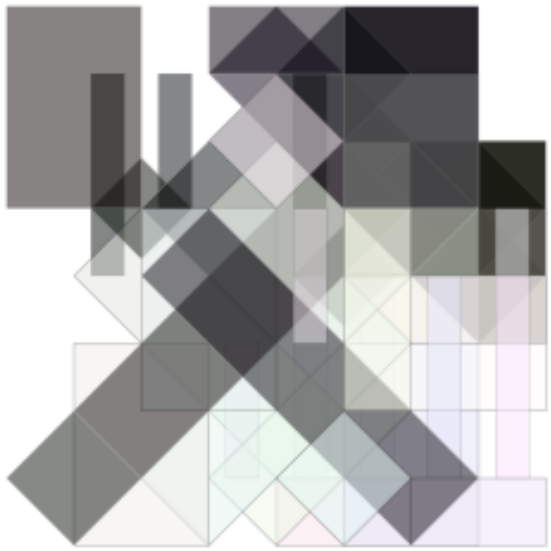
GRAPHING

DR. ALVIN ANG



CONTENTS

PART I

CARTESIAN VS PARAMETRIC

- Example of Cartesian Equation: $y = 2x$
- Example of Parametric Equation:
    - $x = t^2$
    - $y = t^3$

PART II

COMMON FEATURES OF GRAPHS

1. AXIAL INTERCEPTS

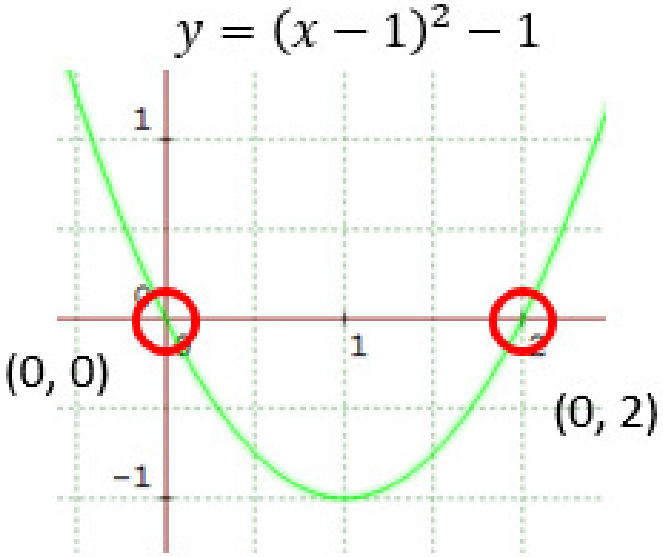


Figure 1: Axial Intercepts

- Graph intercepts x-axis at x = 0 and x = 2 (when y = 0)
- Graph intercepts y-axis at y = 0 when x = 0

2. STATIONARY POINT

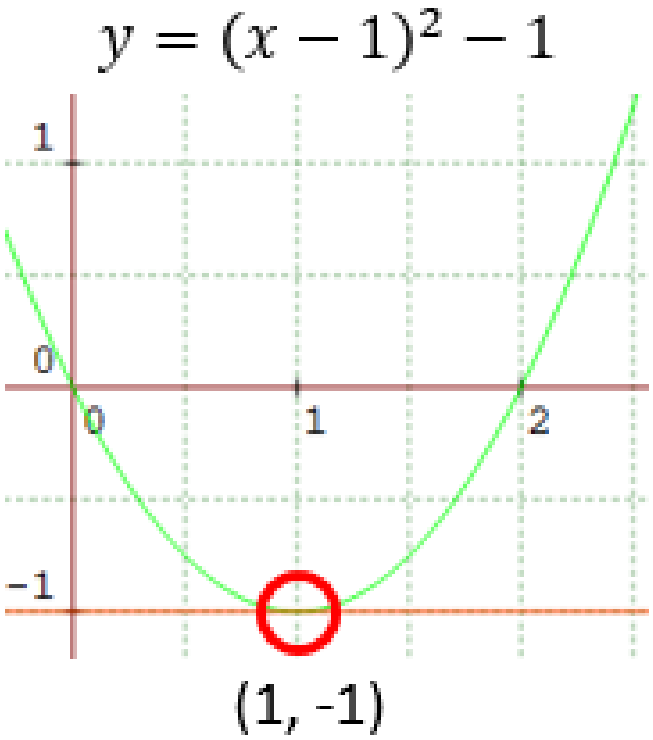


Figure 2: Stationary Point

- Stationary Point is where $\frac{dy}{dx} = 0$.
- Over here is at (1, -1)

