NETWORK MODELING

BY HAND AND EXCEL

DR. ALVIN ANG

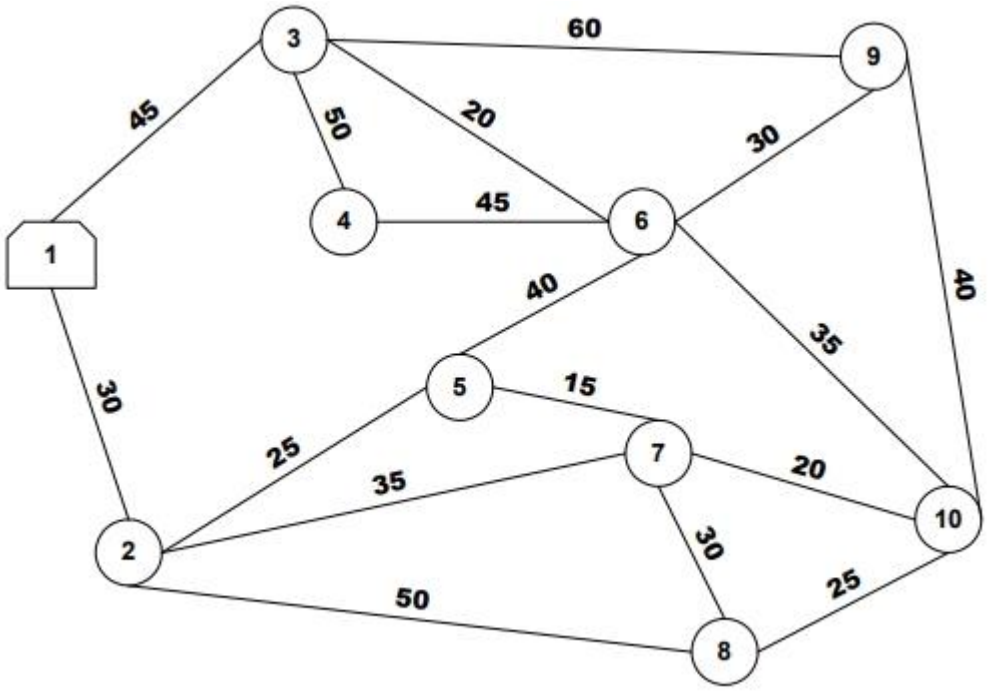
WWW.ALVINANG.SG

CONTENTS

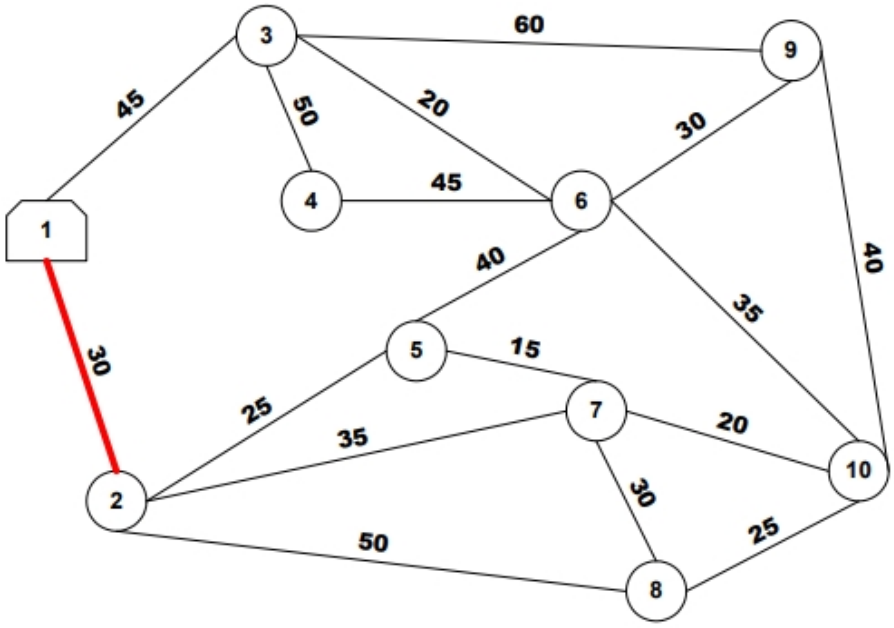
I. MINIMAL SPANNING TREE

- Minimal Spanning Tree
  - Used to determine minimum distance to connect all nodes
  - Used in MRT and telephone companies
  - Minimize length of transit lines and phone cables

