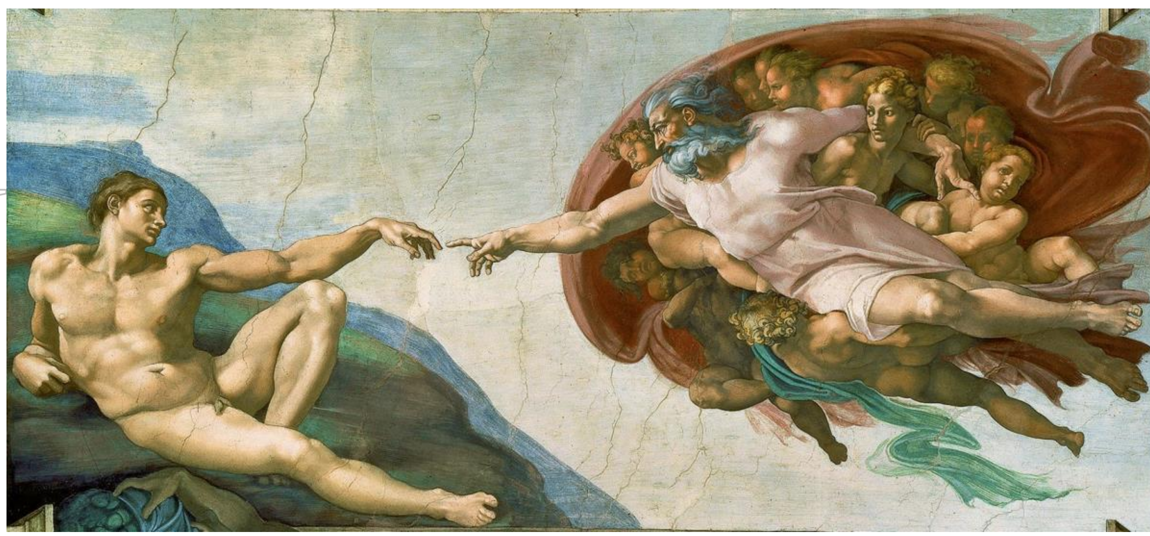
DR. ALVIN'S PUBLICATIONS

HYPOTHESIS TESTING

DR. ALVIN ANG



CONTENTS

PART I

STEPS FOR HYPOTHESIS TESTING

WHAT IS HYPOTHESIS TESTING?

- A hypothesis is a claim.
- Hypothesis testing = verifying the claim → is it true or false?
- In statistics, the claim is the population parameter (usually the population mean).
- In other words, hypothesis testing in statistics is
  - Making a claim/assumption/guess about the population mean
  - Taking a sample to test
  - Verifying is the claim true or false.
- There are generally 6 steps to Statistical Hypothesis Testing:
  - Step 1: Stating the Null and Alternate Hypothesis
  - Step 2: Selecting the Level of Significance
  - Step 3: Selecting the Test Statistics
  - Step 4: Formulating the Decision Rule
  - Step 5: Computing the Value of the Test Statistic and Interpreting the Results
  - Step 6: P-test for double confirmation

STEP 1: STATING THE NULL AND ALTERNATE HYPOTHESIS

Step 1. State the null hypothesis (H_0) and the alternate hypothesis (H_1)

ONE Tailed Test

H_0: $\mu \leq 3.3$ $\quad$ H_0: $\mu \geq 3.3$

H_1: $\mu > 3.3$ $\quad$ H_1: $\mu < 3.3$

TWO Tailed Test

H_0: $\mu = 3.3$

H_1: $\mu \neq 3.3$

Type I and Type II errors

Null Hypothesis	Researcher: Accepts H_0	Researcher: Rejects H_0
H_0 is true	Correct decision	Type I error
H_0 is false	Type II error	Correct decision

Figure 1: Step 1 of Hypothesis Testing

STEP 2: SELECTING THE LEVEL OF SIGNIFICANCE

Step 2. Select the Level of Significance

If Sigma known:
z distribution
Textbook Appendix B1

If Sigma UN-known:
t distribution
Textbook Appendix B2

Level of Significance (Alpha)	Critical value (Z_α) of Z: Two-tailed test	Critical value (Z_α) of Z: Single tailed test
10%	1.645	1.28
5%	1.96	1.645
1%	2.58	2.33

Figure 2: Step 2 of Hypothesis Testing

STEP 3: SELECTING THE TEST STATISTIC

- t-test is used when the Population Standard Deviation σ is unknown.
- Z-test is used when the Population Standard Deviation σ is known.

Step 3. Select the Test Statistic

Testing a Mean, σ Known	$z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$

Where

- z is the value of the test statistic.
- $\overline{X}$ is the sample mean.
- μ is the population mean.
- s is the population standard deviation.
- n is the sample size.

Testing a Mean, σ Unknown	$t = \dfrac{\overline{X} - \mu}{s / \sqrt{n}}$

with n-1 degrees of freedom, where:

- t is the value of the test statistic.
- $\overline{X}$ is the mean of the sample.
- μ is the hypothesized population mean.
- s is the standard deviation of the sample.
- n is the number of observations in the sample.