3. ASYMPTOTE

HORIZONTAL ASYMPTOTE

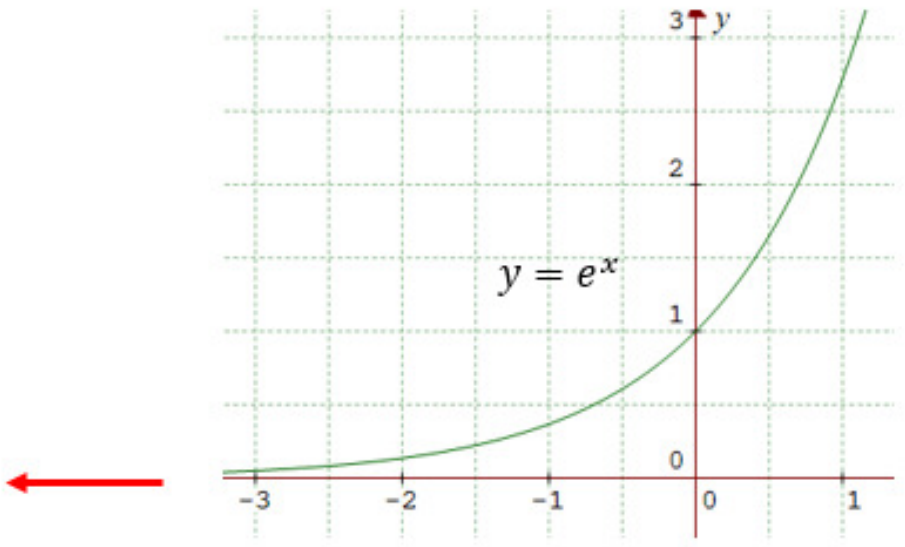


Figure 3: Horizontal Asymptote

- As $x \to -\infty$, $y \to 0$.
- Thus the Horizontal Asymptote is: y = 0.