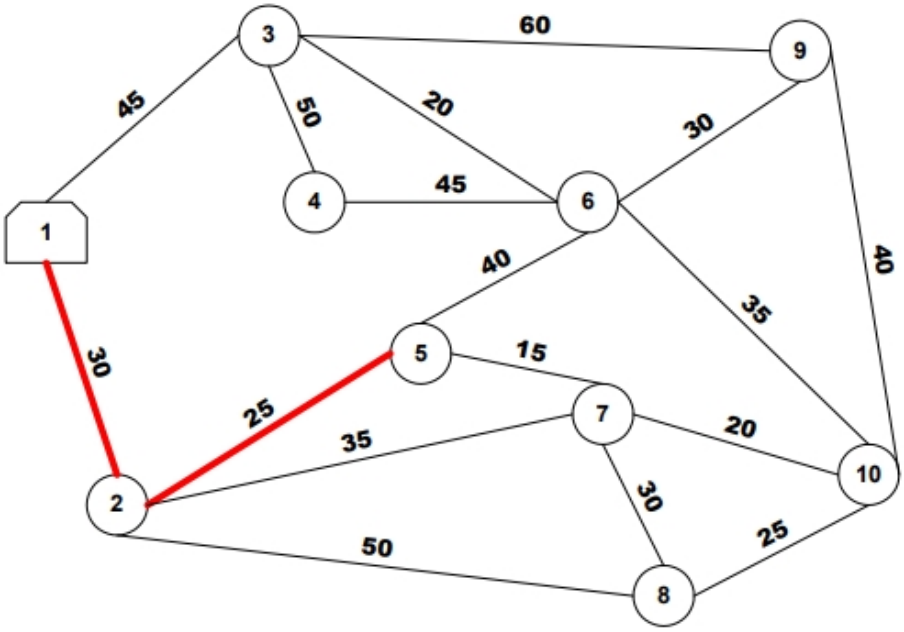
A. EXAMPLE 1



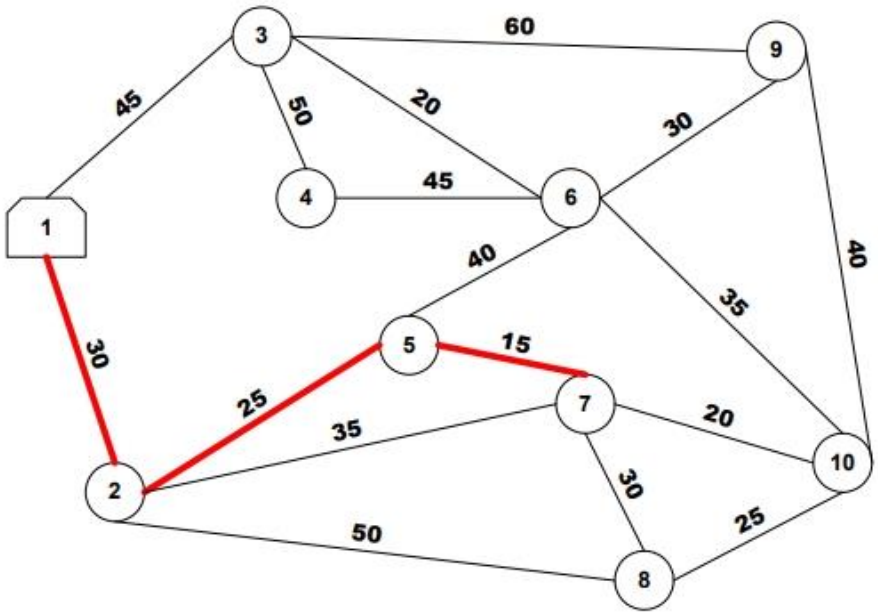
Step 1



Step 2



Step 3



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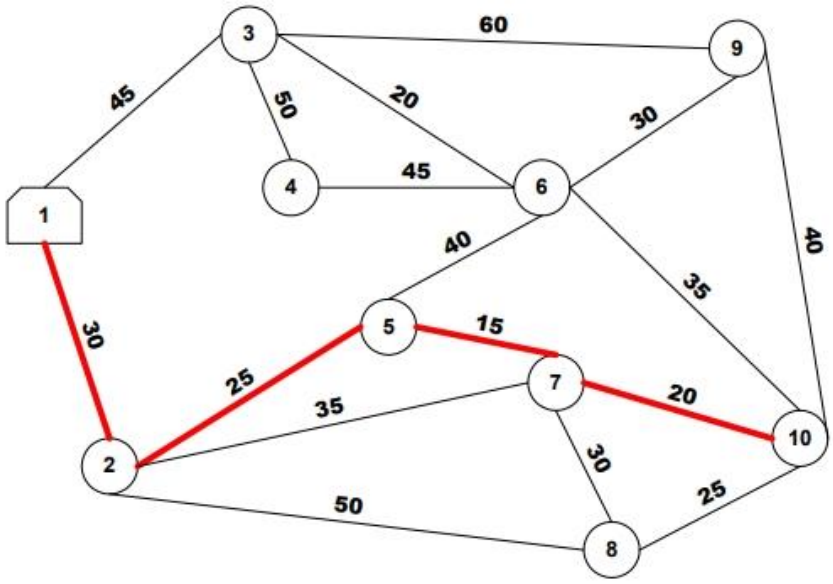
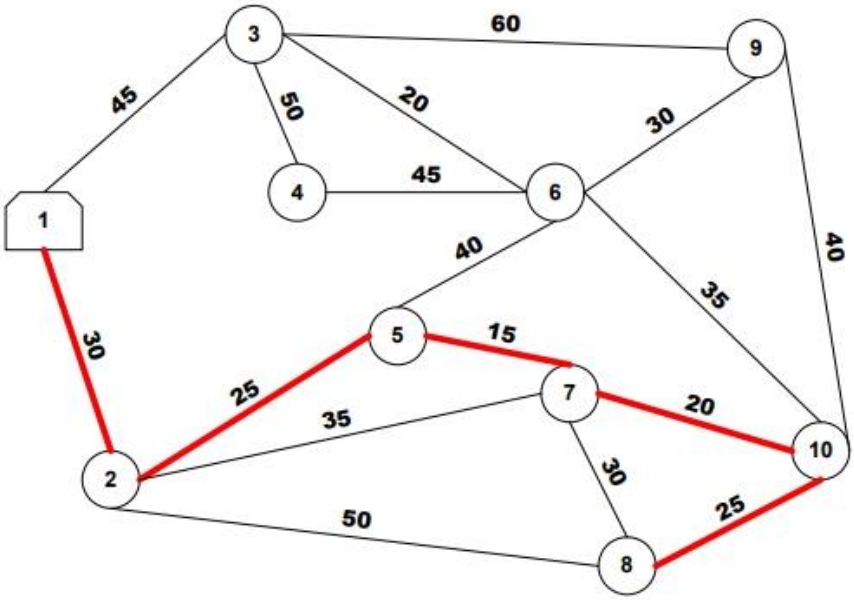
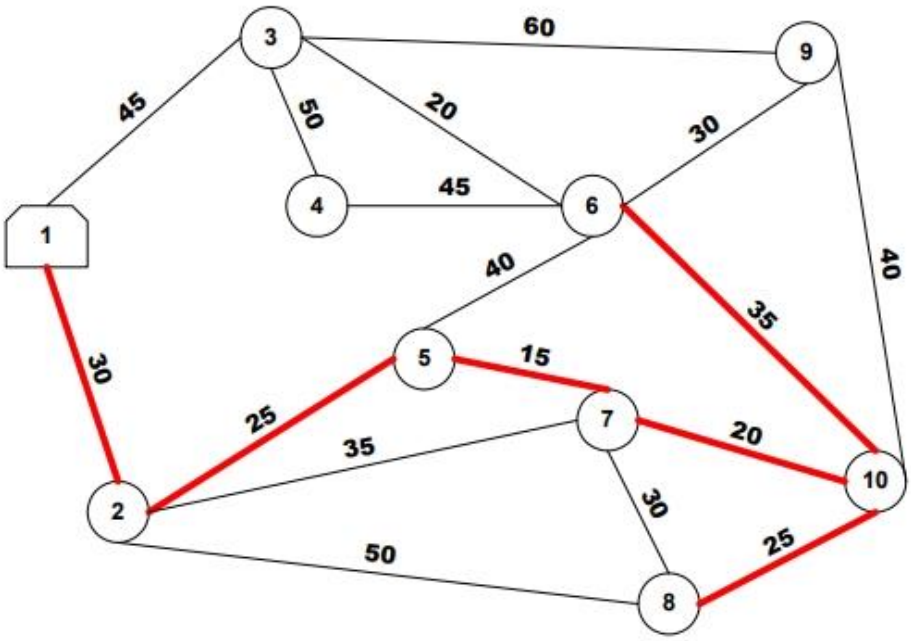