Figure 3: Step 3 of Hypothesis Testing

STEP 4: FORMULATING THE DECISION RULE

Step 4. Formulate the Decision Rule

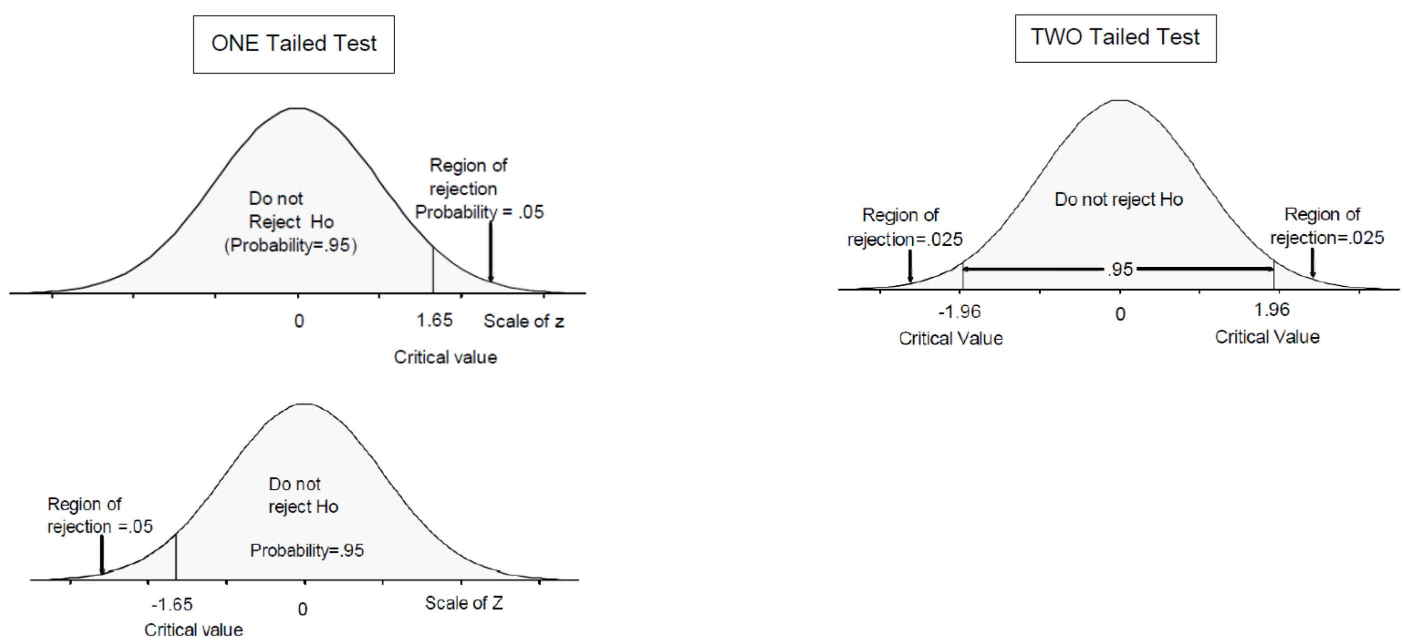


Figure 4: Step 4 of Hypothesis Testing

STEP 5

COMPUTE THE VALUE OF THE TEST STATISTIC, MAKE A DECISION, AND INTERPRET THE RESULTS

STEP 6: P TEST

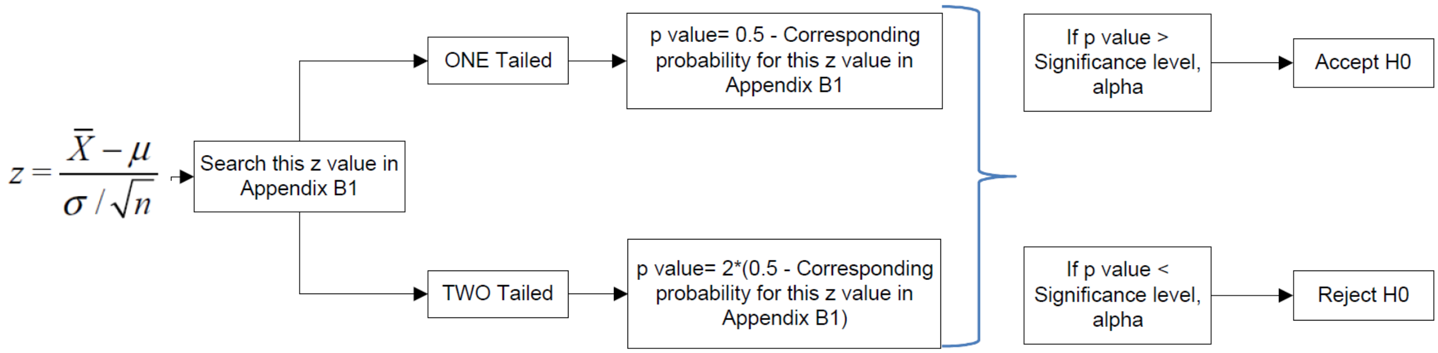


Figure 5: Step 6 of Hypothesis Testing

PART II

Z TEST

A. 1 SAMPLE Z TEST

AN EXAMPLE

Given:

- Population Mean, μ = 40 miles per gallon
- Sample Mean, $\overline{X}$ =38.9 miles per gallon
- Population Std. Dev., σ = 4 miles
- Sample Size, n = 64
- α = 0.01

Find:

- The manufacturers claim: Clipper can go 40 or more miles per gallon
- Is this true?

Answer:

Step 1. State the null hypothesis (H_0) and the alternate hypothesis (H_1)

ONE Tailed Test

$$H_0: \mu \geq 40$$

$$H_1: \mu < 40$$

Step 2. Select the Level of Significance

- Question asks for 0.01 Sig. Level (alpha = 0.01)

If Sigma known:
z distribution
Textbook Appendix B1

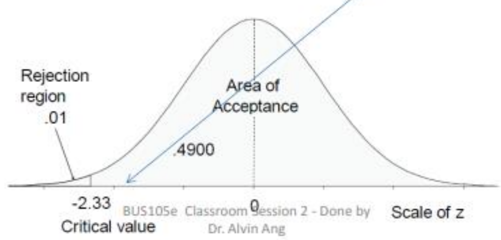
Level of Significance (Alpha)	Critical value (Z_α) of Z Two-tailed test	Single tailed test
10%	1.645	1.28
5%	1.96	1.645
1%	2.58	2.33

Step 3. Select the Test Statistic

Testing a Mean, σ Known $z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$

$$z = \frac{38.9 - 40.0}{4.0 / \sqrt{64}} = \frac{-1.1}{0.5} = -2.20$$

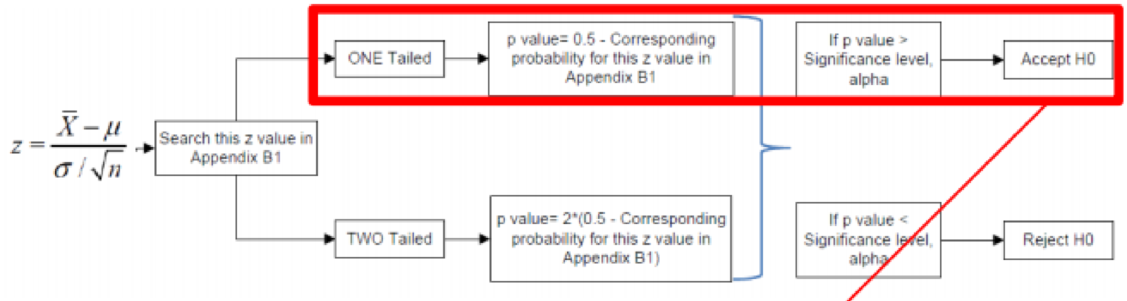
Step 4. Formulate the Decision Rule



Step 5. Compute the value of the test statistic, make a decision, and interpret the results

- Since -2.20 falls in the ACCEPTED region,
- ACCEPT H0
- We are 99% Confident Manufacturer's Claim is true

Step 6: P-test



P = 0.5 – Z (2.20) (appendix B1)
= 0.5 – 0.4861
= 0.0139
Since 0.0139 > 0.01 → ACCEPT H0

B. 1 SAMPLE Z TEST CONCERNING PROPORTION

Test of Hypothesis, One Proportion	$z = \frac{p - \pi}{\sigma_p}$

Where:
- z is the value of the test statistic
- π is the population proportion.
- p is the sample proportion.
- σ_P is the standard error of the population proportion.

σ_P is computed by $\sqrt{\pi(1-\pi)/n}$

AN EXAMPLE

Given:

- π : Population Proportion = 0.30 (30% of students are employed – believed population proportion)
- p: Sample Proportion = 0.25 (25% of students are employed – sampled proportion)
- α: Significance Level = 0.01

Question:

- Is this claim true? That more than 30% of the students are employed?