VERTICAL ASYMPTOTE

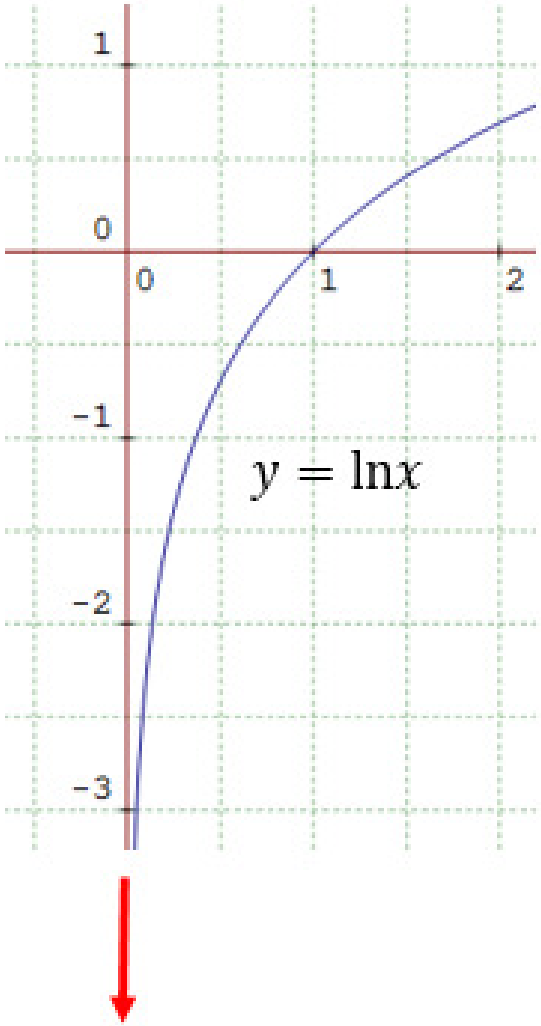


Figure 4: Vertical Asymptote

- As $x \to 0$, $y \to -\infty$
- Thus the Vertical Asymptote is x = 0

OBLIQUE ASYMPTOTE

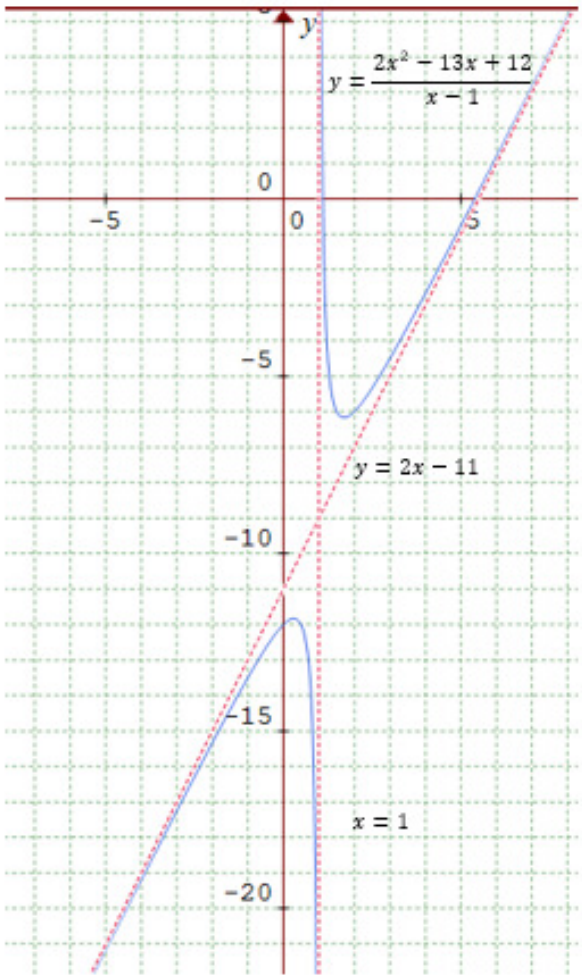


Figure 5: Oblique Asymptote

- The Vertical Asymptote is x = 1
- The Oblique Asymptote is y = 2x – 11
- Special Note:
    - We obtain Vertical Asymptote easily because denominator is x-1
    - But how to obtain Oblique Asymptote?

- Given $y = \dfrac{2x^2 - 13x + 12}{x - 1}$

- $$x-1\overline{)2x^2-13x+12}$$
$$\begin{array}{r} 2x-11 \\ \hline 2x^2-13x+12 \\ -\left(2x^2-2x\right) \\ \hline -11x+12 \\ -\left(-11x+11\right) \\ \hline 1 \end{array}$$

- Now $y = 2x - 11 + \dfrac{1}{x-1}$

- Notice that 2x-11 is the oblique asymptote here!

PART III

CONICS

Conics comprise of:

1. Circles
2. Ellipses
3. Parabolas
4. Hyperbolas

A. CIRCLES

$$(x-h)^2+(y-k)^2=r^2, r\neq 0.$$

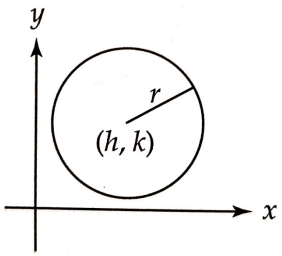


Figure 6: General Equation for Circles (Khin 2019)

B. ELLIPSES

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,\ a,b\neq 0.$$

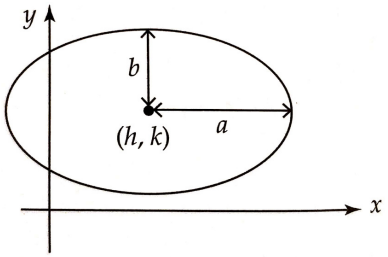


Figure 7: General Equation for Ellipses (Khin 2019)

C. PARABOLAS

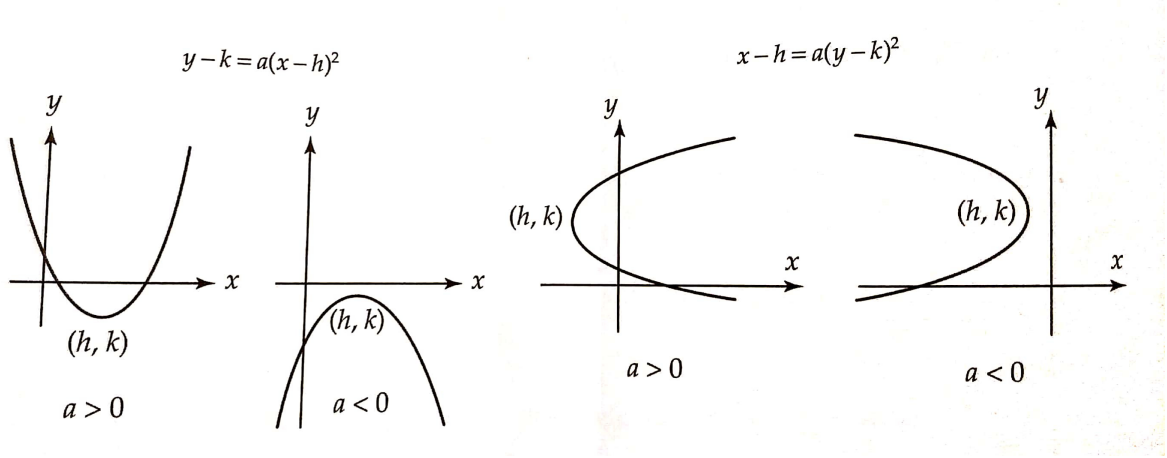


Figure 8: General Equation for Parabolas (Khin 2019)

