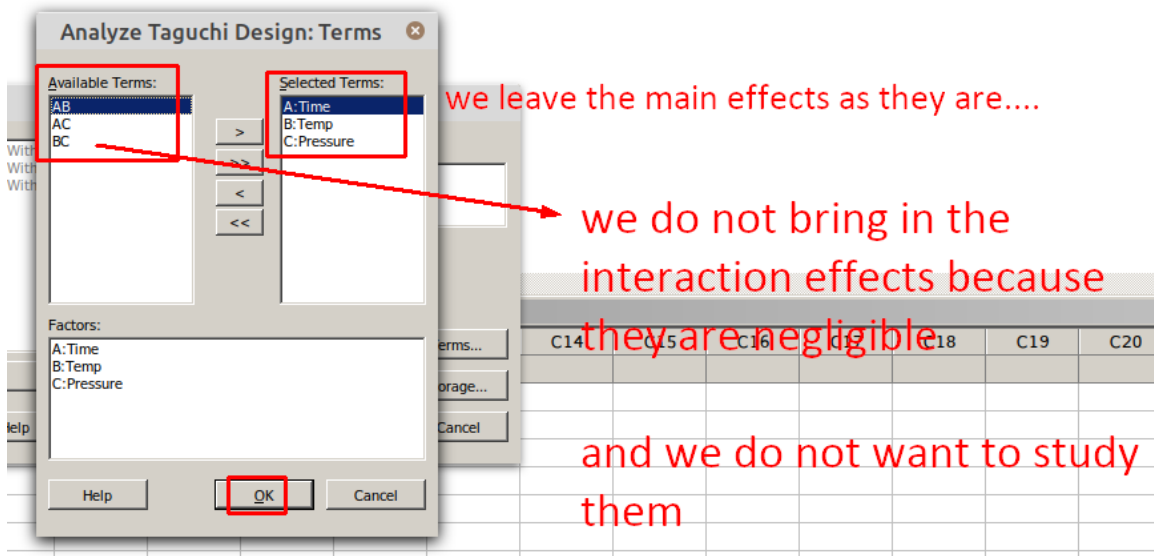
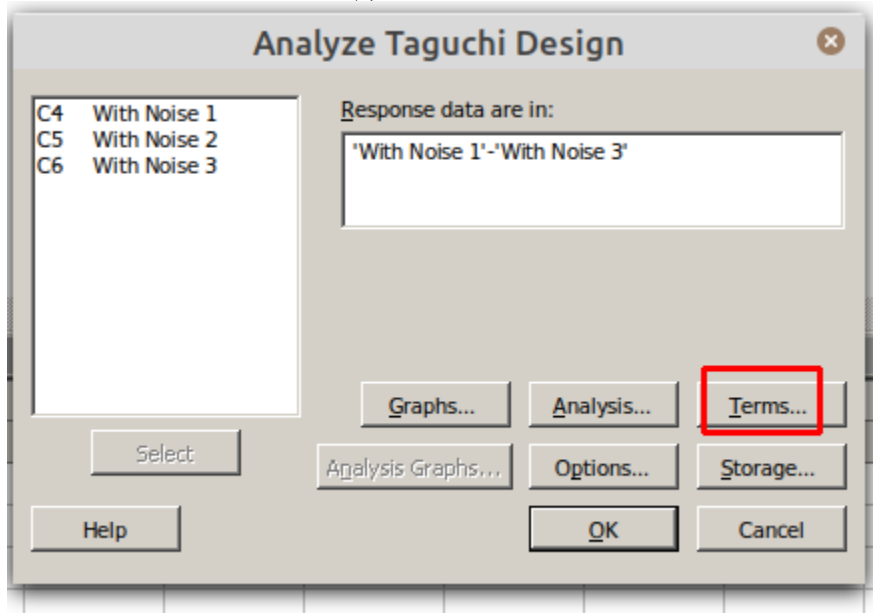
(b) *Graphs*