Answer:

- Actually, this case is a Binomial Distribution, not Normal Distribution.
- Because this case satisfies the assumptions of Binomial:
  - Only Success/Fail (π=0.3)
  - Number of trials is fixed (n=100)
  - Each trail is independent (selecting one student doesn't affect the other)
- But due to the fitting criteria of:
  - $n\pi>5$ and

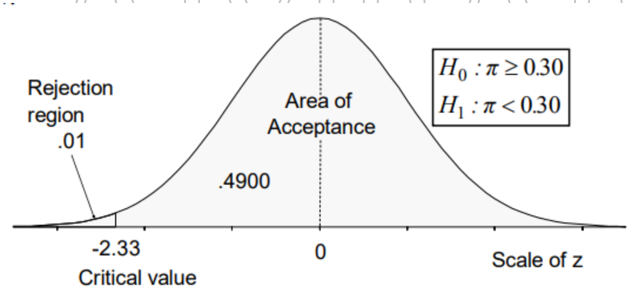
- $n(1-\pi)>5$
- The Normal Distribution can be used to approximate the Binomial Distribution!

- Kindly refer to Ang (2019) for more information.

- Step 1: State the Null and Alternate Hypothesis

$$H_0: \pi \geq 0.30$$
$$H_1: \pi < 0.30$$

- Step 2: State the Significance Level: $\alpha = 0.01$
- Step 3: Select the Test Statistic
  - Since this is Normal Distributed, and σ_p (: the Std. Dev. Of the Population Proportion) can be found
  - Thus Z is the test statistic
- Step 4: Formulate the Decision Rule



$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.25-0.30}{\sqrt{\frac{(0.30)(1-0.30)}{100}}} = -1.09$$

  - Thus Zcritical (1 tail → left tail; alpha = 0.01) = -2.33)
  - Zstat = -1.09
- Step 5: Make the Decision
  - Since Zciritcal < Zstat → Accept H0

C. 2 SAMPLE Z TEST

Test Statistic for No Difference Between Two Sample Means	$z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

where $\bar{X}_1$ and $\bar{X}_2$ refer to the two sample means.
σ_1^2 and σ_2^2 refer to the two sample variances.
n_1 and n_2 refer to the two sample sizes.

AN EXAMPLE

Given:

- 1st Product: Sinus
  - Sample Mean $\overline{X1} = 85.0$
  - Population Std. Dev. $\sigma 1 = 6.0$
  - Sample Size n1 = 100
- 2nd Product: Antidrip
  - Sample Mean $\overline{X2} = 86.2$
  - Population Std. Dev. $\sigma 2 = 6.8$
  - Sample Size n2 = 81
- Alpha = 0.05

Find:

- Is there a significant difference (5% alpha) between the Mean of the 2 products?

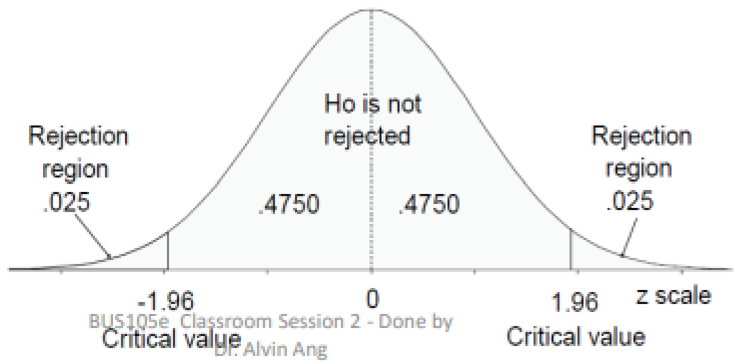
Answer:

- Step 1: State the Null and Alternate Hypothesis

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

- Step 2: Level of Significance, $\alpha = 0.05$
- Step 3: Select the Test Statistic – Z (because the population std. dev. Of both products are known)
- Step 4: Formulate the Decision Rule



- Step 5: Make the Decision

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{85.0 - 86.2}{\sqrt{\frac{(6.0)^2}{100} + \frac{(6.8)^2}{81}}} = -1.24$$

- Since -1.96 < -1.24 < 1.96
- ACCEPT H0
- No difference between X1bar and X2bar

- Step 6: Double confirm with p test

- 2 tailed → p = 2* (0.5 – Z(1.24, Appendix B1)) = 2*(0.5 – 0.3925) = 0.2150
- Since 0.2150 > 0.05
- ACCEPT H0
- No difference between SINUS and ANTIDRIP

D. 2 SAMPLE Z TEST OF PROPORTIONS

Two-Sample Test of Proportions

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p_c(1-p_c)}{n_1} + \frac{p_c(1-p_c)}{n_2}}}$$

where

p_1 is the proportion in the first sample possessing the trait.

p_2 is the proportion in the second sample possessing the trait.

n_1 is the number of observations in the first sample.

n_2 is the number of observations in the second sample.

p_c is the pooled proportion possessing the trait in the combined samples. It is called the ***pooled estimate of the population proportion*** and is found by formula

Pooled Proportion

$$p_c = \frac{X_1 + X_2}{n_1 + n_2}$$

where

X_1 is the number possessing the trait in the first sample.

X_2 is the number possessing the trait in the second sample.

AN EXAMPLE

Given:

	South Side	East Side
Number of working mothers with children under 5	$X_1 = 88$	$X_2 = 57$
Number in sample	$n_1 = 200$	$n_2 = 150$
Proportion with children under 5 and mothers work	$p_1 = 0.44$	$p_2 = 0.38$

- $\alpha = 0.05$

Find:

- Is the proportion on the south side larger than the east side?

Answer:

- Step 1: State the Null and Alternate Hypothesis

The hypotheses are: $H_0: \pi_1 \leq \pi_2$

$H_1: \pi_1 > \pi_2$

where

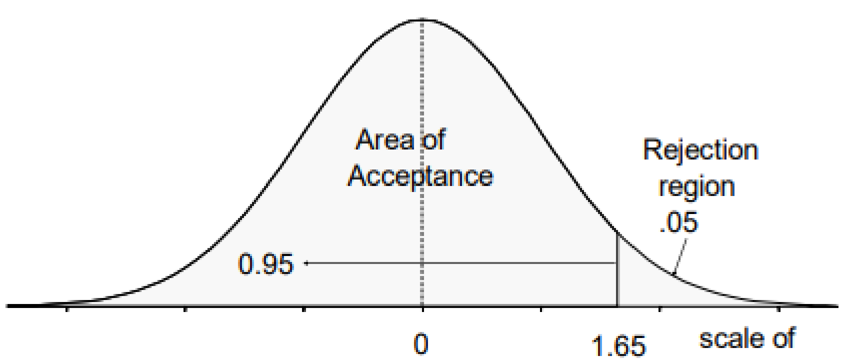
π_1 refers to the proportion of working mothers on the south side.

π_2 refers to the proportion of working mothers on the east side.

