# FUNCTIONS

DR. ALVIN ANG

# CONTENTS

## PART I

## RELATIONS VS FUNCTIONS

### A. FUNCTION

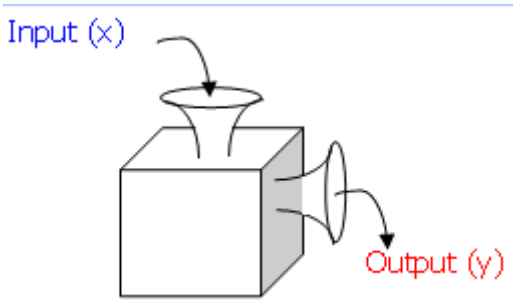



*Figure 1: Function "Machine"*

- One input (x) can only have one output (y)
- One input (x) ***cannot*** have > 1 output (y)
- Similar to saying: For every ***vertical*** line (i.e. x=a) drawn, it can only cut the graph ***f (x)*** once.
- Example 1: $f(x) = x$ is a ***function.*** Example 2:



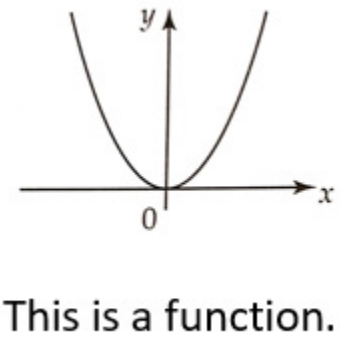
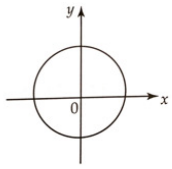



*Figure 2: Example of Function*

## B. RELATION

- One input (x) can have > 1 output (y)
- Similar to saying: For every vertical line (i.e. x=a) drawn, it cuts the graph > once.
- Example: $f^2(x) = x$ is a ***relation, NOT a function***
- Example 2:




This is NOT a function.
It is a relation.

*Figure 3: Example of Relation*

## C. DOMAIN AND RANGE



- $D_f$: Domain of a function (refers to the x axis)
- $R_f$: Range of a function (refers to the y axis)

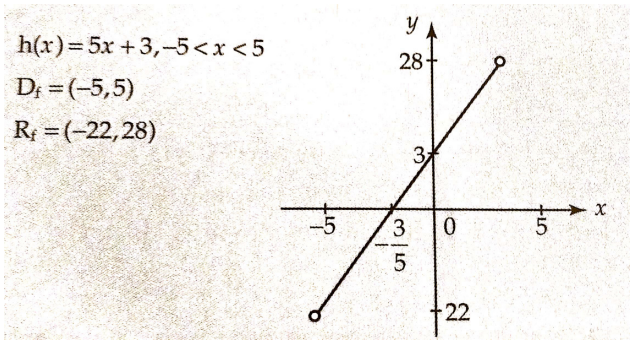



*Figure 4: Domain and Range (Khin 2019)*

## D. DEFINING INEQUALITY

| Inequality form | Set Notation |
|---|---|
| $a < x < b$ | $(a,b) = \{x \in \mathbb{R} \mid a < x < b\}$ |
| $a \leq x \leq b$ | $[a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$ |
| $a \leq x < b$ | $[a,b) = \{x \in \mathbb{R} \mid a \leq x < b\}$ |
| $x \geq a$ | $[a,\infty) = \{x \in \mathbb{R} \mid x \geq a\}$ |

*Figure 5: Inequalities Definition (Khin 2019)*



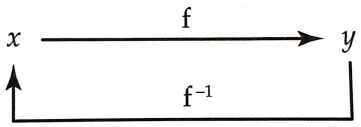
## E. POINT TO NOTE

- Presume f(x) = x, -1 < x < 1
- Presume g(x) = x, -2 < x < 2
- It may appear that f(x) and g(x) are the same function, but they are NOT.
- Because their Domains are different.

## PART II

## INVERSE FUNCTIONS

### A. WHAT IS AN INVERSE FUNCTION?



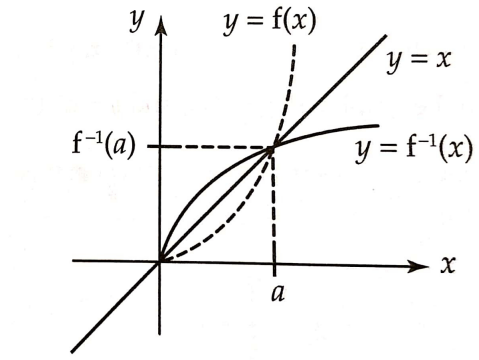




*Figure 6: Inverse Function (Khin 2019)*



- $f^{-1}(x)$ is a reflection of $f(x)$ about the $y = x$ line.