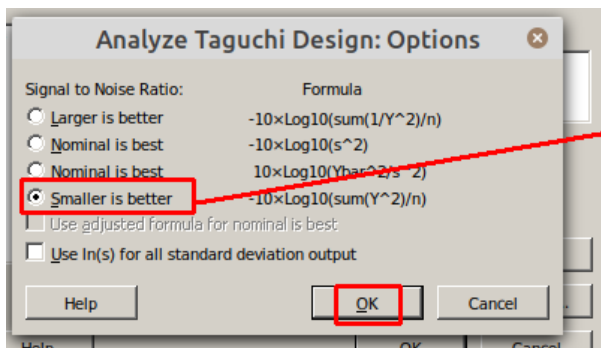
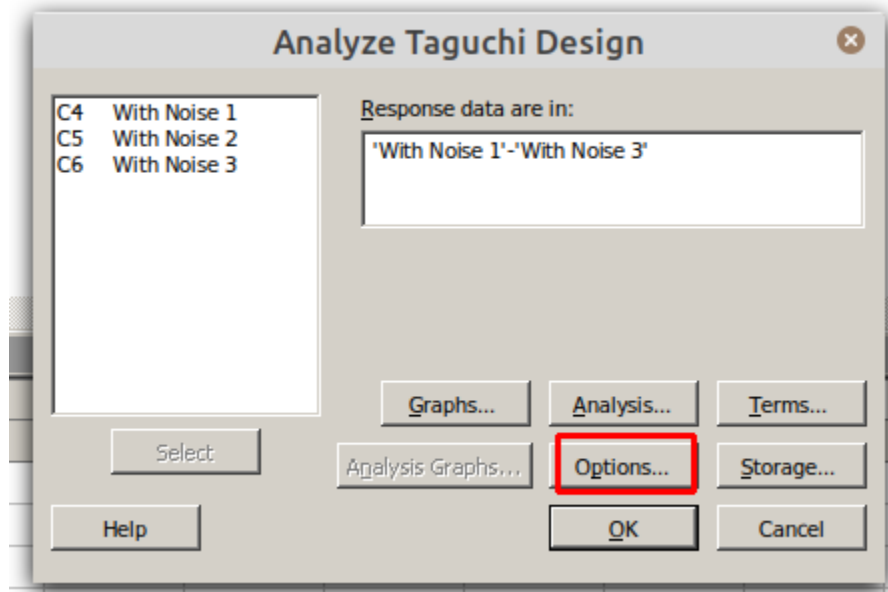
(c) *Analyses*



(d) Terms

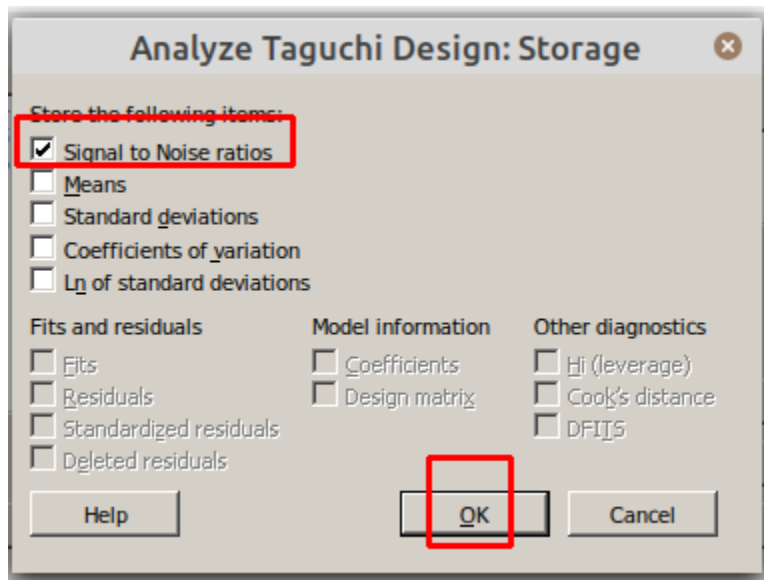
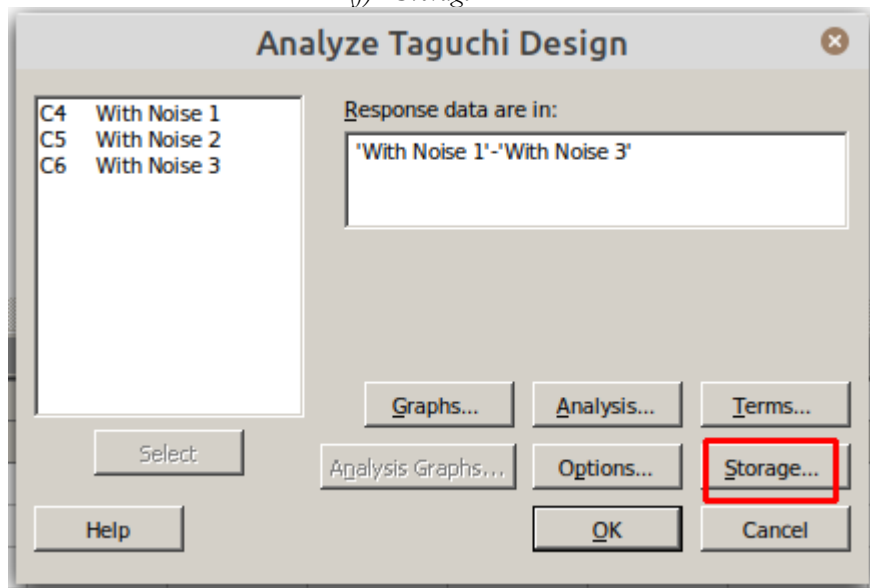