D. HYPERBOLAS

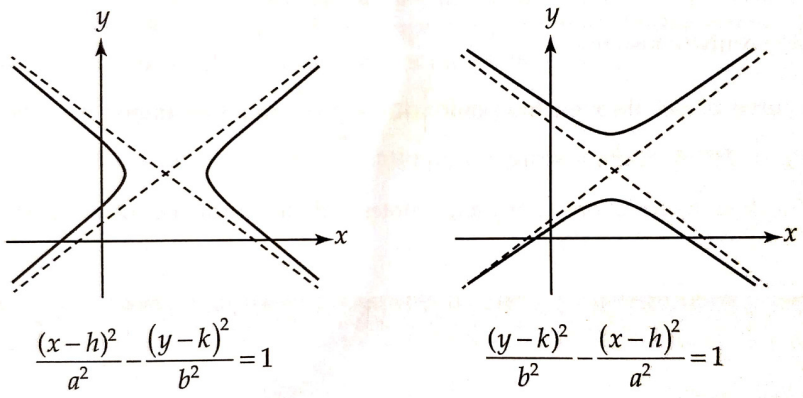


Figure 9: General Equation for Hyperbolas (Khin 2019)

- General Equation for the Asymptotes of the Hyperbolas: $\frac{x-h}{a}=\pm\frac{y-k}{b}$

PART IV

GRAPH TRANSFORMATIONS

A. TRANSLATION

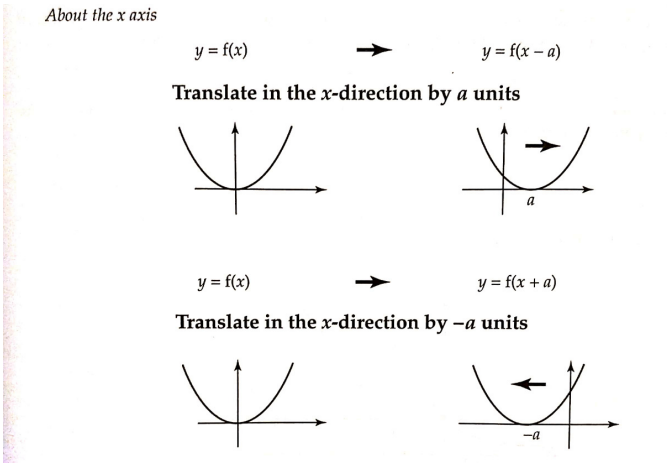


Figure 10: Translating Graphs about the X Axis (Khin 2019)

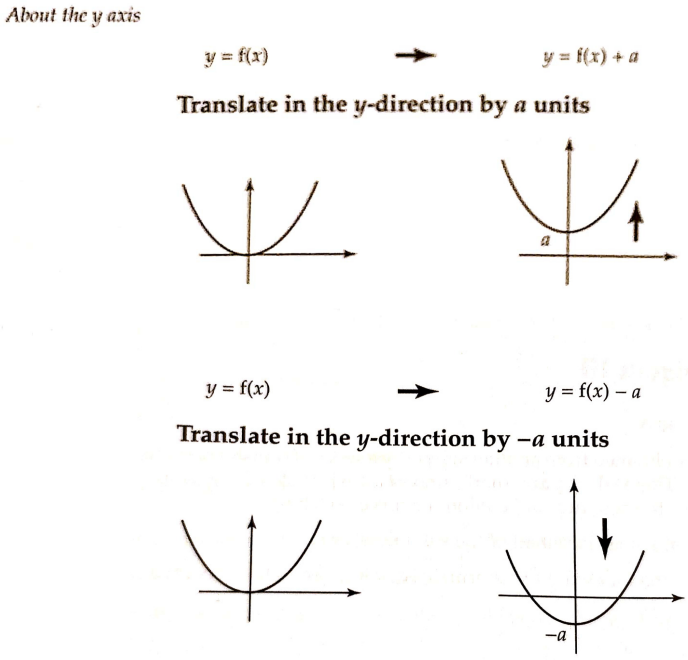


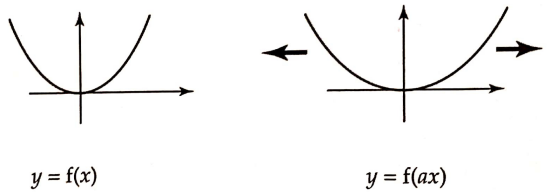
Figure 11: Translating Graph about the Y Axis (Khin 2019)