- Step 2: α = 0.05
- Step 3: The Test Statistic is Z because we have all the necessary variables for the formula:

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p_c(1-p_c)}{n_1} + \frac{p_c(1-p_c)}{n_2}}}$$

- Step 4: Formulate the Decision Rule



- Step 5: Make a Decision

$$p_c = \frac{X_1 + X_2}{n_1 + n_2} = \frac{88 + 57}{200 + 150} = 0.4143$$

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p_c(1-p_c)}{n_1} + \frac{p_c(1-p_c)}{n_2}}} = \frac{0.44 - 0.38}{\sqrt{\frac{(0.4143)(1-0.4143)}{200} + \frac{(0.4143)(1-0.4143)}{150}}} = 1.13$$

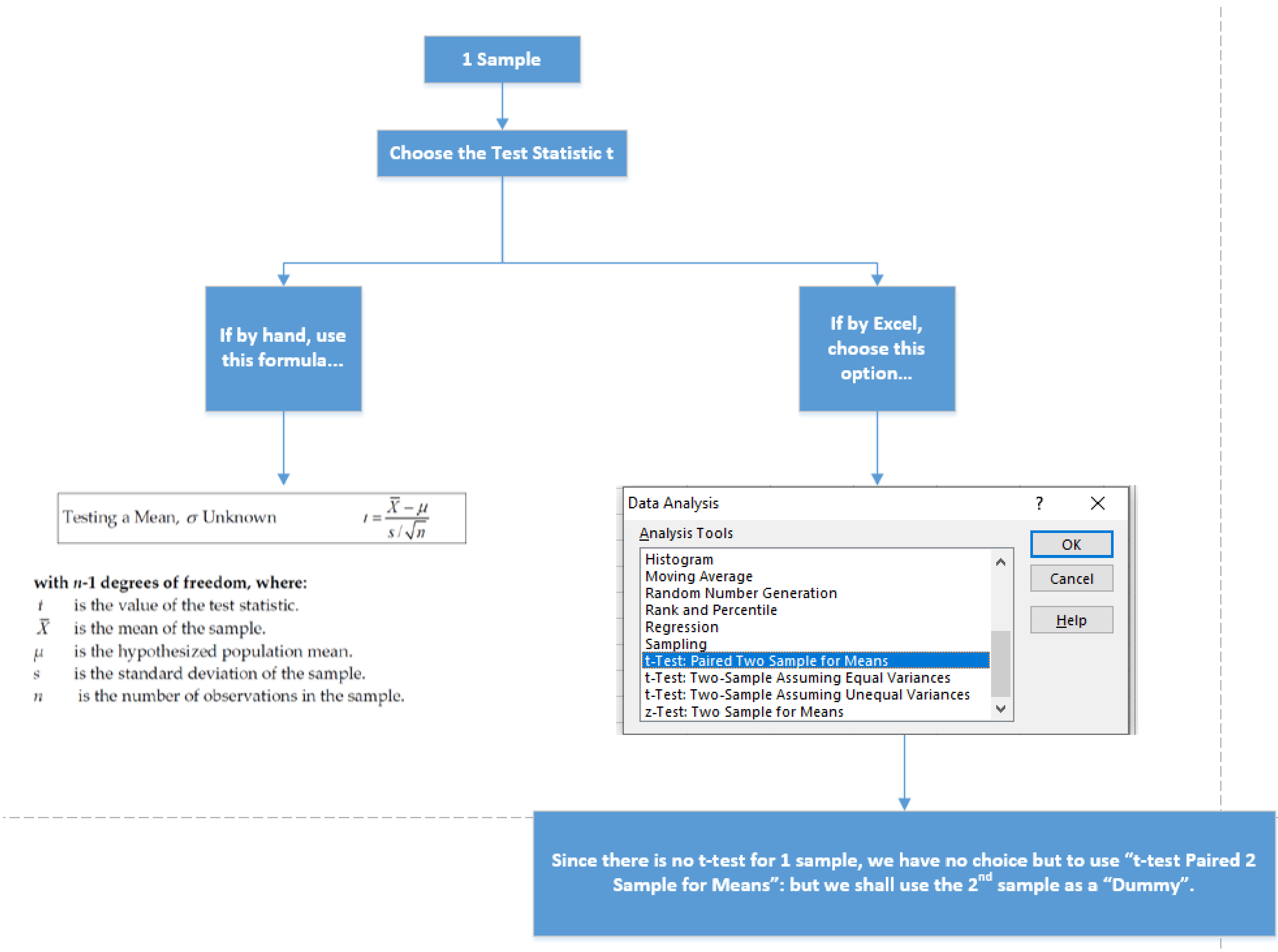
- Since Zstatistic (=1.13) < Zcritical (= 1.65) → Accept H0.
- Conclusion: We do not have enough evidence that the south side proportion is greater than the east side.

PART III

T-TEST

- t-test is used when the Population Standard Deviation σ is unknown.
- It is used to test if there is a difference between means: either 1 sample or 2 samples.

A. 1 SAMPLE T-TEST



Example:

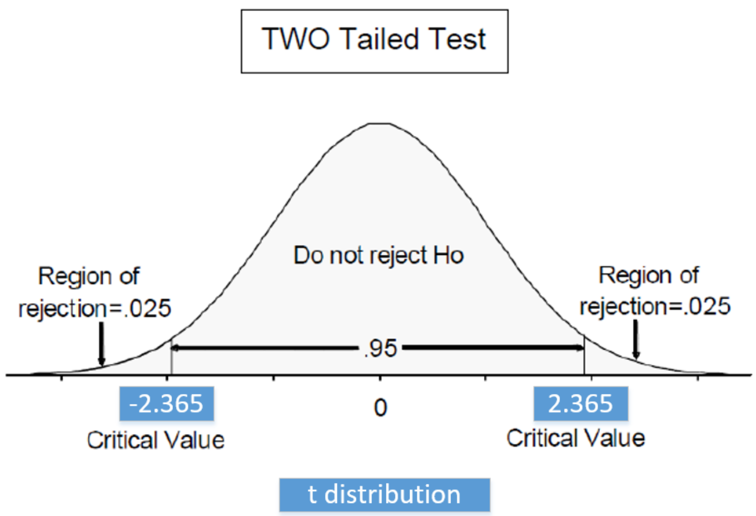
- There are 8 discs:
  - 6.115
  - 6.127
  - 6.129
  - 6.113
  - 6.124
  - 6.121
  - 6.131
  - 6.124
- The mean diamter is supposed to be 6.125.
- They follow Normal Distribution.
- Their Population Std. Dev. σ , is unknown.
- The worker suspects that the machine is misaligned.
- Question: Conduct Hypothesis Test to check is mean diamter = 6.125?

Answer:

- Step 1: State the Null and Alternate Hypothesis
  - $$H_0 : \mu = 6.125$$ $$H_1 : \mu \neq 6.125$$
  - Null Hypothesis: Mean diameter is 6.125
  - Alternate Hypothesis: Mean diameter is not 6.125.
- Step 2: Select Level of Significance
  - α = 5%
- Step 3: Select the Test Statistic

- We choose the t test
- Because it follows the Normal Distribution and the Population Std. Dev. Is unknown.
- Also because the sample size is small.