(e) Options

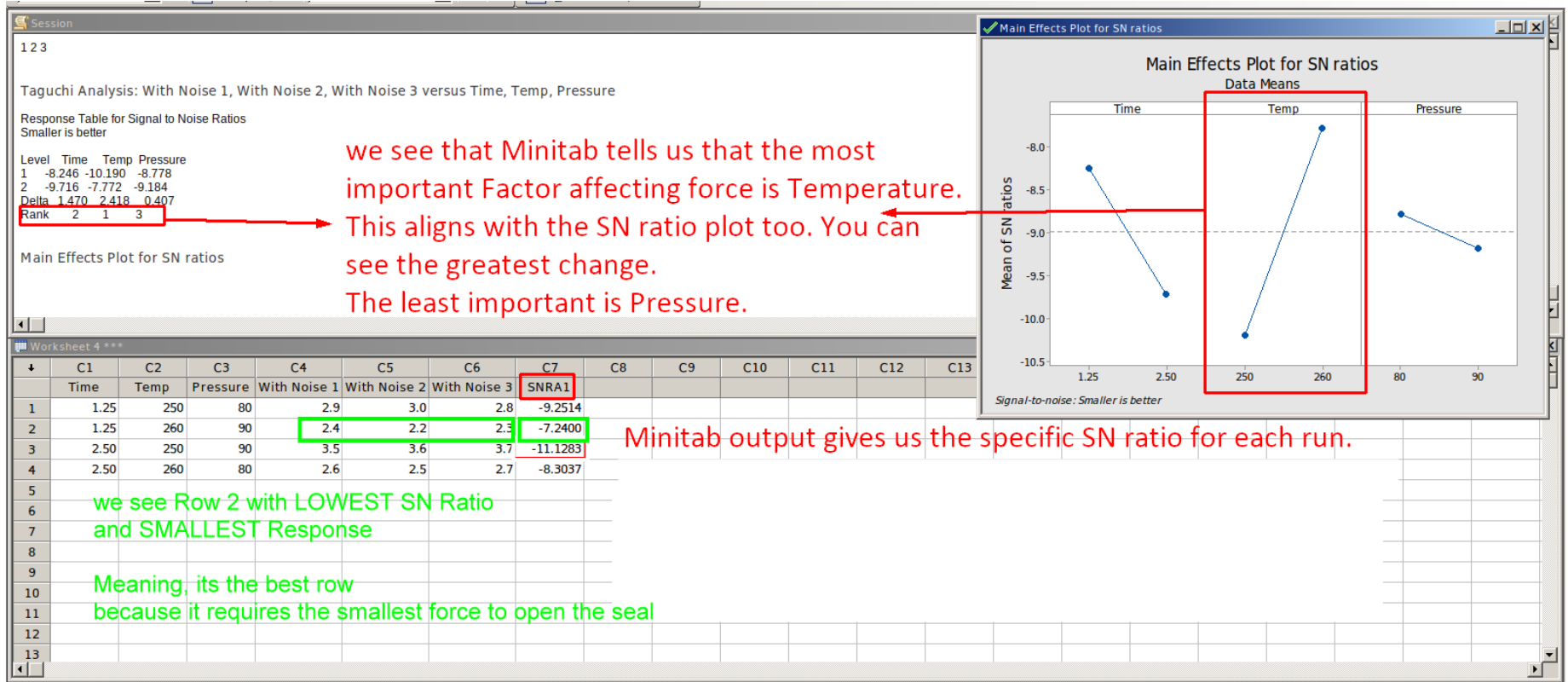


remember earlier we choose smaller is better because we want to minimize the force used to open the package...