B. SCALING

About the x axis

$y = f(x) \rightarrow y = f\left(\frac{x}{a}\right)$

Scale along the x-direction by a factor of a



$y = f(x)$ $\qquad$ $y = f(ax)$

Scale along the x-direction by a factor of $\frac{1}{a}$

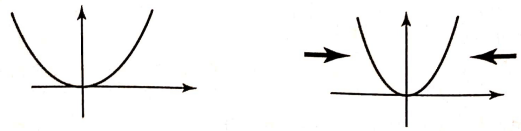
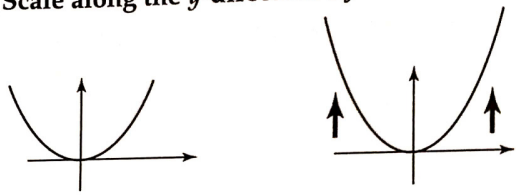


Figure 12: Scaling a Graph About the X Axis (Khin 2019)

About the y axis

$y = f(x) \rightarrow y = a\,f(x)$

Scale along the y-direction by a factor of a



$y = f(x)$ $\qquad$ $y = \frac{1}{a} f(x)$

Scale along the y-direction by a factor of $\frac{1}{a}$

Figure 13: Scaling a Graph About the Y Axis (Khin 2019)