- Step 4: Formulate Decision Rule
  - 2 Tailed Test:
  - We obtain the t-critical values from the t-table.
  - With df = 8-1 = 7 and α = 5%



	Confidence Intervals, c					
	80%	90%	95%	98%	99%	99.9%
	Level of Significance for One-Tailed Test, α					
df	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587

- Step 5: Make the Decision:

BY HAND

We use: $t = \dfrac{\overline{X} - \mu}{s/\sqrt{n}}$

where
t is the value of the test statistic.
$\overline{X}$ is the sample mean.
μ is the population mean. (6.125 inches)
s is the standard deviation of the sample.
n is the sample size. (8)

X	$X-\overline{X}$	$(X-\overline{X})^2$	X^2
6.115	–0.008	0.000064	37.393225
6.127	0.004	0.000016	37.540129
6.129	0.006	0.000036	37.564641
6.113	–0.010	0.000100	37.368769
6.124	0.001	0.000001	37.503376
6.121	–0.002	0.000004	37.466641
6.131	0.008	0.000064	37.589161
6.124	0.001	0.000001	37.503376
Σ 48.984	0.000	0.000286	299.929318

$$\overline{X} = \frac{\Sigma X}{n} = \frac{48.984}{8} = 6.123$$

$$s = \sqrt{\frac{\Sigma(X-\overline{X})^2}{n-1}} = \sqrt{\frac{0.000286}{8-1}}$$
$$= \sqrt{0.000040857} = 0.0063919 = 0.0064$$

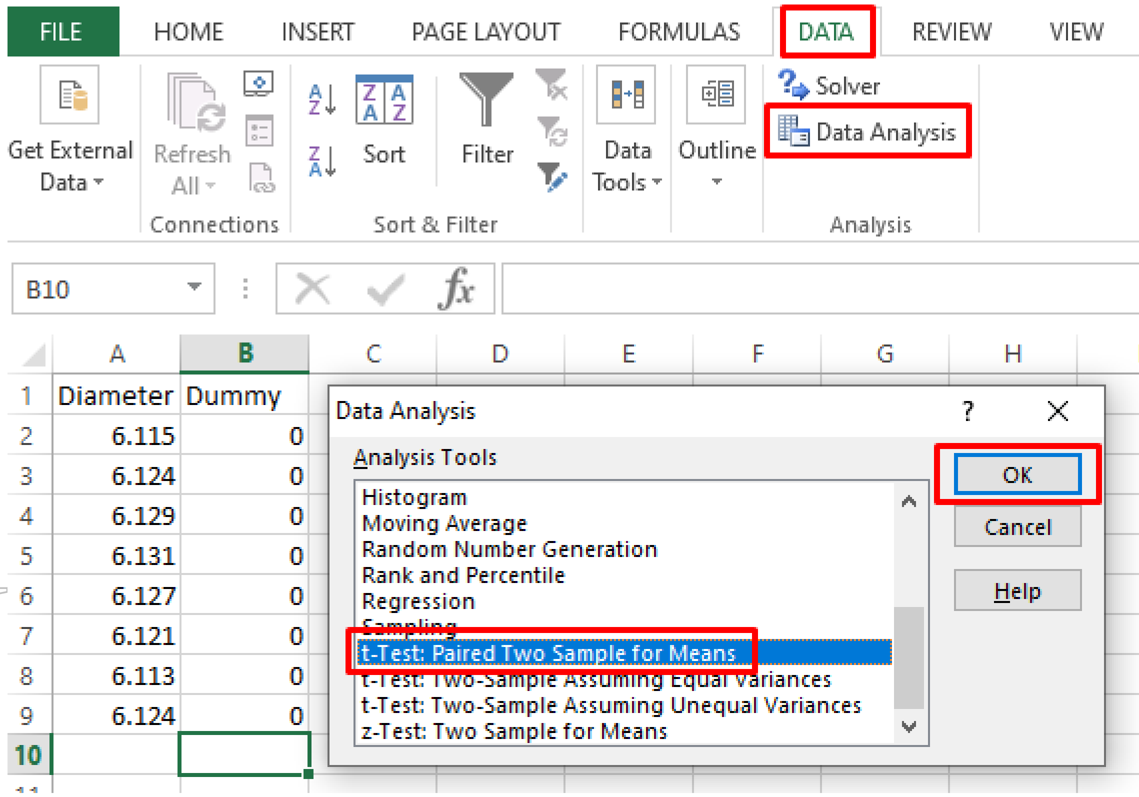
The value of t is computed using:

$$t = \frac{\overline{X}-\mu}{s/\sqrt{n}} = \frac{6.123-6.125}{0.0064/\sqrt{8}} = \frac{-0.002}{0.0022627} = -0.8839$$

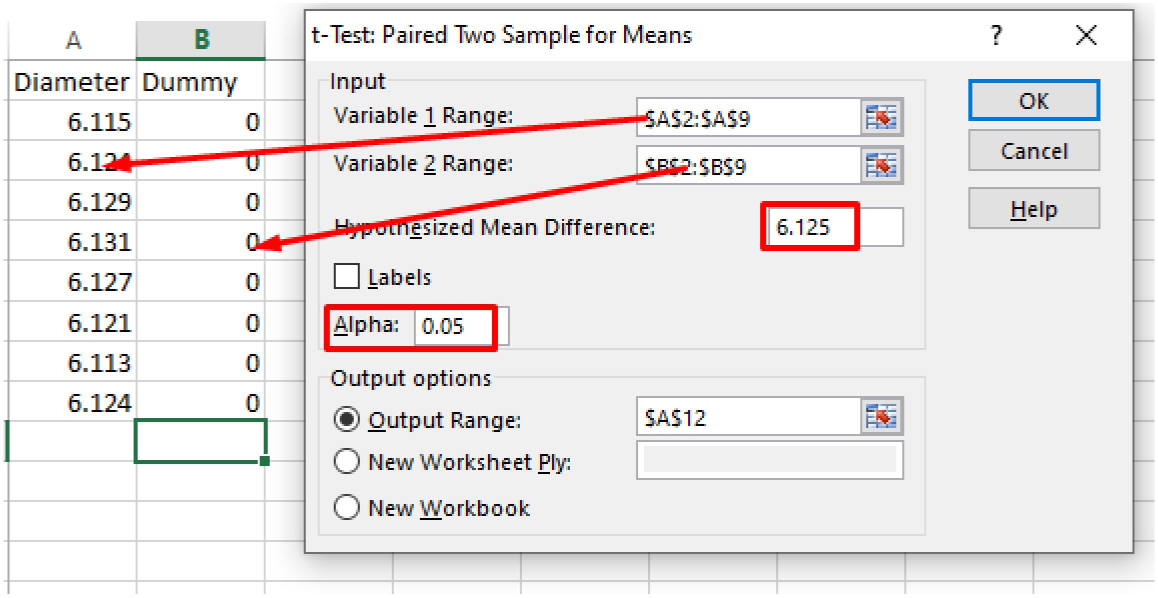
- Since t-stat = -0.8839 which falls in between t-critical
- I.e. -2.365 < -.08839 < 2.365 → Accept H0.