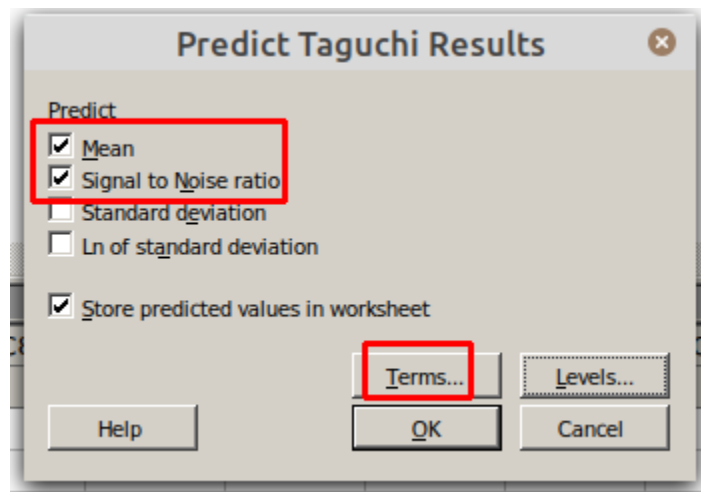
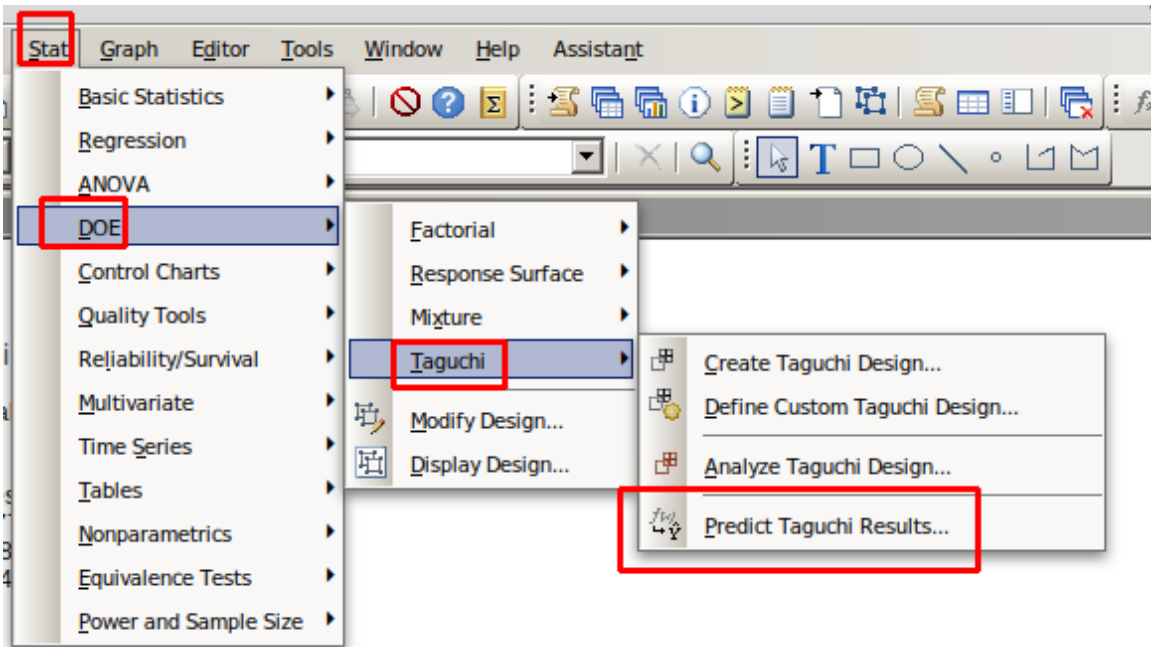
(f) Storage

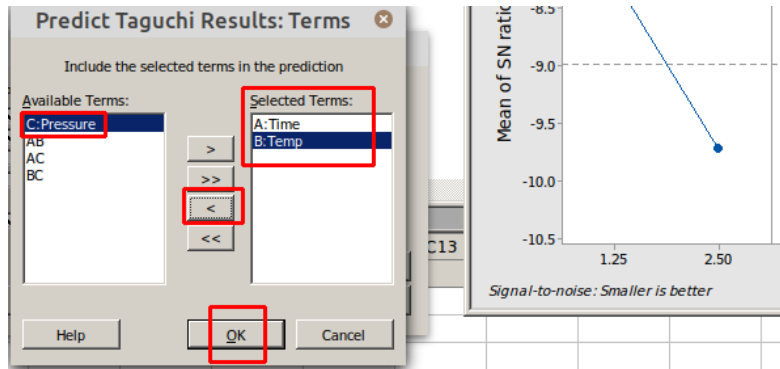


b) Minitab Output

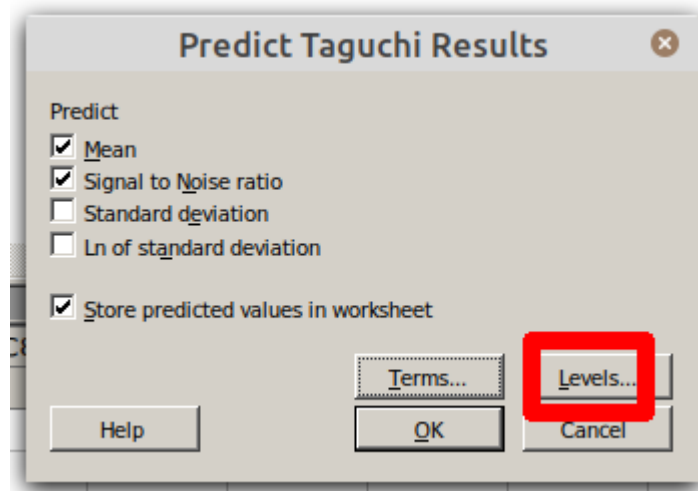


c) Predicting Taguchi Results





as saw earlier, Pressure is the least important Factor. Thus, we remove it from the Prediction.



