INEQUALITIES & SYSTEM OF EQUATIONS

DR. ALVIN ANG

CONTENTS

PART I

INEQUALITIES

A. SINGLE ROOT

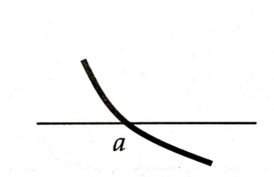
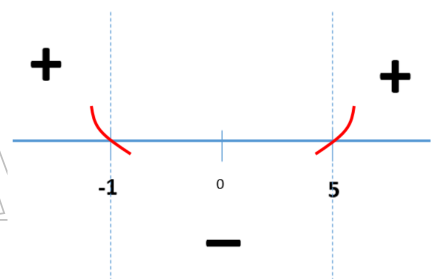


Figure 1: Single Root just cuts the X axis (Khin 2019)

- Single Root means $(x-a)$

Example: $(x-5)(x+1)>0$

- Two key values are 5 and -1.



- There are only 2 options:
    - A: -1 < x < 5 or
    - B: x < -1 and x > 5
- Test A → Sub 4 inside to test → you get negative (nil)
- Test B → Sub 6 (or -2) inside to test → you get positive (correct)
- Thus answer is B.

B. EVEN POWERED ROOTS

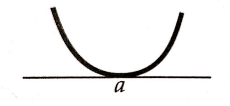
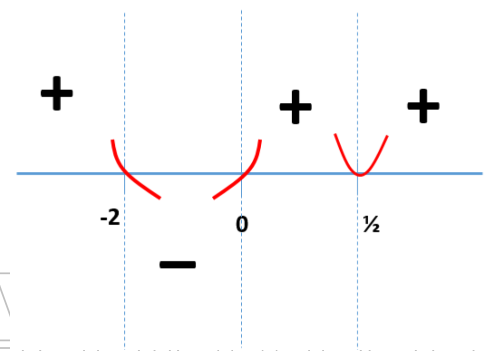


Figure 2: Even Powered Roots turn at the X axis (Khin 2019)

- Even Powered Roots mean $(x-a)^2, (x-a)^4, etc...$

Example: $x(x+2)(2x-1)^2 > 0$

- Three key values are -2, 0 and ½.



- Test -3 → you get positive (correct)
- Test -1 → you get negative (wrong)
- Test ¼ → you get positive (correct)
- Test 1 → you get positive (correct)
- Note that at ½, it's an "Even Powered Root", thus it curves up at the axis.
- The answer is:
    - X < -2
    - 0 < X < ½
    - X > ½

C. ODD POWERED ROOTS

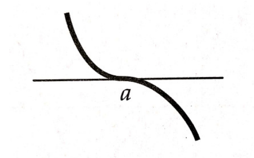
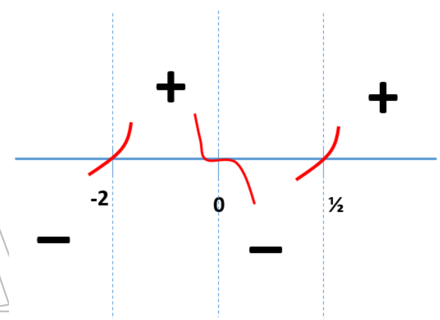


Figure 3: Odd Powered Roots have Inflexion Point at X Axis (Khin 2019)

- Even Powered Roots mean $(x-a)^3, (x-a)^5, etc...$

Example: $x^3(x+2)(2x-1) > 0$

- Three key values are -2, 0 and ½.



- Test -3 → you get -ve (wrong)
- Test -1 → you get +ve (correct)
- Test ¼ → you get -ve (wrong)
- Test 1 → you get +ve (correct)
- Note that at 0, it's an "Odd Powered Root", thus it has an inflexion point at the axis.
- The answer is:
  - -2 < X < 0
  - X > ½

D. RATIONAL FUNCTIONS

- Rational Functions have the form: $\frac{f(x)}{g(x)}$
- How to solve $\frac{f(x)}{g(x)} \geq or \leq 0$?
- ***MUST*** multiply throughout by +ve term (e.g. $\left[g(x)\right]^2$) so that the sign (≤, ≥) doesn't change.
- ***CANNOT*** simply multiply throughout by $g(x)$ because we don't know if $g(x)$ is +ve or –ve.

Example 1: Solve $\frac{(x+4)(x-3)}{x+1} \geq 0$

- ***MUST*** multiply throughout by $(x+1)^2$
- ***CANNOT*** simply multiply throughout by $(x+1)$
- $(x+4)(x-3)(x+1) \geq 0$
- Thus -4 ≤ X < -1 ; X ≥ 3

Example 2: Solve $\frac{(x-1)(x+2)}{(x+1)^2(x-2)^3} \geq 0$

- ***MUST*** multiply throughout by $(x+1)^2(x-2)^4$ → Notice that all powers are even!
- ***CANNOT*** simply multiply throughout by $(x+1)^2(x-2)^3$
- $(x-2)(x-1)(x+2) \geq 0$

- Thus -2 ≤ X < -1 ; -1 < X < 1 ; X > 2

E. GRAPHICAL VS ALGEBRAIC APPROACHES

Example: Solve $x-2 \geq \frac{1}{x}$

GRAPHICAL APPROACH

- There are two functions here: x-2 and 1/x
- If using GC, plot the two functions:

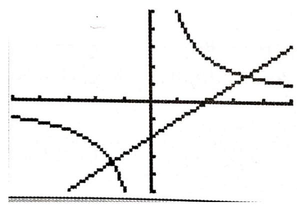


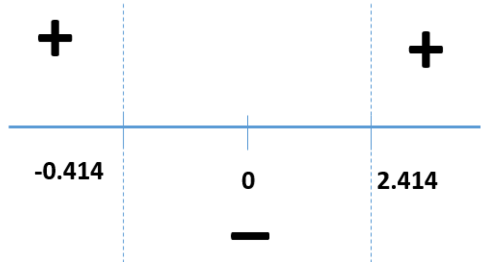
Figure 4: Graphical Approach (Khin 2019)

- Use GC to find their intersection points, then from the graph find out where $x-2 \geq \frac{1}{x}$

ALGEBRAIC APPROACH

- $$x-2 \geq \frac{1}{x}$$
  $$\frac{x^2-2x-1}{x} \geq 0$$
  $$x^4-2x^3-x^2 \geq 0$$
  $$x^2\left(x^2-2x-1\right) \geq 0$$
- Since $x^2 \geq 0$, $x^2-2x-1 \geq 0$
- $x=\frac{2 \pm \sqrt{4+4}}{2}=1 \pm \sqrt{2}$

- Two key values: $1-\sqrt{2}=-0.414$ & $1+\sqrt{2}=2.414$



- Test -3 → you get +ve (correct)
- Test 0 → you get -ve (wrong)
- Test 5 → you get +ve (correct)
- Thus X ≤ -0.414 or X ≥ 2.414

F. INVOLVING INTERSECTION (SPLITTING)

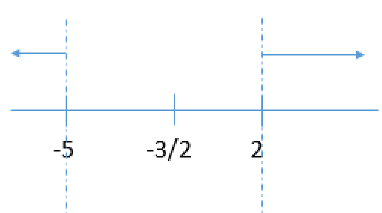
Example: Solve $1<\dfrac{4x-4}{2x+3}<3$

- Split into

$$1<\frac{4x-1}{2x+3} \qquad\qquad \frac{4x-1}{2x+3}<3$$

$$\therefore x<-\frac{3}{2}\ or\ x>2 \qquad\qquad \therefore x<-5\ or\ x>-\frac{3}{2}$$

$$\therefore x<-5\ or\ x>2$$



G. MODULUS

- $|x|^2 = |x^2| = x^2$
- $\sqrt{x^2} = |x|$
- $|x| < a$ means $-a < x < a$
- $|x| > a$ means $x > a$ or $x < -a$

Any x falling outside the modulus?

Yes

Example 1

$|2-3x| > 1+3x-x^2$

Resolve the stuff inside the modulus first

$2-3x<0$

$x>\frac{2}{3}$

$2-3x>0$

$x<\frac{2}{3}$

Then do splitting

$-(2-3x) > 1+3x-x^2$

$\therefore x>\sqrt{3}\ or x<-\sqrt{3}\left(rejected \because x>\frac{2}{3}\right)$

$2-3x > 1+3x-x^2$

$\therefore x<3-2\sqrt{2}\ or x>3+2\sqrt{2}\left(rejected \because x<\frac{2}{3}\right)$

$\therefore x<3-2\sqrt{2}\ or x>\sqrt{3}$

Example 2

$x|x-2| \geq 3$

Resolve the stuff inside the modulus first

$x>2$

$x<2$

Then do splitting

$x(x-2)>3$

$x>3\ or x<-1(rejected \because x>2)$

$x(x-2)<3$

Complex Roots – No Solution

$\therefore x>3$

No

Example 3

$|2x^2+x|<3$

Then do splitting

$-3<2x^2+x$

$2x^2+x<3$

$-\frac{3}{2}<x<1$

H. SQUARING BOTH SIDES

- If modulus is on both sides of the inequality, simply square both sides to solve.
- Example: $|x| > |2x+3|$
- $x^2 > (2x+3)^2$
- Answer: $-3 < x < -1$

I. SUBSTITUTION

- The trick is to find a suitable replacement for x, so as to help solve easier and faster.
- Example: $\frac{2x+10}{x^2+2} > 4$ → Answer: $-\frac{1}{2} < x < 1$
- Now solve $\frac{2x+6}{x^2-4x+6} > 4$ → It's easier to replace x with x – 2
- $\frac{2x+6}{x^2-4x+6} = \frac{2(x-2)+10}{(x-2)^2+2}$ → Therefore $-\frac{1}{2} < (x-2) < 1$
- Thus $1\frac{1}{2} < x < 3$

PART II

SYSTEM OF EQUATIONS

- Unfortunately, there is no way to describe this topic other than experience thru practice.
- However, two important GC functions are required:
    - How to solve 2 equations 2 unknowns? (or more) (Ang 2019)
    - How to solve polynomial / higher order functions (Ang 2019)
- Refer to sample practices with solutions for examples.

REFERENCES

Ang, A. (2019). Some Useful TI 84 GC Functions.

Khin, S. B. (2019). Effective Guide (H2) Mathematics, Fairfield Book Publishers.

ABOUT MR SONG BOON KHING

Mr. Song Boon Khing graduated from NUS with a Bachelor of Science (2nd Upper Hons) degree, majoring in Applied Mathematics. Imbued with the passion to help and positively influence the young, Mr. Song applied and was awarded the MOE teaching award after graduating from Hwa Chong Junior College. Upon receiving his Post Graduate Diploma in Education (PGDE) with Credit, Mr. Song taught at National Junior College (NJC), teaching H1 and H2 A Level Mathematics.

ABOUT DR. ALVIN ANG

Dr. Alvin Ang earned his Ph.D., Masters and Bachelor degrees from NTU, Singapore. He is a scientist, entrepreneur, as well as a personal/business advisor. More about him at www.AlvinAng.sg.