Let $y = f(x) = \sqrt{1-x}$,
$\Rightarrow 1 - x = y^2$
$\Rightarrow x = 1 - y^2$
$\Rightarrow f^{-1}(x) = 1 - x^2$

*Figure 7: Example of Inverse Function (Khin 2019)*

## B. EXISTENCE TEST

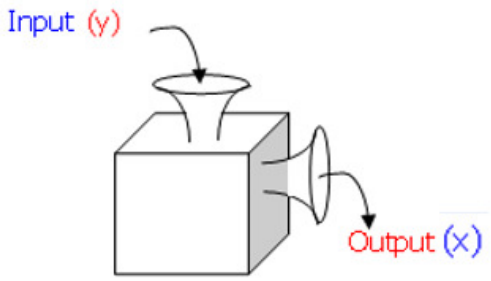



*Figure 8: Swapping the Function Machine*

- One input (y) can only have one output (x)
- One input (y) ***cannot*** have > 1 output (x)
- Similar to saying: For every ***horizontal*** line (i.e. y=a) drawn, it can only cut the graph ***of f (x)*** once.
- I.e. if every ***horizontal*** line cut the graph ***of f (x) only*** once. → f (x) is a one to one function → thus ***$f^{1}$ (x)*** exists!

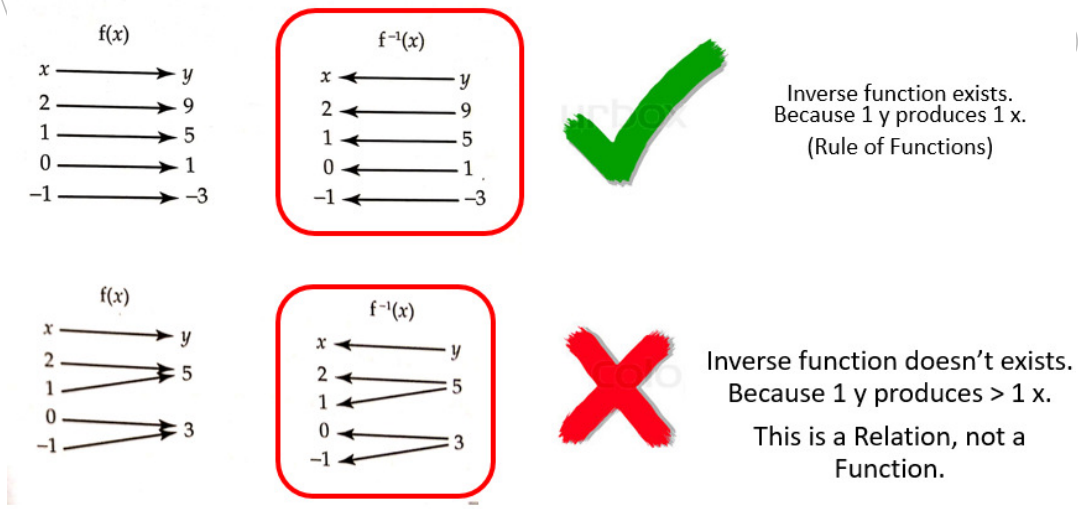



*Figure 9: Existence Test of Inverse Function*

## C. DOMAIN AND RANGE OF INVERSE FUNCTION

$$R_f = D_{f^{-1}}$$

- The Range of the Function (Y axis) = Domain of an Inverse Function (Y axis)

$$D_f = R_{f^{-1}}$$

- The Domain of the Function (X axis) = Range of an Inverse Function (X axis)
- Example:

$$g(x) = x + 1\,,\ -5 < x < 5$$

$$g^{-1}(x) = x - 1\,,\ -4 < x < 6$$

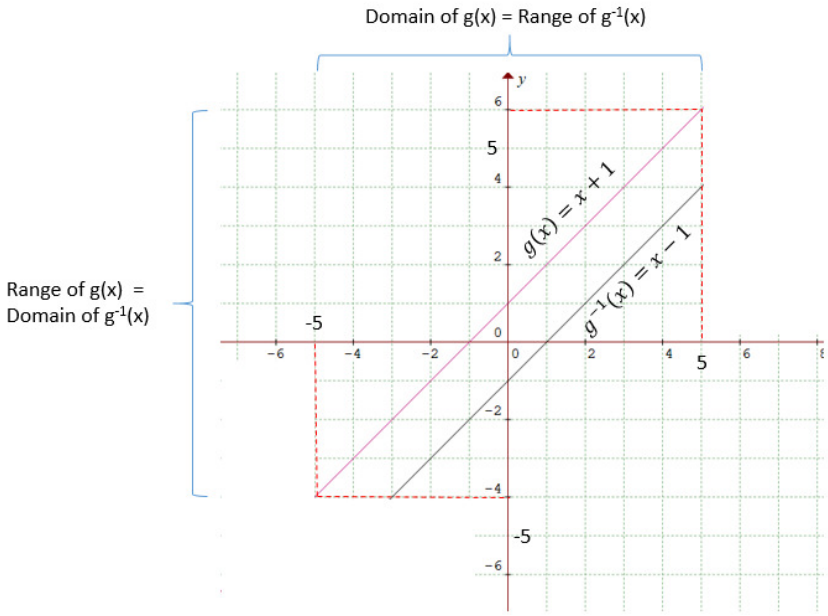



*Figure 10: Example of Domain and Range of Inverse Functions*

$$R_g = D_{g^{-1}} = (-4, 6)$$

$$D_g = R_{g^{-1}} = (-5, 5)$$

## PART III

## COMPOSITE FUNCTIONS

### A. WHAT ARE COMPOSITE FUNCTIONS?

Example of a Composite Function:

g (f(x))