Predict Taguchi Results: Levels

Specify new levels or factors in

- Uncoded units
- Coded units

Method of specifying new factor levels

- Select variables stored in worksheet
- Select levels from a list

| Factor | Levels |
|--------|--------|
| Time | 1.25 |
| Temp | 260 |

Signal-to-noise: Smaller is better

we are trying to predict the Response (Mean) by setting the Time ~ 1.25 and Temp ~ 260.....(ignoring Pressure)

Taguchi Analysis: With Noise 1, With Noise 2, With Noise 3 versus Time, Temp, Pressure

Predicted values

| | |
|-----------|------|
| S/N Ratio | Mean |
| -7.03674 | 2.2 |

Factor levels for predictions

| | |
|------|------|
| Time | Temp |
| 1.25 | 260 |

we see that the predicted response is 2.25 when Time ~ 1.25 Temp ~ 260

E. SUMMARY OF TAGUCHI'S EXAMPLE

With reference to Example: Minimizing the Response (Smaller is Better), we can repeat this example for:

1. NOMINAL RESPONSE (NOMINAL IS BEST)

| | <i>Temp</i> | <i>Time</i> | <i>Pressure</i> | <i>Polymer</i> |
|----------------|-------------|-------------|-----------------|----------------|
| Level 1 – 1 Lo | 125 | 80 | 2 | 0.5 |
| Level 2 – 2Mid | 150 | 85 | 4 | 1 |
| Level 3 – 3 Hi | 200 | 90 | 6 | 1.5 |

- We want to nominalize the product specifications
- Choose a L9 (4 Factor 3 Levels) Taguchi Experiment

| Run | 1 Temp | 2 Time | 3 Pressure | 4 Polymer | Y ₁ | Y ₂ | Y ₃ | S/N |
|-----|-----------|-----------|---------------|--------------|----------------|----------------|----------------|-------|
| 1 | 1 | 1 | 1 | 1 | 88.2 | 82.4 | 70.3 | 18.88 |
| 2 | 1 | 2 | 2 | 2 | 74.7 | 69.2 | 64.1 | 22.33 |
| 3 | 1 | 3 | 3 | 3 | 56.4 | 53.7 | 44.9 | 18.69 |
| 4 | 2 | 1 | 2 | 3 | 80.2 | 78.9 | 63.2 | 17.88 |
| 5 | 2 | 2 | 3 | 1 | 77.4 | 76.2 | 53.9 | 14.37 |
| 6 | 2 | 3 | 1 | 2 | 88.9 | 88.1 | 82.9 | 28.48 |
| 7 | 3 | 1 | 3 | 2 | 64.3 | 61.9 | 56.1 | 23.17 |
| 8 | 3 | 2 | 1 | 3 | 98.6 | 92.6 | 88.8 | 25.53 |
| 9 | 3 | 3 | 2 | 1 | 75.9 | 73.4 | 62.8 | 20.11 |

- Produce the table above (where Y1, Y2 and Y3 are the Responses with Noise).

2. LARGEST RESPONSE (LARGER IS BETTER)

| | <i>Preheat</i> | <i>Equalize</i> | <i>Austenize</i> | <i>Temper</i> | <i>Quench</i> |
|---------------|----------------|-----------------|------------------|---------------|---------------|
| Level 1 – Low | 1250 | 1350 | 1725 | 750 | 150 |
| Level 2 – Hi | 1450 | 1450 | 1775 | 950 | 200 |