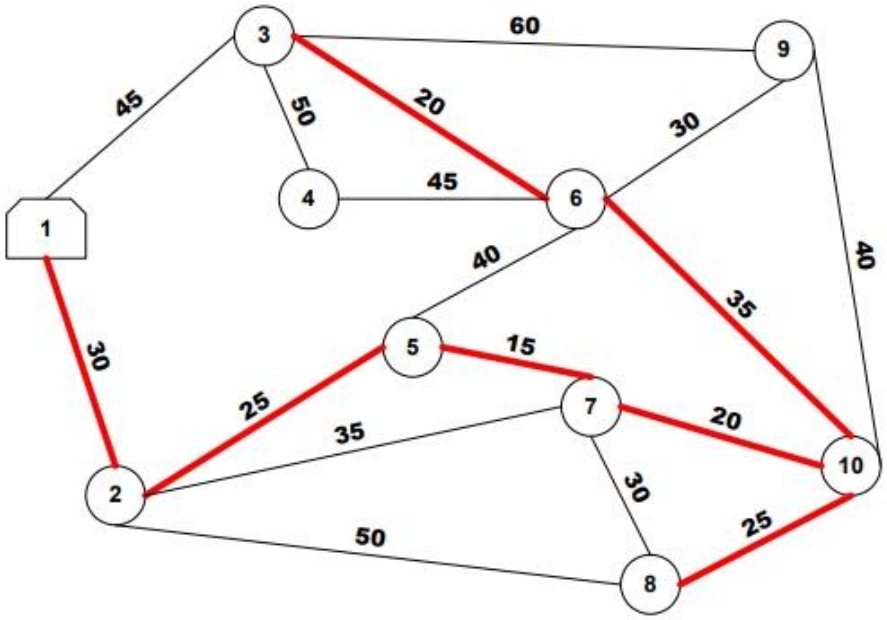
Step 5



Step 6



Step 7



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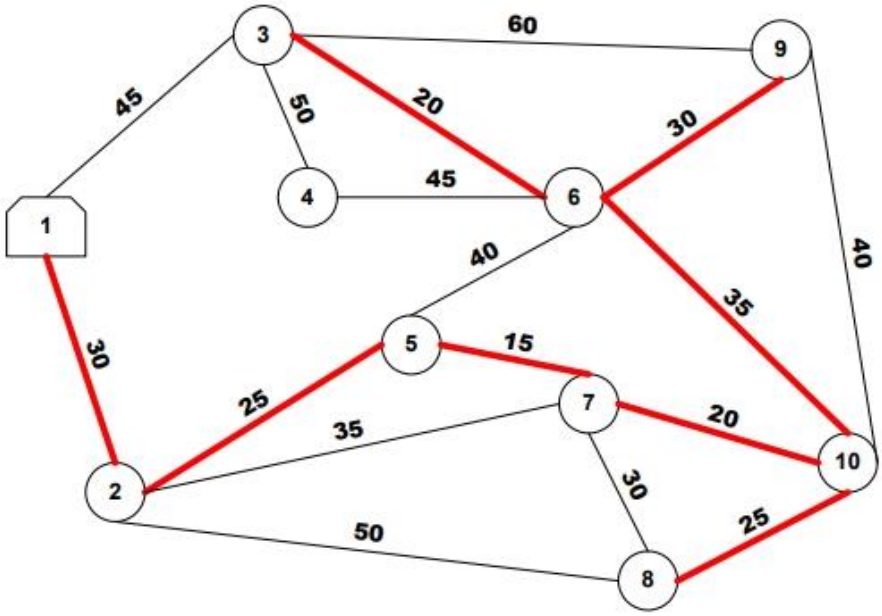
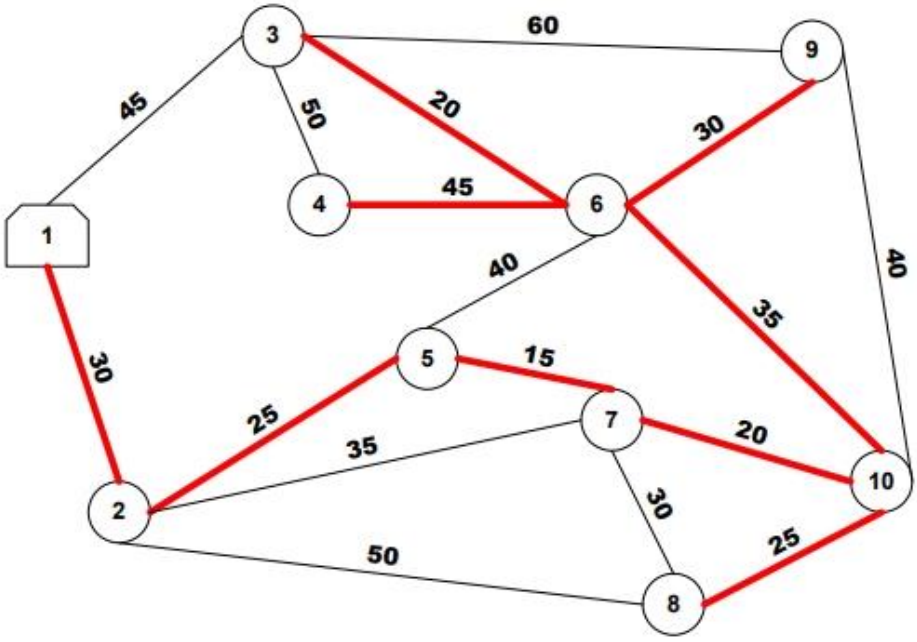


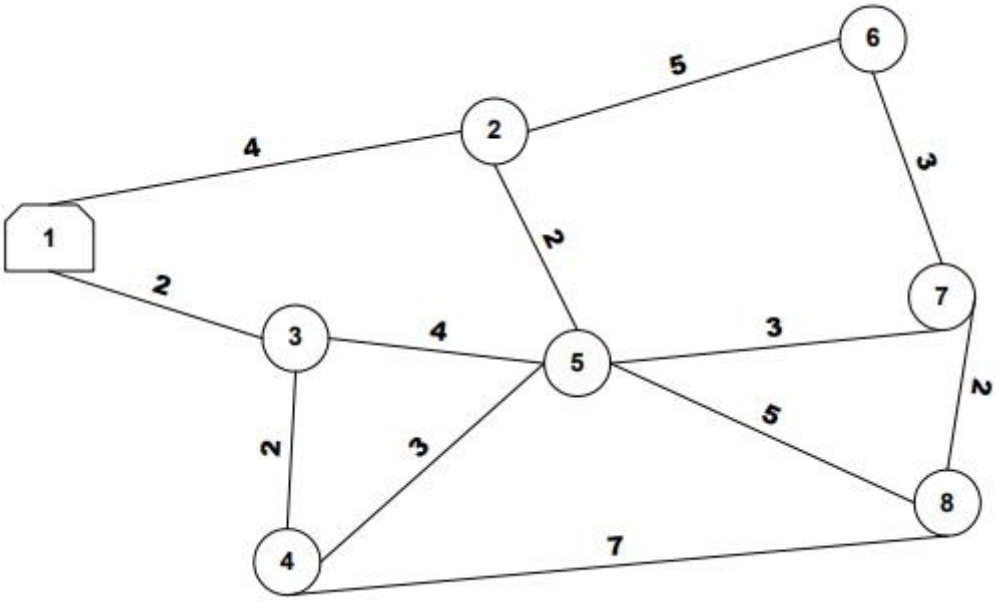
Step 9



Step 10

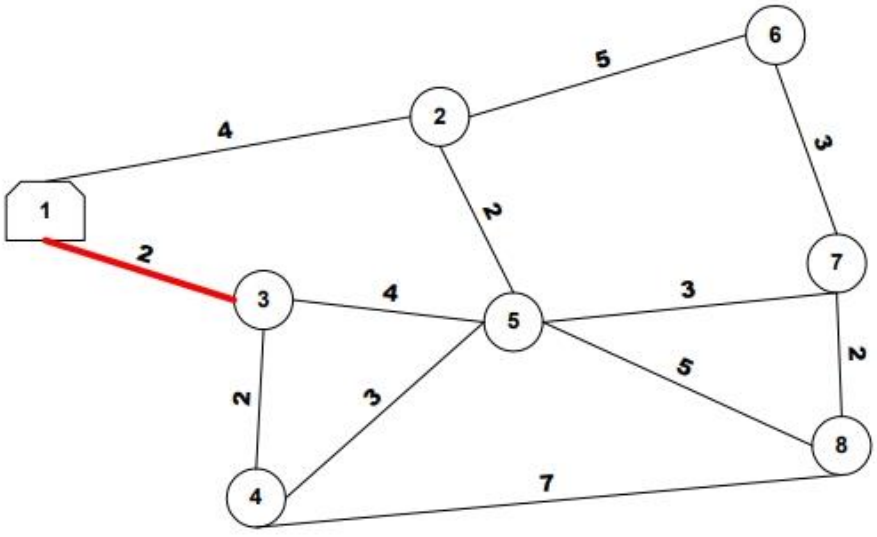
Arcs	Values
1 --2	30
2 -- 5	25
5 -- 7	15
7 -- 10	20
10 -- 8	25
10 -- 6	35
6 -- 3	20
6 -- 9	30
6 --4	45
Total	245