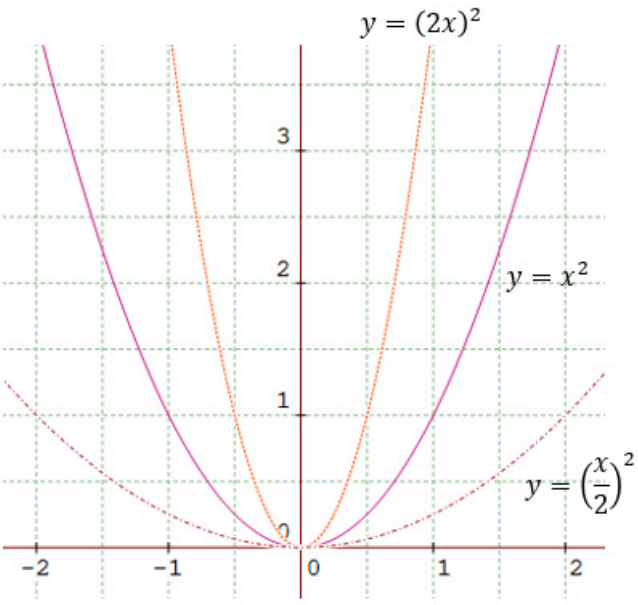


Figure 14: Example of Scaling about the X Axis

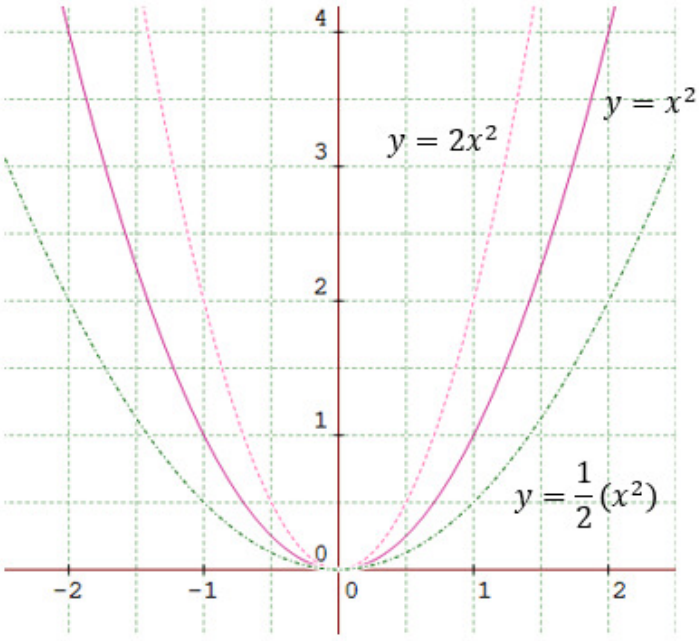


Figure 15: Example of Scaling about the Y Axis

C. REFLECTION

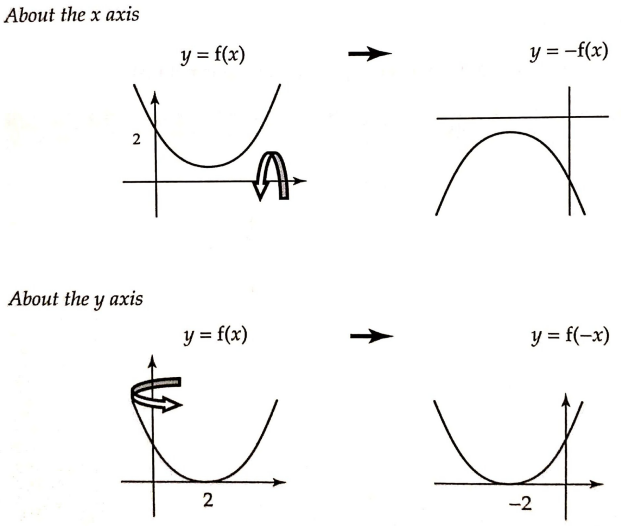


Figure 16: Reflection about the Axis (Khin 2019)

D. MODULUS

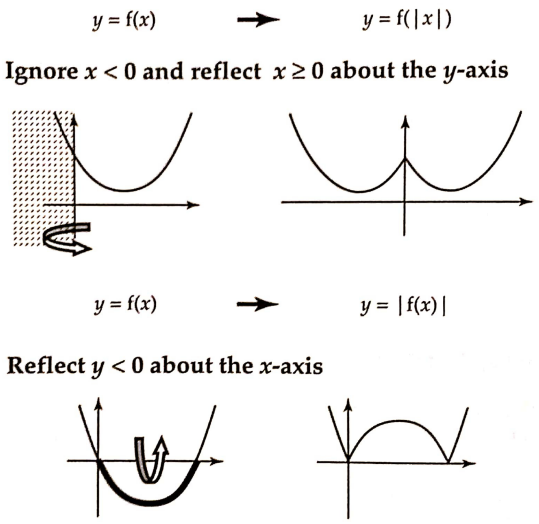


Figure 17: Modulus about the Axis (Khin 2019)

PART V

ADVANCED GRAPH TRANSFORMATIONS

A. COMPOUND / MULTIPLE

- Since we have learnt Translation, Scaling and Reflection earlier, we will now combine them.
- But there must be an order to how it is done.
- Transformation in ***sequence*** of ***Priority***

1. For X
    - Addition
    - Multiplication
2. For Y
    - Multiplication
    - Addition

Example:

- Given y = f (x)
- Sketch y = 2 f(-3x + 4) – 5
- For X:
    - Step 1: Convert $f(x) \to f(x+4)$, then sketch
    - Step 2: Convert $f(x+4) \to f(3x+4)$, then edit the sketch
    - Step 3: Convert $f(3x+4) \to f(-3x+4)$, then edit the sketch
- For Y:
    - Step 4: Convert $f(-3x+4) \to 2f(-3x+4)$, then edit the sketch

- Step 5: Convert $2f\left(-3x+4\right)\rightarrow 2f\left(-3x+4\right)-5$, then edit the sketch

B. SQUARE ROOT[1]

- Example given $y=f\left(x\right)$
- Sketch $y^2=f\left(x\right)$
- Steps to sketch "Square Root" are outlined below...

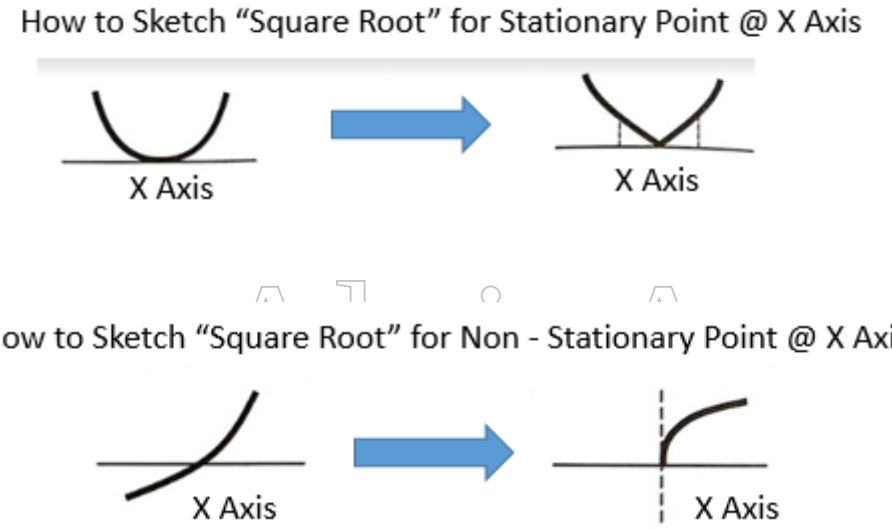


Figure 18: Sketching "Square Root" at the X Axis (Khin 2019)