- We want to increase the hardness of steel alloy.
- Choose a L8 (5 Factor 2 Levels) Taguchi Experiment

| Run | 1 Preheat | 2 Equalize | 3 Austenize | 4 Temper | 5 Quench | Y ₁ | Y ₂ | Y ₃ | S/N |
|-----|--------------|---------------|----------------|-------------|-------------|----------------|----------------|----------------|-------|
| 1 | 1 | 1 | 1 | 1 | 1 | 38.0 | 36.3 | 38.3 | 31.48 |
| 2 | 1 | 1 | 1 | 2 | 2 | 39.9 | 56.2 | 35.1 | 32.32 |
| 3 | 1 | 2 | 2 | 1 | 1 | 45.1 | 65.7 | 32.1 | 32.48 |
| 4 | 1 | 2 | 2 | 2 | 2 | 41.5 | 46.8 | 29.7 | 31.40 |
| 5 | 2 | 1 | 2 | 1 | 2 | 62.1 | 53.9 | 61.3 | 35.38 |
| 6 | 2 | 1 | 2 | 2 | 1 | 23.4 | 19.8 | 42.9 | 27.85 |
| 7 | 2 | 2 | 1 | 1 | 2 | 33.7 | 43.0 | 61.8 | 32.51 |
| 8 | 2 | 2 | 1 | 2 | 1 | 31.4 | 47.7 | 31.2 | 30.82 |

- Produce the following table above (where Y1, Y2 and Y3 are the Responses with Noise).
- The S/N ration should be the same as above.

VII. DOE FAQs

A. WHAT IF MANY / ALL FACTORS BECOME SIGNIFICANT?

- There could be outliers or special causes that distort each run.
- Check if there are any unusual values.
- Outliers should be removed or replaced.

B. WHAT IF THERE'S MISSING DATA?

- Replace missing data with average.

C. MUST WE USE ALL FACTORS?

- Yes, try to. it is to your advantage.
- If you don't use them, the unused factors will be called "dummy" factors.

D. WHAT IF I CAN'T REPLICATE ALL RUNS?

- Randomly select runs to be replicated

E. MUST I RANDOMIZE ALL TRIALS?

- No, but it would be best.
- Sometimes it's not possible to re-setup e.g., setting up a furnace temperature is only done once.
- You can't randomize and must carry things out in certain order at one go.
- This may lead to confounding the temperature with another unknown factor. (Due to no randomization).

VIII. SUCCESSFUL STEPS TO IMPLEMENT DOE

1. Define objectives
2. Assemble a small knowledge team
3. Review all pertinent relevant data
4. Brainstorm to generate potential factors.
 - a. Be creative and do not accept existing theories without data.
5. Segregate (from the list) those factors that can be controlled vs uncontrollable.
6. Separate the expensive factors vs cheap factors.
 - a. Always include factors that are cheap, quick and easy to study.
7. For every factor, set levels boldly but NOT carelessly.
 - a. You need levels to be as wide as possible to force effects out of them.
 - b. But you must avoid dangerous or unfeasible conditions.
8. Design your study.
 - a. Narrow down the factors + review the total cost + complexity and control of the experiment + need for replications.
9. Randomize but if you can't, be aware of drawing false conclusions due to unknown external influence affecting the DOE.
10. Run the DOE + ensure correct levels + ensure materials are correct and keep good records.
11. Use Graphs to communicate findings.
 - a. Report conclusions in simple language for the audience, not statistical terminologies.

IX. OBSTACLES TO DOE

A. RESISTANCE INERTIA

- "We have always used OFAT and no time to learn new approach!"
- Answer: You will always get what you have always gotten if you keep doing what you keep doing!

B. EXPENSIVE COST

- Nothing is free.
- DOE is an investment with a payback.
- Normally, the main cost is not Material Cost nor Processing Cost.
- Its the cost of using People's time.
- Use it carefully.

C. LACK OF MANAGEMENT SUPPORT

- Management must be involved.
- They must be educated and understanding what the objectives are, what the costs will be, what is expected, and believe the power of DOE.

D. LACK OF TRAINING

- You need educated people who knows DOE techniques to carry it out.

E. WRONG THINKING THAT DOE IS ONLY USED FOR MANUFACTURING

- DOE can be used for any area that can define factors for study.
- Including strategy and sales.

A. SELECTED FACTORIAL DESIGNS

1. 2^2 FULL FACTORIAL

2^2 Full Factorial

| | A | B |
|----------|----------|----------|
| 1 | - | - |
| 2 | + | - |
| 3 | - | + |
| 4 | + | + |

All terms are free from aliasing.

Full Resolution

2. 2^3 FULL FACTORIAL

2^3 Full Factorial

| | A | B | C |
|---|----------|----------|----------|
| 1 | -1 | -1 | - |
| 2 | 1 | -1 | - |
| 3 | -1 | 1 | - |
| 4 | 1 | 1 | - |
| 5 | -1 | -1 | + |
| 6 | 1 | -1 | + |
| 7 | -1 | 1 | + |
| 8 | 1 | 1 | + |

All terms are free from aliasing.