B. EXAMPLE 2

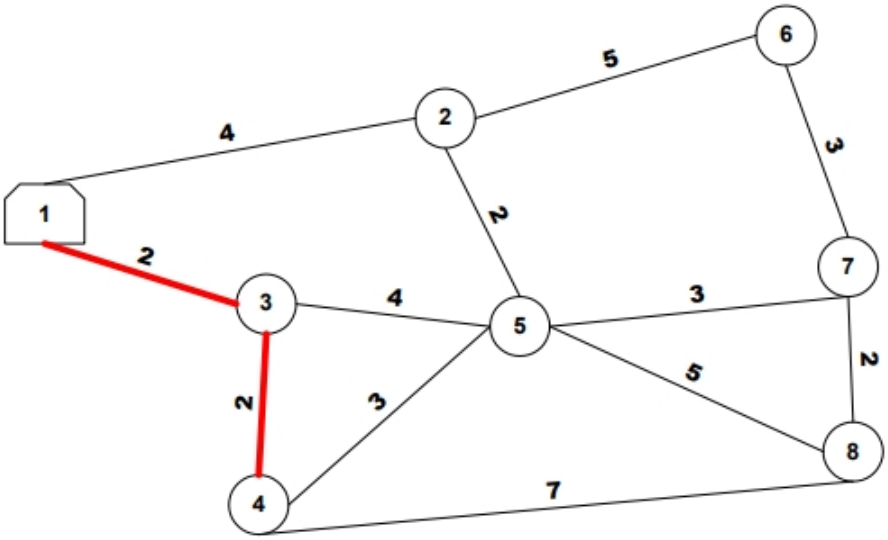


Step 1

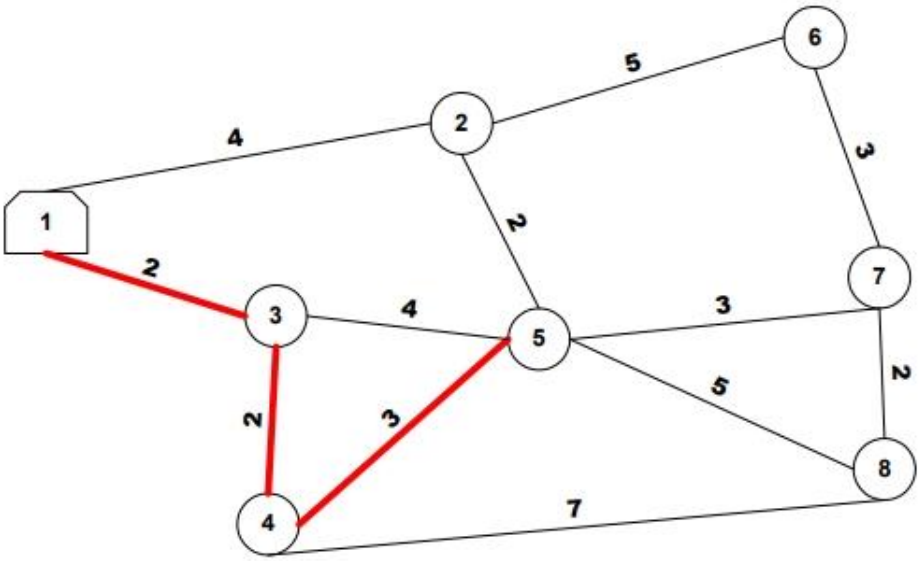
Algorithm



Step 2

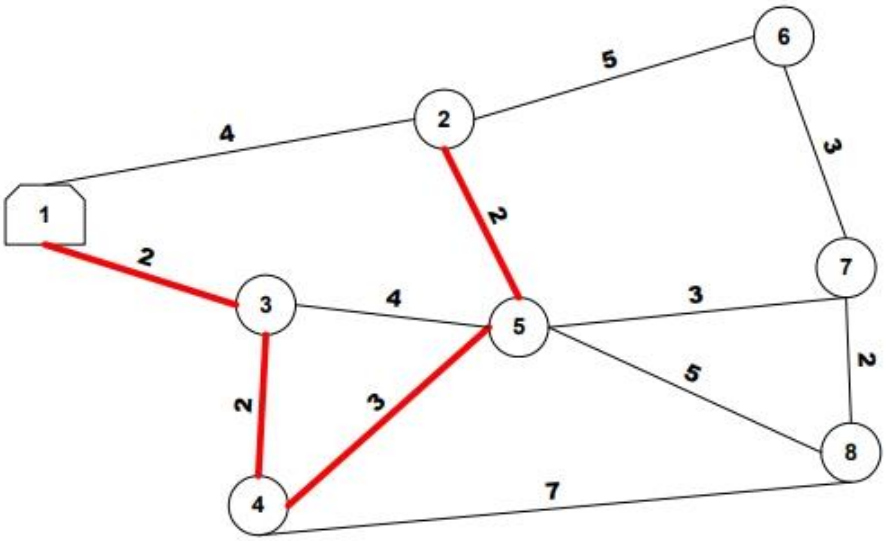


Step 3



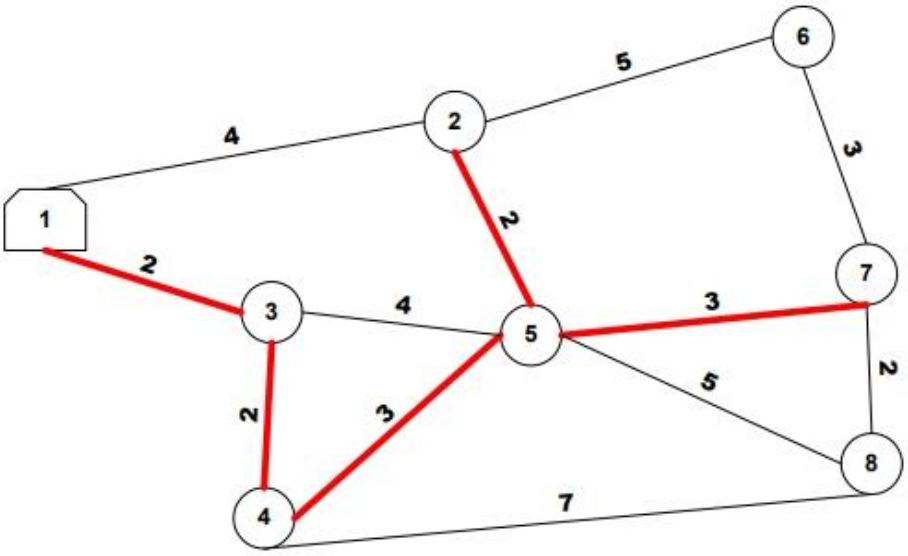
Step 4

Algorithm



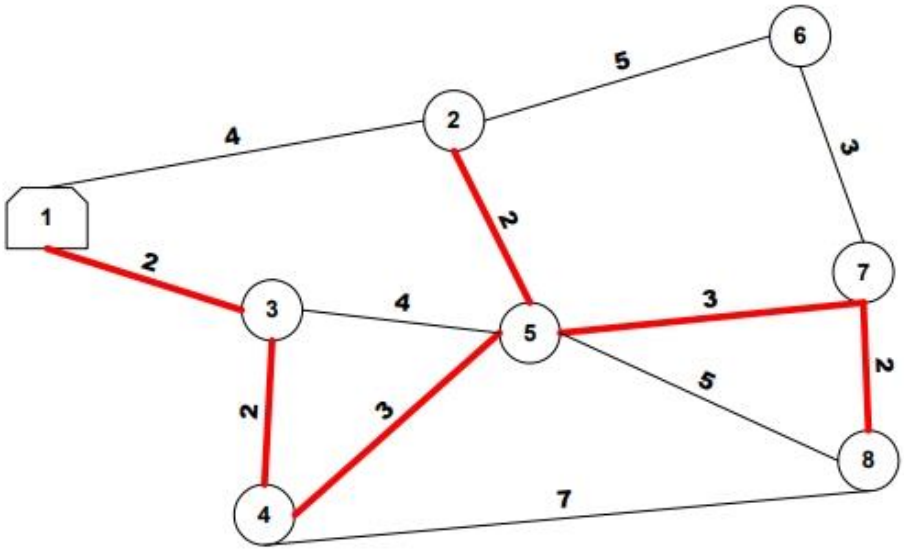
Step 5

Algorithm



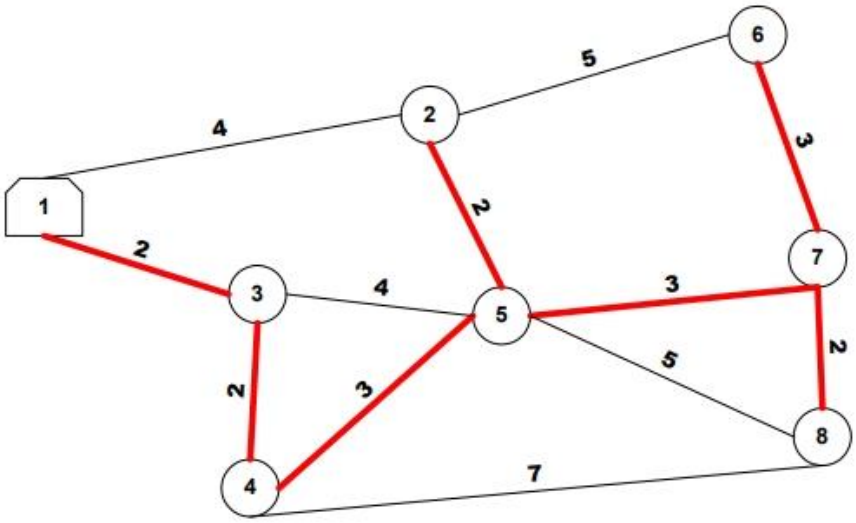
Step 6

Algorithm



Step 7

Algorithm