BY EXCEL

- In Excel → Data → Data Analysis, we do not have t-test 1 Sample test.
- Thus, we need to create a Dummy (pretending as the 2nd Sample).



- Choose Variable 1 to be the Diameter Column.
- Choose Variable 2 to be the Dummy Column.
- Ensure that the Hypothesized Mean is 6.125
- Ensure that Alpha is 0.05.

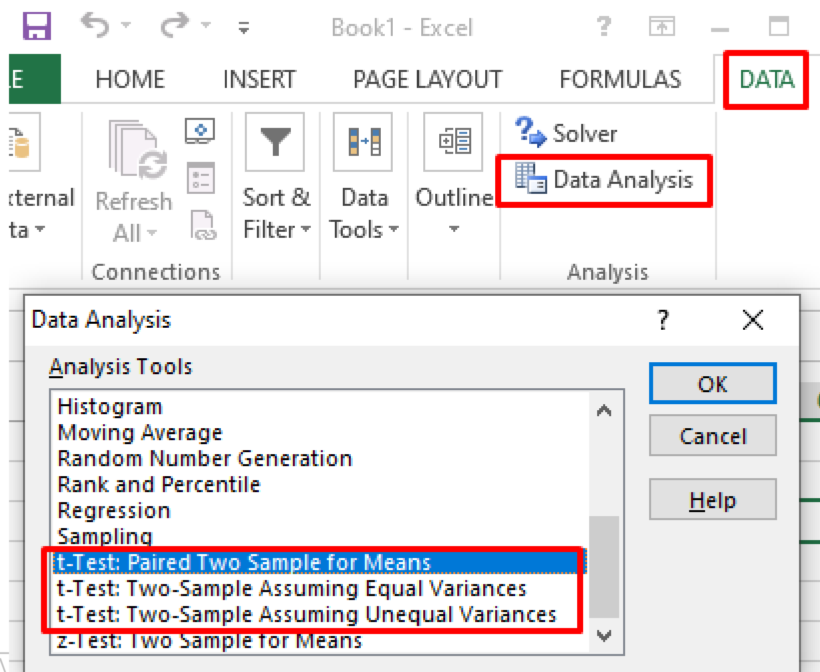


t-Test: Paired Two Sample for Means		
	Variable 1	*Variable 2*
Mean	6.123	0
Variance	4.08571E-05	0
Observations	8	8
Pearson Correlation	#DIV/0!	
Hypothesized Mean Difference	6.125	
df	7	
t Stat	-0.884995358	
P(T<=t) one-tail	0.202765452	
t Critical one-tail	1.894578605	
P(T<=t) two-tail	0.405530903	
t Critical two-tail	2.364624252	

- Since the p-value (P(T<=t) two-tail) is = 0.4, which is bigger than Alpha (0.05), it means that the p-value has spread into H0 region → Accept H0.

B. 2 SAMPLES T-TEST

- Under Excel → Data → Data Analysis, we have 3 options for t-test 2 samples (see below picture):

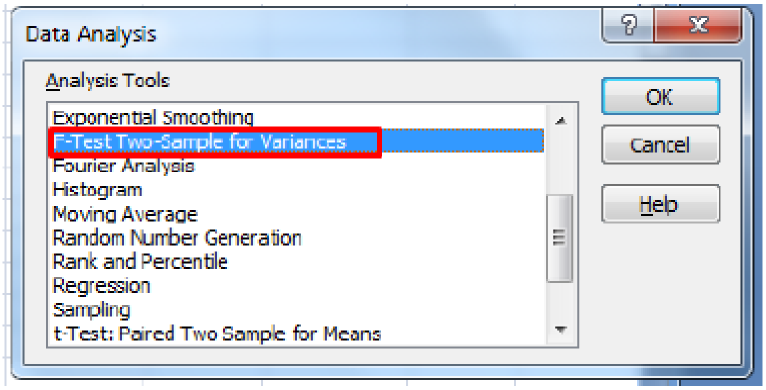


- t-test: Paired 2 Sample for Means
    - This means that the 2 samples are related/dependent on each other.
    - "Before and after" studies e.g. Weight Loss Program.
    - The samples are dependent because they are from the same individuals.
- t-test: 2 Sample Assuming Equal Variances
    - This means that the 2 samples are unrelated to each other.
    - But they each have the same variance.
- t-test: 2 Samples Assuming Unequal Variances
    - This means that the 2 samples are unrelated to each other.

        - But they have different variances.

  - Under Excel → Data → Data Analysis, we do a F-Test: 2 sample for Variances to check whether are they Equal / Unequal variances.

C. USING F-TEST TO DETERMINE EQUAL OR UNEQUAL VARIANCES



- After determining that the 2 samples are independent, we need to check if they have Equal / Unequal variances.

- After selecting all the 2 samples data, under Excel → Data → Data Analysis, click on F-Test: 2 Sample for Variances.

F-Test Two-Sample for Variances		
	Rating1	*Rating0*
Mean	16.86206897	14.1372549
Variance	16.83743842	28.16078431
Observations	29	51
df	28	50
F	0.597903746	
P(F<=f) one-tail	0.07268509	
F Critical one-tai	0.558717536	

- In this example, we have 2 samples: Rating 1 (Variance = 16.8) and Rating 0 (Variance = 28.2).