Full Resolution

3. 2^3 HALF FRACTIONAL FACTORIAL

2^3 Half Fractional Factorial

| | A | B | C |
|----------|----------|----------|----------|
| 1 | - | - | + |
| 2 | + | - | - |
| 3 | - | + | - |
| 4 | + | + | + |

I + ABC

A + BC

B + AC

C + AB

Resolution III

F. SELECTED PLACKET BURMAN DESIGNS

Eight-Run 2^7 Plackett-Burman

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | + | - | - | + | - | + | + |
| 2 | + | + | - | - | + | - | + |
| 3 | + | + | + | - | - | + | - |
| 4 | - | + | + | + | - | - | + |
| 5 | + | - | + | + | + | - | - |
| 6 | - | + | - | + | + | + | - |
| 7 | - | - | + | - | + | + | + |
| 8 | - | - | - | - | - | - | - |

12-Run 2^{11} Plackett-Burman

| | A | B | C | D | E | F | G | H | I | J | K |
|----|---|---|---|---|---|---|---|---|---|---|---|
| 1 | + | - | + | - | - | - | + | + | + | - | + |
| 2 | + | + | - | + | - | - | - | + | + | + | - |
| 3 | - | + | + | - | + | - | - | - | + | + | + |
| 4 | + | - | + | + | - | + | - | - | - | + | + |
| 5 | + | + | - | + | + | - | + | - | - | - | + |
| 6 | + | + | + | - | + | + | - | + | - | - | - |
| 7 | - | + | + | + | - | + | + | - | + | - | - |
| 8 | - | - | + | + | + | - | + | + | - | + | - |
| 9 | - | - | - | + | + | + | - | + | + | - | + |
| 10 | + | - | - | - | + | + | + | - | + | + | - |
| 11 | - | + | - | - | - | + | + | + | - | + | + |
| 12 | - | - | - | - | - | - | - | - | - | - | - |

16-Run 2^{15} Plackett-Burman

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | + | - | - | - | + | - | - | + | + | - | + | - | + | + | + |
| 2 | + | + | - | - | - | + | - | - | + | + | - | + | - | + | + |
| 3 | + | + | + | - | - | - | + | - | - | + | + | - | + | - | + |
| 4 | + | + | + | + | - | - | - | + | - | - | + | + | - | + | - |
| 5 | - | + | + | + | + | - | - | - | + | - | - | + | + | - | + |
| 6 | + | - | + | + | + | + | - | - | - | + | - | - | + | + | - |
| 7 | - | + | - | + | + | + | + | - | - | - | + | - | - | + | + |
| 8 | + | - | + | - | + | + | + | + | - | - | - | + | - | - | + |
| 9 | + | + | - | + | - | + | + | + | + | - | - | - | + | - | - |
| 10 | - | + | + | - | + | - | + | + | + | + | - | - | - | + | - |
| 11 | - | - | + | + | - | + | - | + | + | + | + | - | - | - | + |
| 12 | + | - | - | + | + | - | + | - | + | + | + | + | - | - | - |
| 13 | - | + | - | - | + | + | - | + | - | + | + | + | + | - | - |
| 14 | - | - | + | - | - | + | + | - | + | - | + | + | + | + | - |

G. SELECTED TAGUCHI DESIGNS

Taguchi L4 (2^3)

| | 1 | 2 | 3 |
|----------|----------|----------|----------|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 |
| 3 | 2 | 1 | 2 |
| 4 | 2 | 2 | 1 |

| | 1 | 2 | 3 |
|----------|----------|----------|----------|
| 1 | | 3 | 2 |
| 2 | | | 1 |

Taguchi L8 (2^7)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 3 | 1 | 2 | 2 | 1 | 1 | 2 | 2 |
| 4 | 1 | 2 | 2 | 2 | 2 | 1 | 1 |
| 5 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| 6 | 2 | 1 | 2 | 2 | 1 | 2 | 1 |
| 7 | 2 | 2 | 1 | 1 | 2 | 2 | 1 |
| 8 | 2 | 2 | 1 | 2 | 1 | 1 | 2 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | | | 1 | 6 | 7 | 4 | 5 |
| 3 | | | | 7 | 6 | 5 | 4 |
| 4 | | | | | 1 | 2 | 3 |
| 5 | | | | | | 3 | 2 |
| 6 | | | | | | | |
| 7 | | | | | | | |

Taguchi L8 (1^4 with up to 2^4)

| | 1 | 2 | 3 | 4 | 5 |
|----------|----------|----------|----------|----------|----------|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 | 2 |
| 3 | 2 | 1 | 1 | 2 | 2 |
| 4 | 2 | 2 | 2 | 1 | 1 |
| 5 | 3 | 1 | 2 | 1 | 2 |
| 6 | 3 | 2 | 1 | 2 | 1 |
| 7 | 4 | 1 | 2 | 2 | 1 |
| 8 | 4 | 2 | 1 | 1 | 2 |