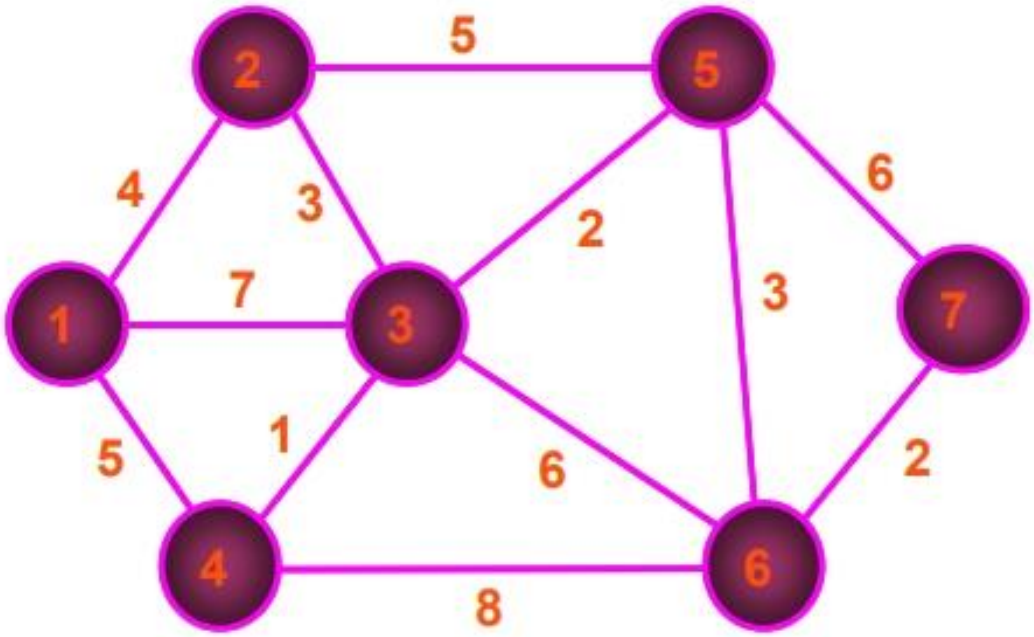
Step 8

1 -- 3	2
3 -- 4	2
4 --5	3
5 --2	2
5 --7	3
7 --8	2
7 --6	3
Total	17

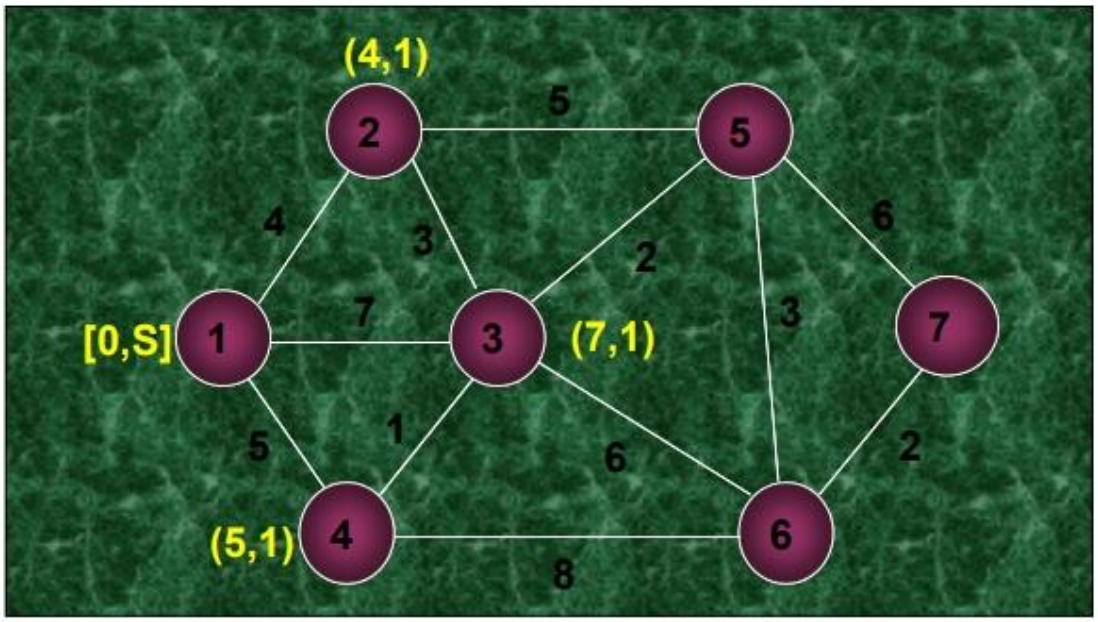
A. BY HAND

- Shortest Route Algorithm
  - To find Shortest Path from 1 node to another node
  - Or Shortest Path from 1 node to ALL other nodes
  - Practical Use:
    - GPS System / Garmin Technologies
    - You want to find shortest path from home to work?