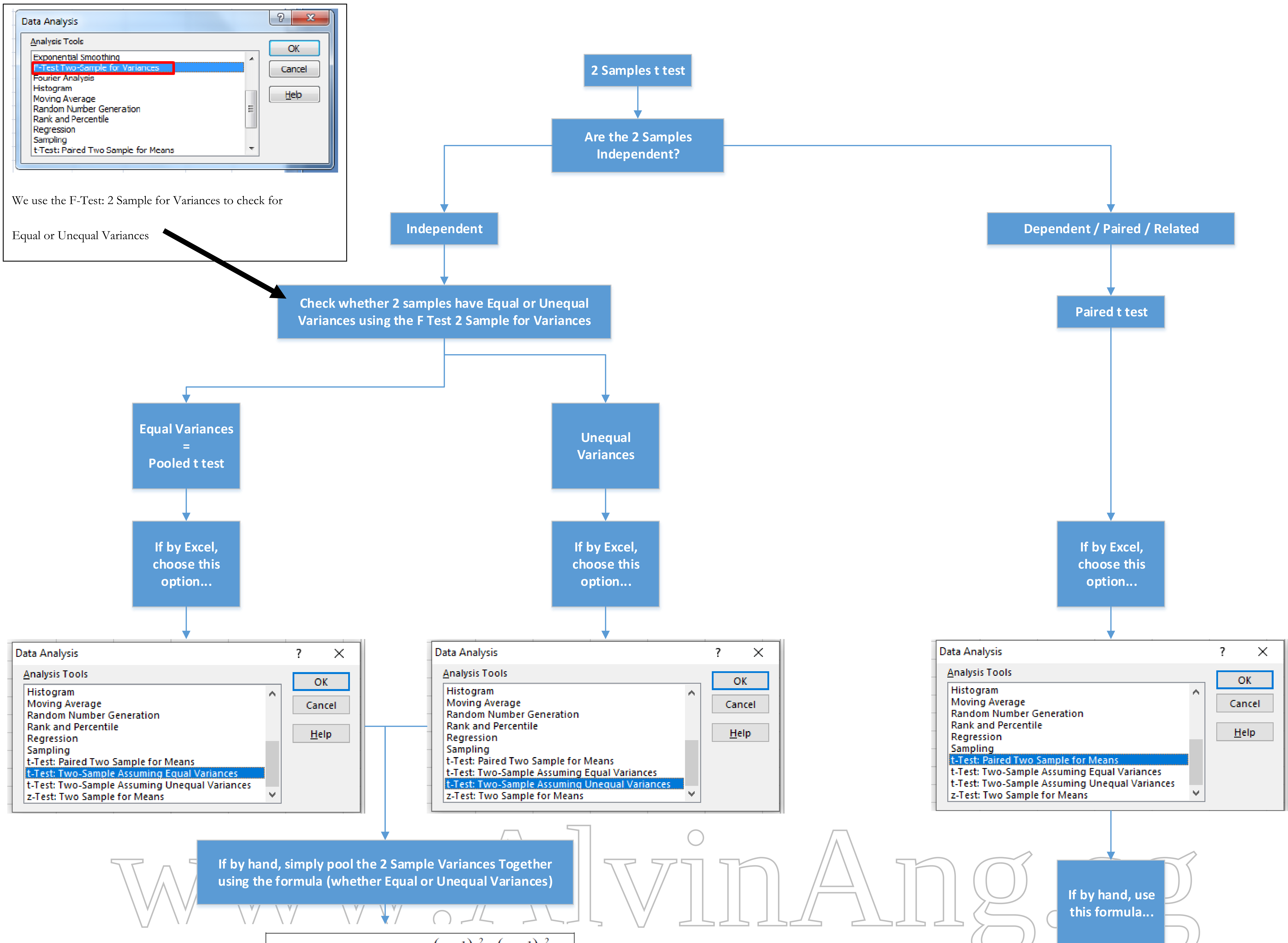
- Can we conclude that their Variance is different because they are far apart?

- The hypothesis is:
    - H0: Both Variances are Equal
    - H1: Variance of Rating 0 is larger than Variance of Rating 1

- We see that the p-value (P(F<=f) one-tail) = 0.07.

- If our α = 10%, then the p-value (0.07) < α (0.1)

- This means that we accept H1: they have Unequal Variances.



Pooled Variance $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

where

s_p^2 is the pooled estimate of the population variance.

s_1^2 is the variance of the first sample.

s_2^2 is the variance of the second sample.

n_1 is the number of observations in the first sample.

n_2 is the number of observations in the second sample.

Step 2
Calculate Pooled Test Statistic t

Two-Sample Test of Means—Unknown σ₁ and σ₂ $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Where:

$\bar{X}_1$ is the mean of the first sample.

$\bar{X}_2$ is the mean of the second sample.

n_1 is the number of observations in the first sample.

n_2 is the number of observations in the second sample.

s_p^2 is the polled estimate of the population variance.

Paired t test $t = \frac{\bar{d}}{s_d / \sqrt{n}}$

where $\bar{d}$ is the mean of the difference between the paired or related observations.

s_d is the standard deviation of the differences between the paired or related observations.

n is the number of paired observations.

For a paired difference test, there are $(n - 1)$ degrees of freedom.

The standard deviation of the differences s_d is computed using the familiar formula for the standard deviation except that d is substituted for X.

The formula is: $s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n - 1}}$

D. 2 SAMPLE POOLED T-TEST

1. ASSUMING EQUAL VARIANCES EXAMPLE

Given:

Accounting major

$33,000	$29,000	$31,000	$30,000	$32,000
$28,000	$32,000	$27,000	$28,000	$30,000

General Business major

$30,000	$31,500	$29,000	$29,500
$28,000	$29,500	$28,000	$26,500

- Accounting Major
  - Sample Mean $\overline{X1} = \$30k$
  - Sample Std. Dev. $S1 = \$2k$
  - Sample Size n1 = 10

- Business Major
  - Sample Mean $\overline{X2} = \$29k$
  - Samples Std. Dev. $S2 = \$1,512$
  - Sample Size n2 = 8

Find:

- Alpha = 0.05
- Does Accounting earn more?

Answer:

- Step 1: State the Null and Alternate Hypothesis

H_0: $\mu_1 \leq \mu_2$

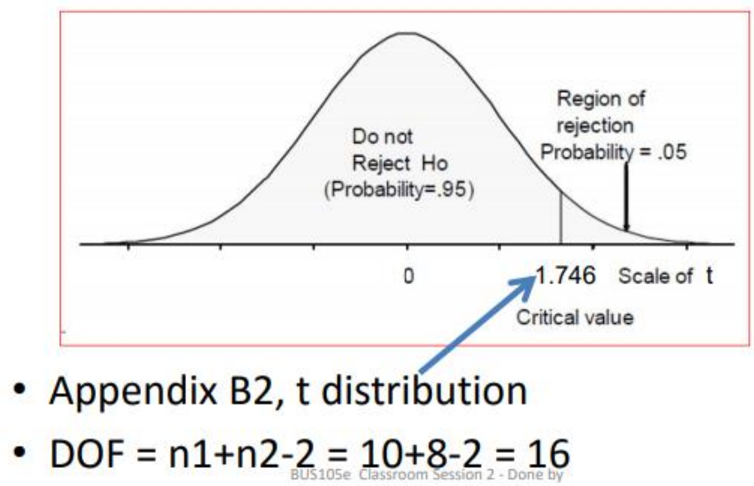
H_1: $\mu_1 > \mu_2$

where

μ_1 refers to accounting majors (graduates).

μ_2 refers to general business majors (graduates).