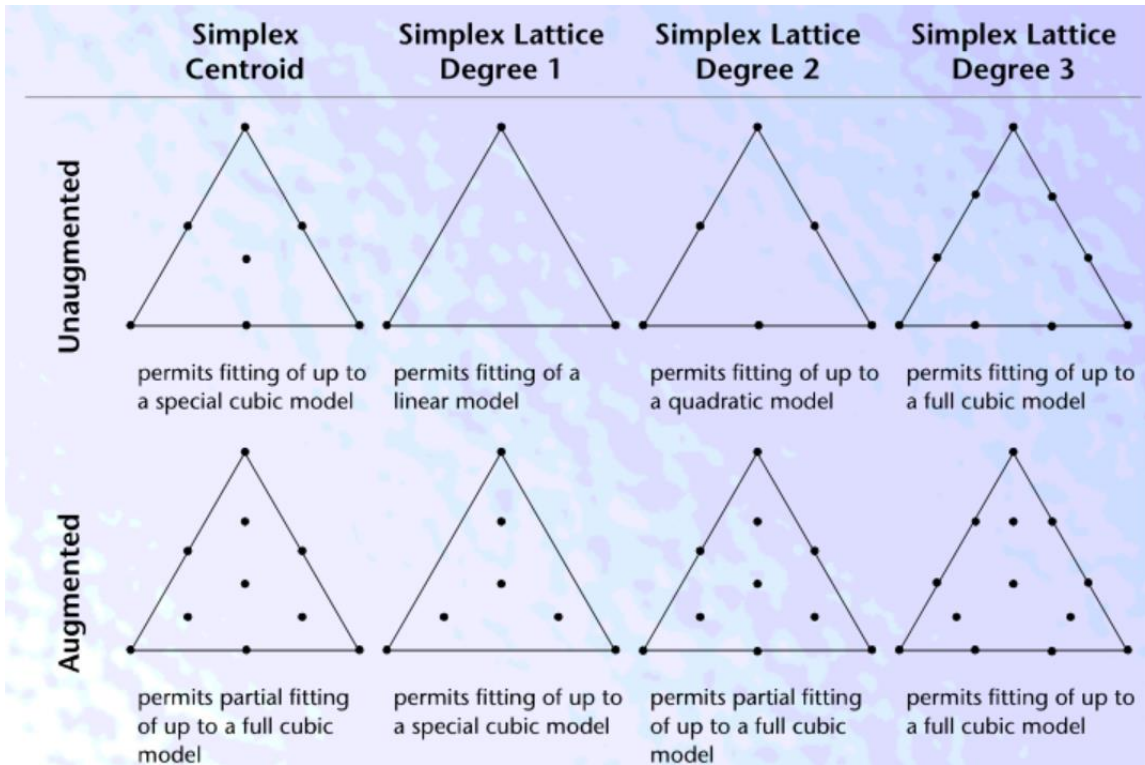
The L8 (1^4 with up to 2^4) array does not have an interaction table.

Taguchi L9 (3⁴)

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 |
| 4 | 2 | 1 | 2 | 3 |
| 5 | 2 | 2 | 3 | 1 |
| 6 | 2 | 3 | 1 | 2 |
| 7 | 3 | 1 | 3 | 2 |
| 8 | 3 | 2 | 1 | 3 |
| 9 | 3 | 3 | 2 | 1 |

The L9 (3⁴) array does not have an interaction table.

H. SELECTED MIXTURE DESIGNS



Three-Factor Simplex Centroid Design

| Run | A | B | C |
|-----|-------|-------|-------|
| 1 | 1 | 0 | 0 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 |
| 4 | 0.5 | 0.5 | 0 |
| 5 | 0.5 | 0 | 0.5 |
| 6 | 0 | 0.5 | 0.5 |
| 7 | 0.333 | 0.333 | 0.333 |

Three-Factor Simplex Lattice Design

| Run | A | B | C |
|-----|-------|-------|-------|
| 1 | 1 | 0 | 0 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 |
| 4 | 0.333 | 0.333 | 0.333 |
| 5 | 0.667 | 0.167 | 0.167 |
| 6 | 0.167 | 0.667 | 0.167 |
| 7 | 0.167 | 0.167 | 0.667 |

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ABOUT DR. ALVIN ANG



Dr. Alvin Ang earned his Ph.D., Masters and Bachelor degrees from NTU, Singapore. He is a scientist, entrepreneur, as well as a personal/business advisor. More about him at www.AlvinAng.sg.