[1] Square Root Graphing has been EXCLUDED from Exams since 2017

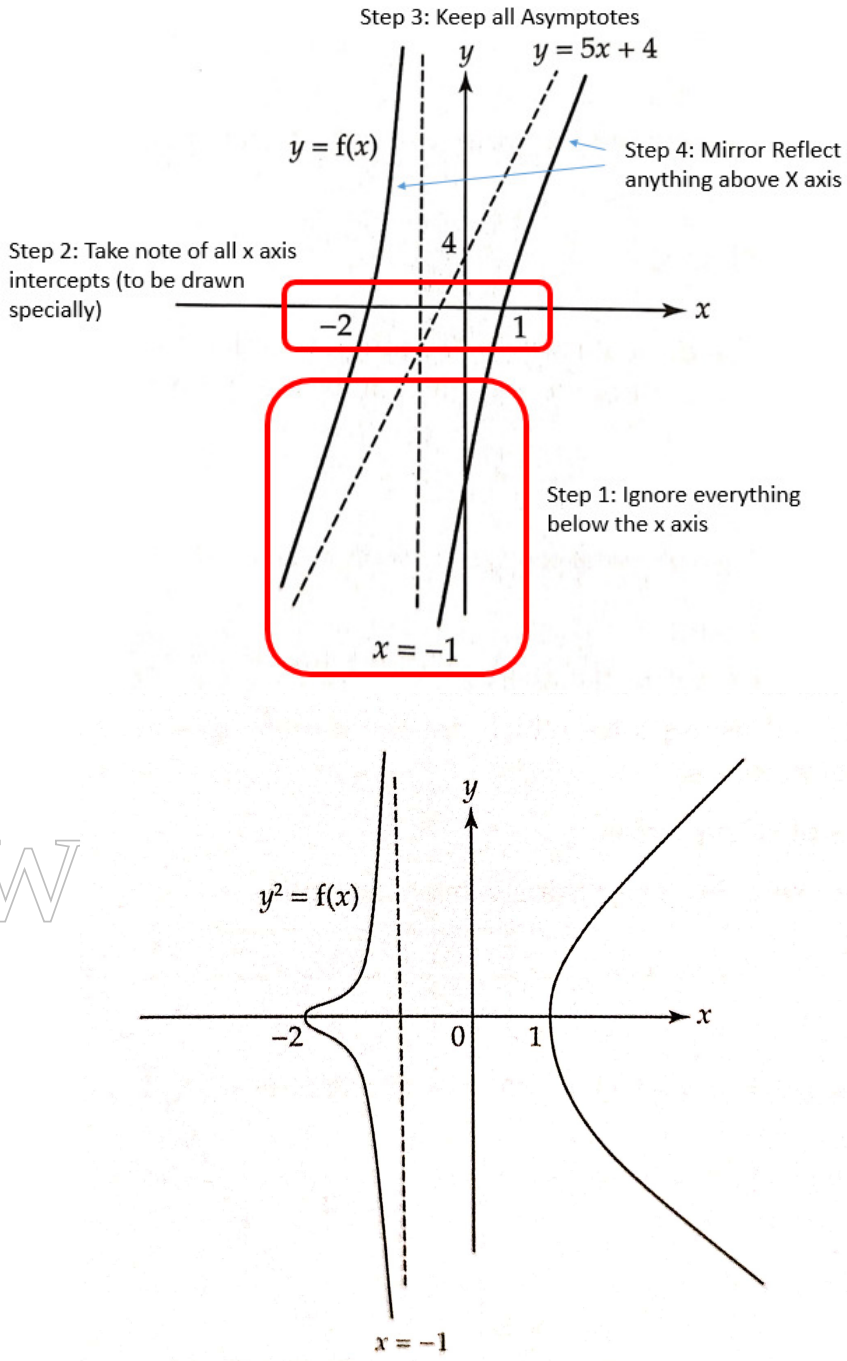
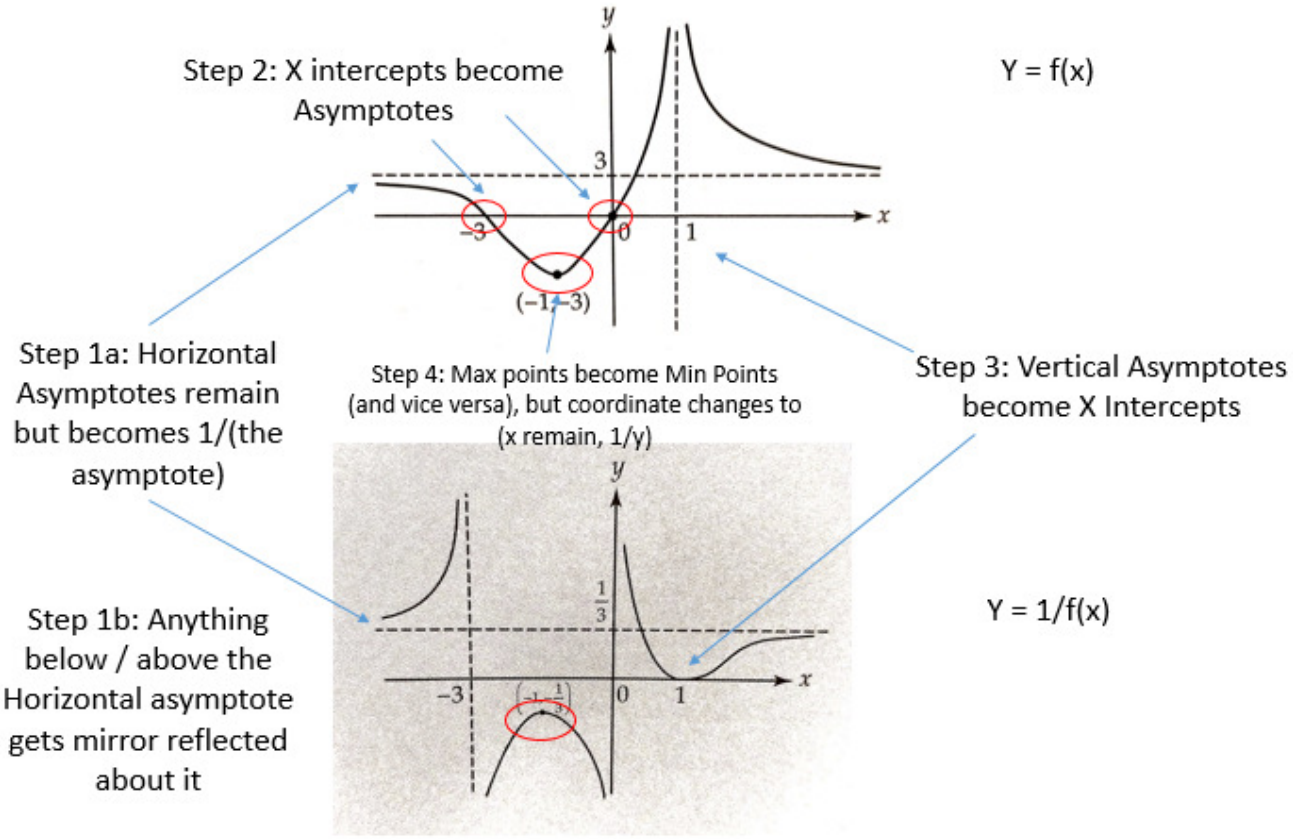


Figure 19: Example of How to Sketch Square Root (Khin 2019)

C. RECIPROCAL

- Example given $y = f(x)$
- Sketch $y = \dfrac{1}{f(x)}$



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Khin, S. B. (2019). Effective Guide (H2) Mathematics, Fairfield Book Publishers.

ABOUT MR SONG BOON KHING

Mr. Song Boon Khing graduated from NUS with a Bachelor of Science (2nd Upper Hons) degree, majoring in Applied Mathematics. Imbued with the passion to help and positively influence the young, Mr. Song applied and was awarded the MOE teaching award after graduating from Hwa Chong Junior College. Upon receiving his Post Graduate Diploma in Education (PGDE) with Credit, Mr. Song taught at National Junior College (NJC), teaching H1 and H2 A Level Mathematics.

ABOUT DR. ALVIN ANG

Dr. Alvin Ang earned his Ph.D., Masters and Bachelor degrees from NTU, Singapore. He is a scientist, entrepreneur, as well as a personal/business advisor. More about him at www.AlvinAng.sg.