- Step 2: Alpha = 0.05
- Step 3: Select the Test Statistic
  - Test Statistic chosen: t for 2 samples
  - Assume Equal Variances
  - In this question, we just assume that both their Population Variations are Equal without needing to do the F test because we are going to pool both their Sample Variances together later on.
- Step 4: Formulate the Decision Rule



- Step 5: Calculate t statistic and t critical and make a decision
  - By Hand:

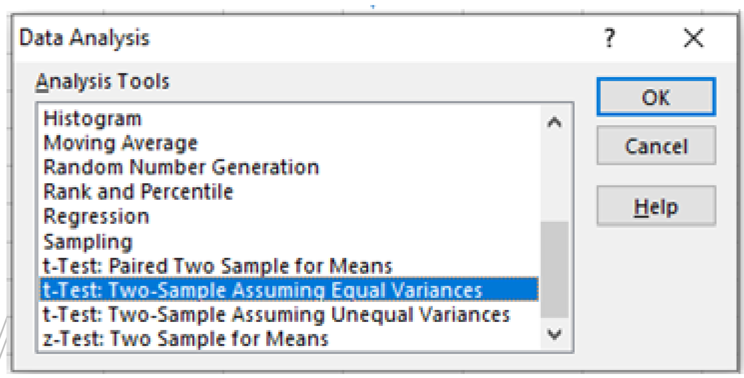
$$s_p^2 = \frac{(n_1-1)(s_1^2)+(n_2-1)(s_2^2)}{n_1+n_2-2} = \frac{(10-1)(2000)^2+(8-1)(1512)^2}{10+8-2} = 3{,}250{,}188$$

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{30{,}000-29{,}000}{\sqrt{(3{,}250{,}188)\left(\frac{1}{10}+\frac{1}{8}\right)}} = \frac{1{,}000}{\sqrt{3{,}250{,}188(0.225)}} = 1.169$$

- Since 1.169 < 1.746
- ACCEPT H0
- Accounting Majors does not earn more!

BUS105e Classroom Session 2 - Done bv

- By Excel:



t-Test: Two-Sample Assuming Equal Variances

	Accounting	*General Biz*
Mean	30000	29000
Variance	4000000	2285714.286
Observations	10	8
Pooled Variance	3250000	
Hypothesized Mean Difference	0	
df	16	
t Stat	1.169410692	
P(T<=t) one-tail	0.129682486	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.259364971	
t Critical two-tail	2.119905299	

- Since p-value (=0.1297) > alpha (=0.05) → Accept H0.

2. ASSUMING UNEQUAL VARIANCES EXAMPLE

- We do not have such an example.
- Rather, should we face this situation, we simply follow the steps above for "1. Assuming Equal Variances".
- But the only exception is:
    - When we use Excel, select "t-test: 2 Samples Assuming Unequal Variances"
- Other than that, all other steps remain the same.
- Reason: Because whether or not Equal/Unequal Variances, we end up with "Pooling both Variances together" → Which ends up with the same results.

E. 2 SAMPLES PAIRED T-TEST EXAMPLE

Given:

Student	Fall Semester	Spring Semester
A	2.7	3.1
B	3.4	3.3
C	3.5	3.3
D	3.0	2.9
E	2.1	1.8
F	2.7	2.4

Find:

- Alpha = 0.05
- Did the grades decline? I.e. Drop from Fall to Spring?

Answer:

- Step 1: State the Null and Alternate Hypothesis

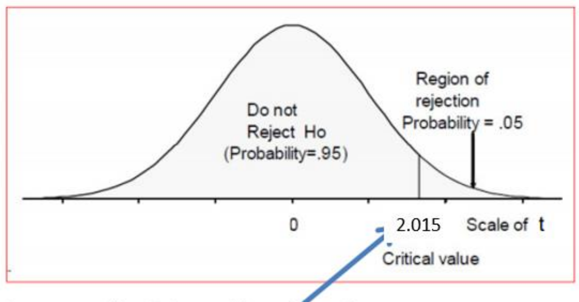
$$H_0: \mu_d \leq 0$$
$$H_1: \mu_d > 0$$

  - H0: There is No Difference
  - H1: There is a difference → Fall – Spring > 0

- Step 2: Level of Significance Alpha = 0.05
- Step 3: Select the Test Statistic
  - t test 2 sample : PAIRED is chosen

- They are Paired = Dependent because we are testing back the same group of students
- I.e. Exams Before and Exams After

- Step 4: Formulate the Decision Rule



- Appendix B2, t distribution
- n-1 = 6-1 = 5 degree of freedom

n: number of PAIRED observations

- Step 5: Make the Decision
- By Hand:

Paired t test $t = \dfrac{\bar{d}}{s_d / \sqrt{n}}$

where $\bar{d}$ is the mean of the difference between the paired or related observations.

s_d is the standard deviation of the differences between the paired or related observations.

n is the number of paired observations.

For a paired difference test, there are $(n - 1)$ degrees of freedom.

The standard deviation of the differences s_d is computed using the familiar formula for the standard deviation except that d is substituted for X.

The formula is: $s_d = \sqrt{\dfrac{\Sigma(d - \bar{d})^2}{n - 1}}$

Student	Fall	Spring	d	$(d - \bar{d})$	$(d - \bar{d})^2$
A	2.7	3.1	–0.4	–0.5	0.25
B	3.4	3.3	0.1	0	0
C	3.5	3.3	0.2	0.1	0.01
D	3.0	2.9	0.1	0	0
E	2.1	1.8	0.3	0.2	0.04
F	2.7	2.4	0.3	0.2	0.04
			0.6		0.34

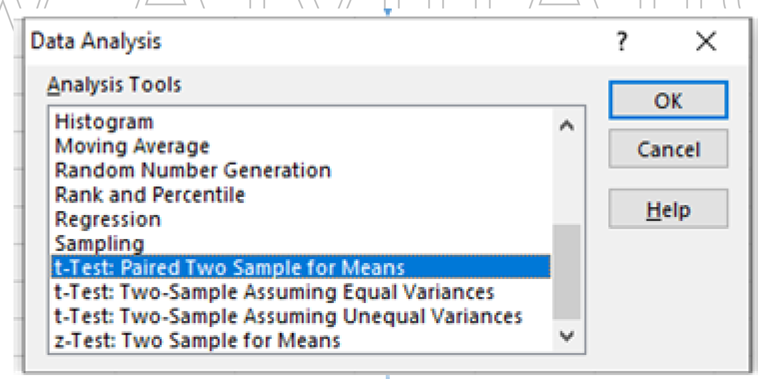
$$\bar{d} = \frac{\Sigma d}{N} = \frac{0.6}{6} = 0.10$$

$$s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n-1}} = \sqrt{\frac{0.34}{6-1}} = 0.2608$$

The t statistic is computed by: $t = \dfrac{\bar{d}}{s_d / \sqrt{n}} = \dfrac{0.10}{0.2608 / \sqrt{6}} = \dfrac{0.10}{0.1065} = 0.94$

- Since t statistic (=0.94) < t critical (=2.015) → Accept H0.
- There is no difference, no drop in grades.

- By Excel:



t-Test: Paired Two Sample for Means

	Fall Semester	Spring Semester
Mean	2.9	2.8
Variance	0.268	0.352
Observations	6	6
Pearson Correlation	0.89860815	
Hypothesized Mean Difference	0	
df	5	
t Stat	0.939336437	
P(T<=t) one-tail	0.195342008	
t Critical one-tail	2.015048373	
P(T<=t) two-tail	0.390684017	
t Critical two-tail	2.570581836	

- Since p-value (=0.195) > alpha (=0.05) → Accept H0.

REFERENCES

Ang, A. (2019). *Probability Models*. Singapore.

ABOUT DR. ALVIN ANG

Dr. Alvin Ang earned his Ph.D., Masters and Bachelor degrees from NTU, Singapore. He is a scientist, entrepreneur, as well as a personal/business advisor. More about him at www.AlvinAng.sg.