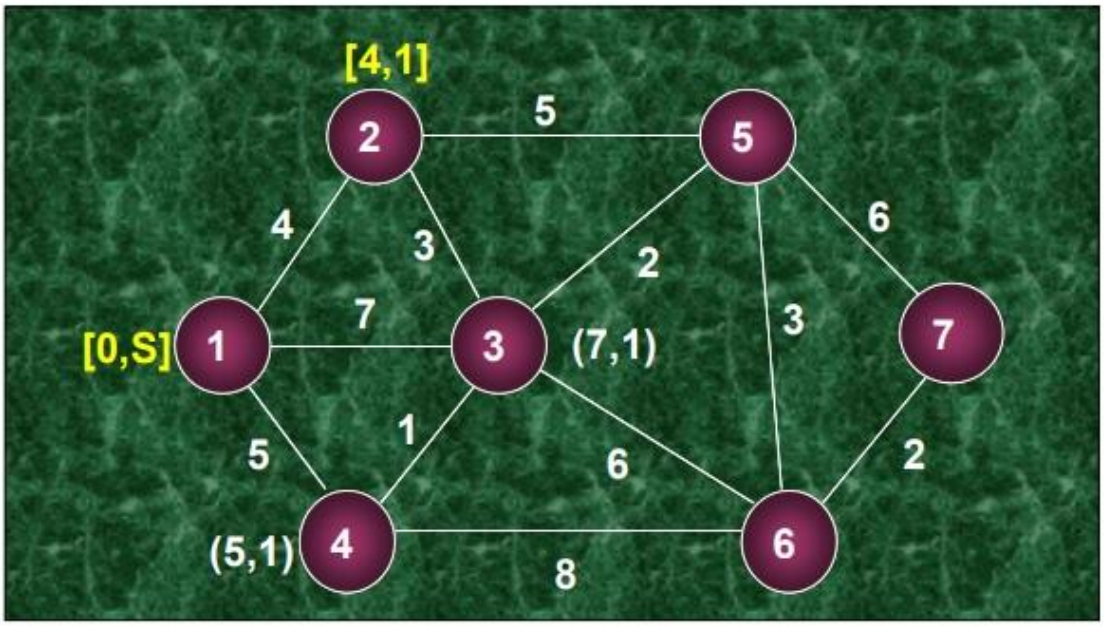
1. EXAMPLE 1



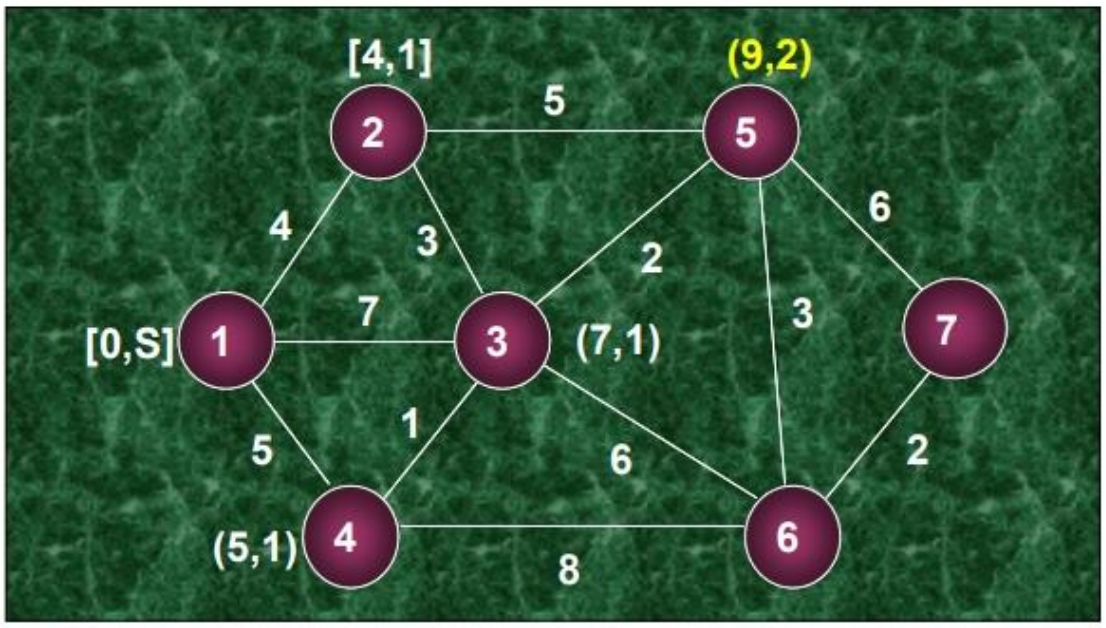
Illustrated Problem 6.1: Step 1



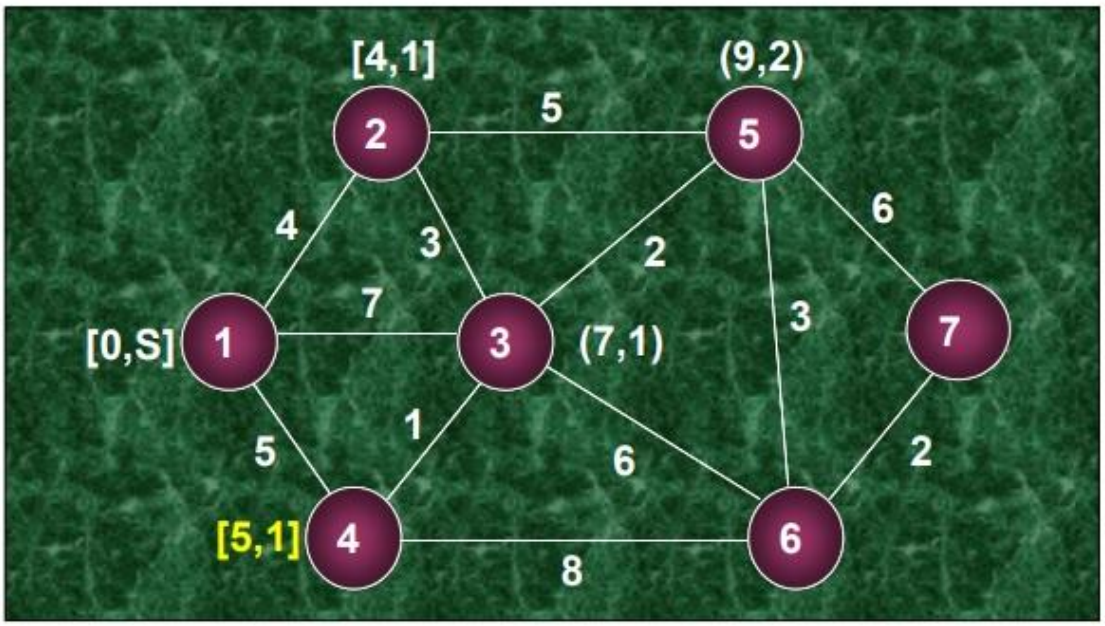
Illustrated Problem 6.1: Step 2



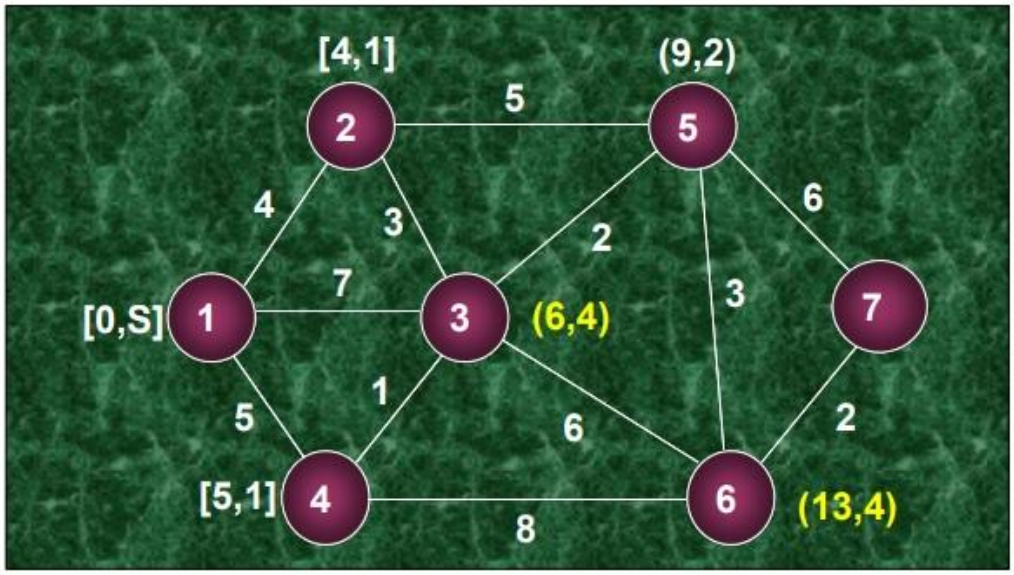