- f (x) = x + 1, -2 < x < 2
- g (x) = x + 1, -4 < x < 4
- Thus g (f(x)) = (x+1) + 1
    - = x + 2, -2 < x < 2

### B. RULES OF COMPOSITE FUNCTIONS



1) $D_{f(x)} = D_{g(f(x))}$
2) $R_{f(x)} \subset D_{g(x)}$ → (Just check for this to show that the Composite Function exists.)
3) $R_{g(f(x))} \subset R_{g(x)}$
4) Any violation of the above rules = the Composite Function does NOT exist.

Example:

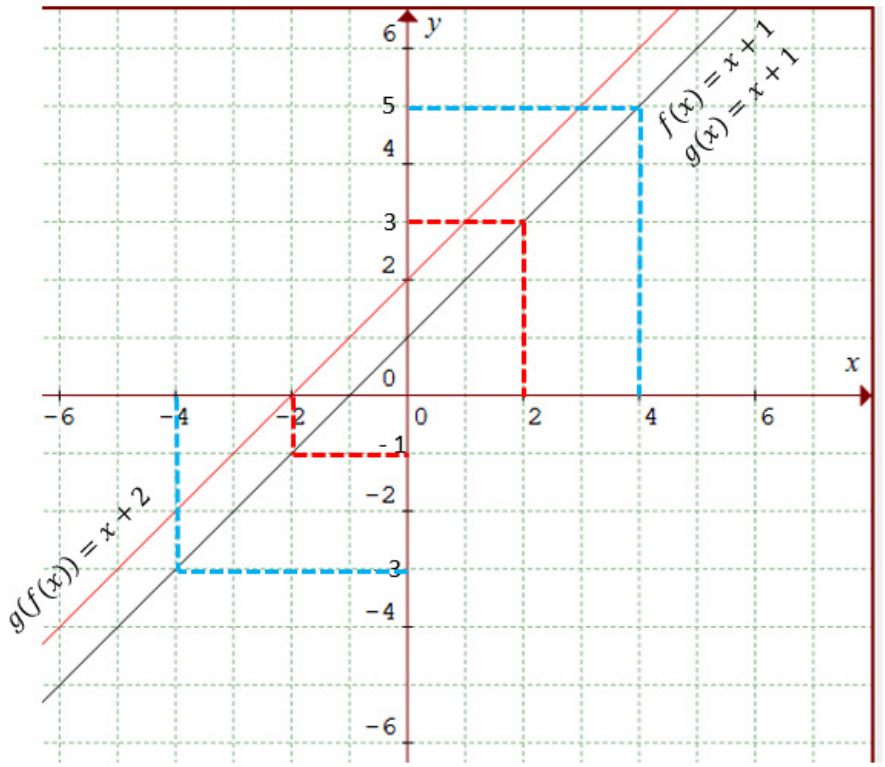



*Figure 11: The Composite Function of g (f(x)) exists.*



### C. SPECIAL CASE: COMPOSING A FUNCTION WITH ITS INVERSE

Rules:

- $f\, f^{-1}(x) = f^{-1}\, f(x) = x$
- Domain of $f^{-1}\, f(x)$ = Domain of $f(x)$
- Domain of $f\, f^{-1}(x)$ = Domain of $f^{-1}(x)$

## REFERENCES

Khin, S. B. (2019). Effective Guide (H2) Mathematics, Fairfield Book Publishers.

### ABOUT MR SONG BOON KHING

Mr. Song Boon Khing graduated from NUS with a Bachelor of Science (2nd Upper Hons) degree, majoring in Applied Mathematics. Imbued with the passion to help and positively influence the young, Mr. Song applied and was awarded the MOE teaching award after graduating from Hwa Chong Junior College. Upon receiving his Post Graduate Diploma in Education (PGDE) with Credit, Mr. Song taught at National Junior College (NJC), teaching H1 and H2 A Level Mathematics.

### ABOUT DR. ALVIN ANG

Dr. Alvin Ang earned his Ph.D., Masters and Bachelor degrees from NTU, Singapore. He is a scientist, entrepreneur, as well as a personal/business advisor. More about him at www.AlvinAng.sg.