LINEAR PROGRAMMING PART I

MODEL FORMULATION
BY DR. ALVIN ANG

I. INTRODUCTION

- Linear programming (LP, also called linear optimization) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships.

- Linear programming is a special case of mathematical programming (mathematical optimization) (Wikipedia 2018).

- LP is widely mentioned in undergraduate textbooks and is also a popular technique used in industry (Anderson, Sweeney et al. 2005)

- In this publication, we will be using a simple example to demonstrate how linear programming can be used to solve a baking problem.

II. CONTENTS

III. SCENARIO

- Lucy, a Foreign Domestic Worker (FDW), or maid for short, is tasked by her Madame (or employer) to bake 2 types of cupcakes: Round Shaped Vanilla, and Heart Shaped Chocolate.

- This is because her Madame's son's annual school fun fare (1 day only) is coming up and he needs to raise funds for the school – so he can bring the cupcakes to sell at the fun fare.

- (Her Madame has paid for Lucy to go baking classes but she's not happy that Lucy hasn't done any baking till date – wasting her precious money, so she thinks that this is the perfect opportunity to utilize her).

Figure 1: Round Shaped Vanilla Cupcake

Figure 2: Heart Shaped Chocolate Cupcake

Lucy's Madame has come up with the following logistical requirements:

Table 1: Requirements for Vanilla Cupcakes

Round Shaped Vanilla Cupcake (X_1)	Requirements
Baking Powder needed per cupcake	0.1 kg
Man-hour per Cupcake	1/20th hour (3 min)
Demand for Vanilla Cupcakes at the Fun Fare	less than 25
Profit per Vanilla Cupcake Sold	$2

Table 2: Requirements for Chocolate Cupcakes

Heart Shaped Chocolate Cupcake (X_2)	Requirements
Baking Powder needed per cupcake	0.2 kg
Man-hour per Cupcake	1/10th hour (6 min)
Demand for Chocolate Cupcakes at the Fun Fare	less than 60
Profit per Chocolate Cupcake Sold	$3

Table 3: Resources Available in the Kitchen

Resources	Available
Baking Powder	10 kg
Man-hours from Maid	10 hours

IV. OBJECTIVES

Lucy's Madame wants to know, given the requirements above:

1. How many Chocolate and Vanilla cupcakes should she make in order to maximize profits?
2. What is the Maximum profit that her son can earn at the fun fair per day?
3. Should the requirements change or deviate, what will the new maximum profit be?
4. And subsequently, the new optimal number of cupcakes to be baked?

V. MODEL FORMULATION

A. STEP 1: NAME THE VARIABLES

- Let
- X_1 represent the Number of Vanilla Cupcakes to be sold.
- X_2 represent the Number of Chocolate Cupcakes to be sold.
- Z represent the Potential Total Profit that can be earned.

B. STEP 2: STATE THE OBJECTIVE EQUATION

$$\mathbf{Max\ Z = \$2X_1 + \$3X_2}$$

- The equation states the objective of Lucy's Madame – where Max means Maximum.
- Since each Vanilla Cupcake (X_1) is sold at a profit for $2, and likewise Chocolate Cupcake (X_2) at profit of $3, a maximum profit can be earned by finding out the number of X_1 and X_2.

C. STEP 3: STATE THE CONSTRAINTS

Using the requirements in Table 1, 2 and 3,

Constraint 1: Baking Powder

$$\mathbf{(0.1)*(X_1) + (0.2)*(X_2) \leq 10\ kg}$$

- Since there is only 10kg of Baking Powder left in the kitchen, yet each Vanilla cupcake uses 0.1 kg, and Chocolate cupcake uses 0.2 kg, of Baking Powder, the maximum overall amount of Baking Powder that can be used is 10 kg!

Constraint 2: Man-hours

$$(0.05)*(X_1) + (0.1)*(X_2) \leq 10 \text{ hours}$$

- Since Lucy's Madame only allows Lucy to use a maximum of 10 hours to complete baking everything, yet the Vanilla cupcake takes 3 minutes, while Chocolate cupcake takes 6 minutes, the maximum allowable time to complete the task is 10 hours!

Constraint 3: Demand for Vanilla Cupcakes

$$X_1 \leq 25 \text{ Vanilla cupcakes}$$

- It is wasteful to overproduce.
- Since Lucy's Madame anticipates a maximum of 25 Vanilla cupcakes to be sold at the fun fair, the maximum number of Vanilla cupcakes baked should not exceed 25!

Constraint 4: Demand for Chocolate Cupcakes

$$X_2 \leq 60 \text{ Chocolate cupcakes}$$

- It is wasteful to overproduce.
- Since Lucy's Madame anticipates a maximum of 60 Chocolate cupcakes to be sold at the fun fair, the maximum number of Chocolate cupcakes baked should not exceed 25!

Constraint 5: Non-negativity

$X_1, X_2 \geq 0$ cupcakes

- It is well understood that there cannot be a negative number of cupcakes baked, thus this constraint exists.

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VI. REFERENCES

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VII. ABOUT THE AUTHOR

A. ABOUT DR ALVIN ANG

Dr. Alvin Ang earned his Ph.D., Masters and Bachelor degrees from NTU, Singapore. He was a Professor as well as a personal/business advisor. More about him at www. AlvinAng.sg.