Illustrated Problem 6.1: Step 3



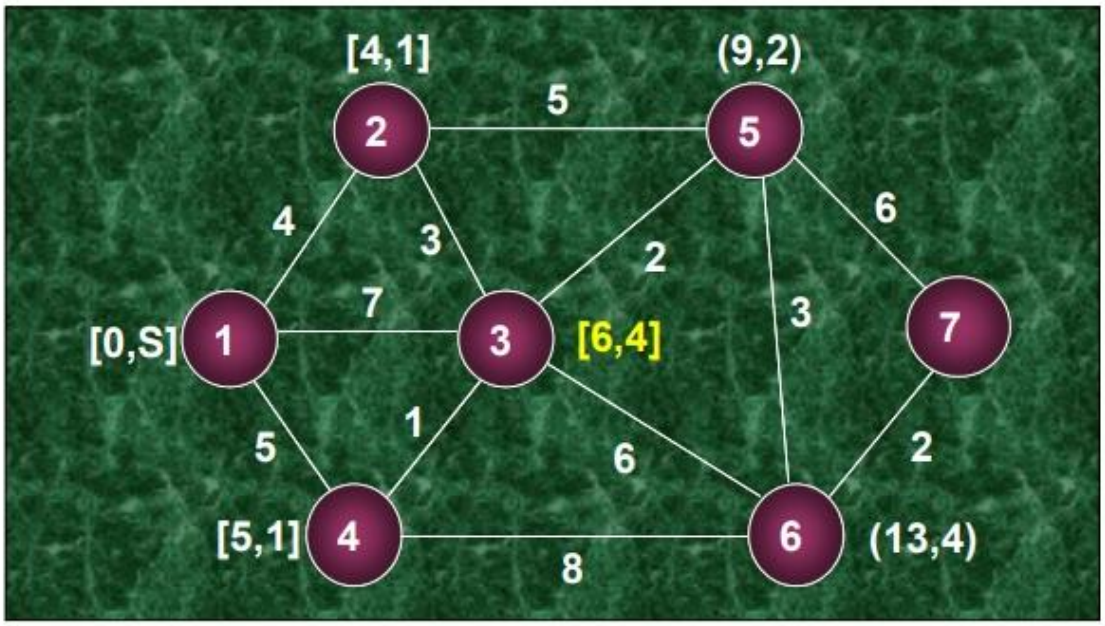
Illustrated Problem 6.1: Step 4



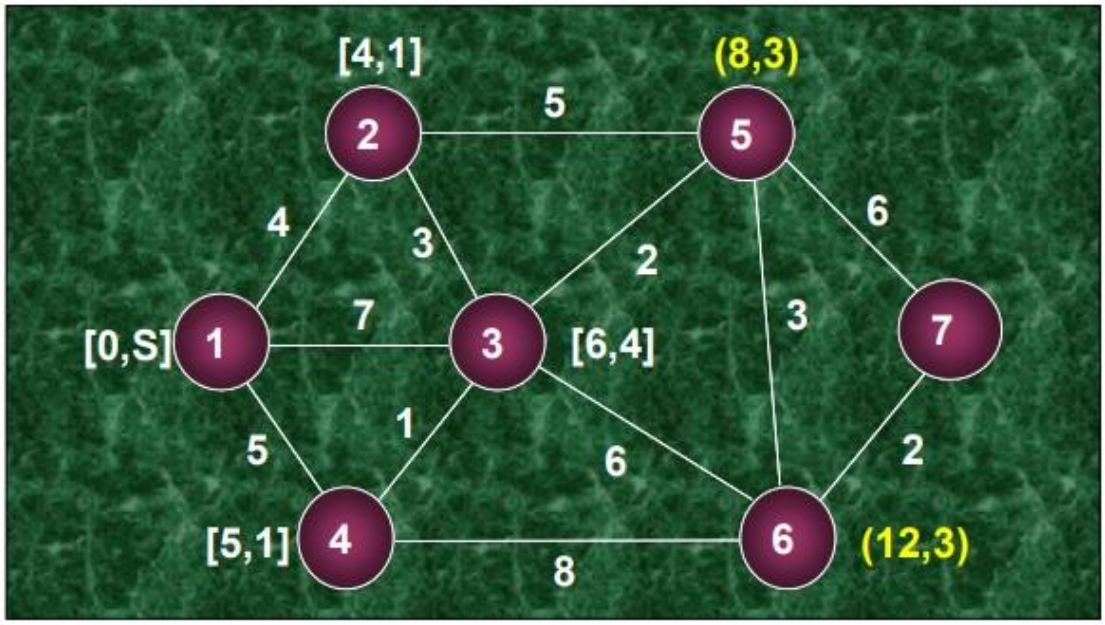
Illustrated Problem 6.1: Step 5



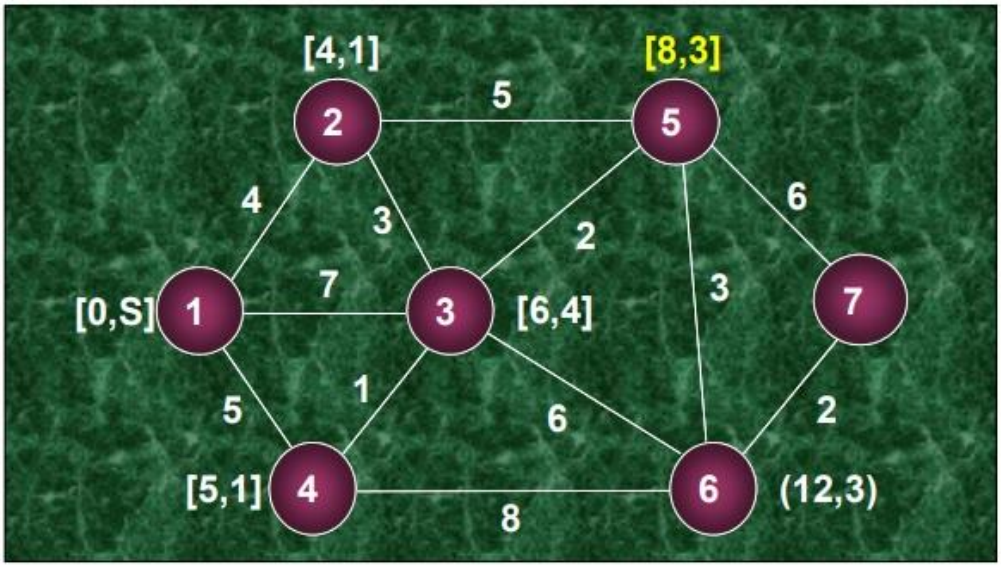
Illustrated Problem 6.1: Step 6



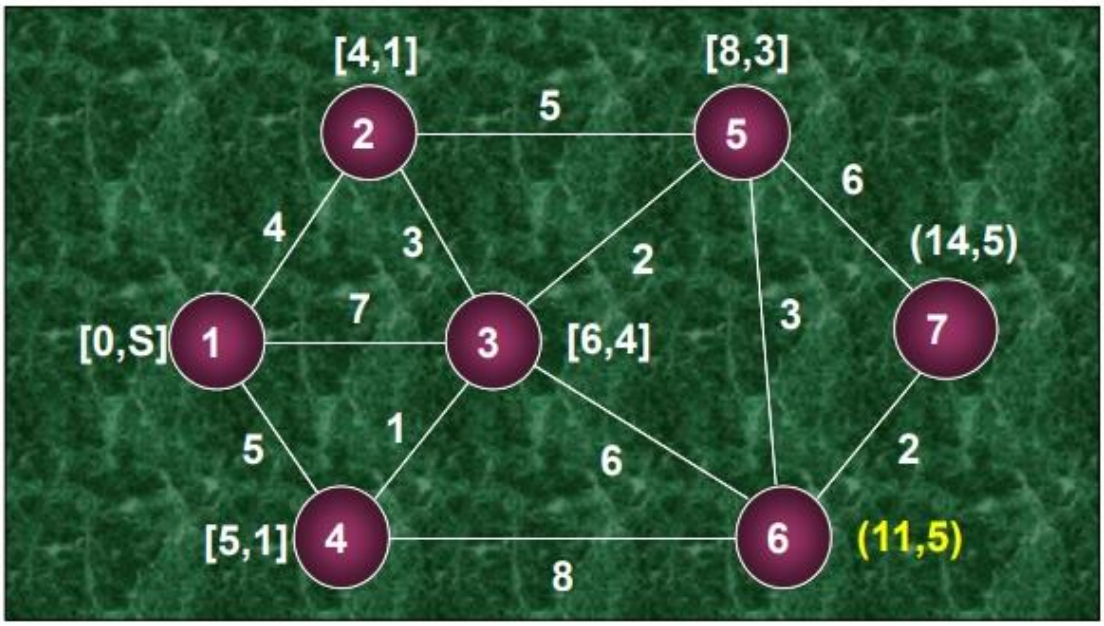
Illustrated Problem 6.1: Step 7



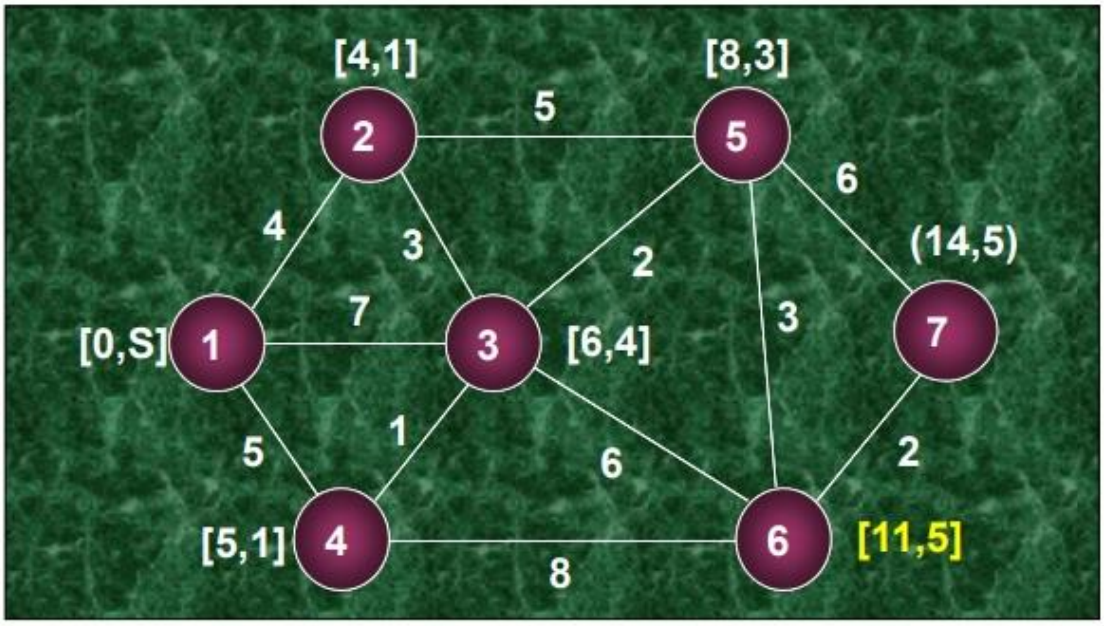
Illustrated Problem 6.1: Step 8



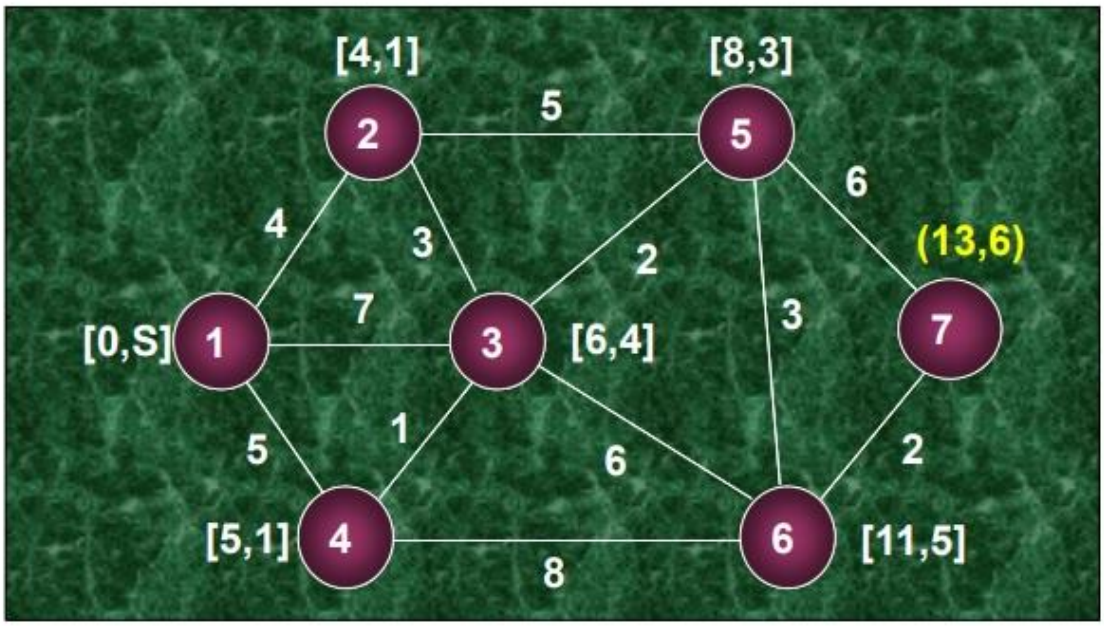
Illustrated Problem 6.1: Step 9



Illustrated Problem 6.1: Step 10



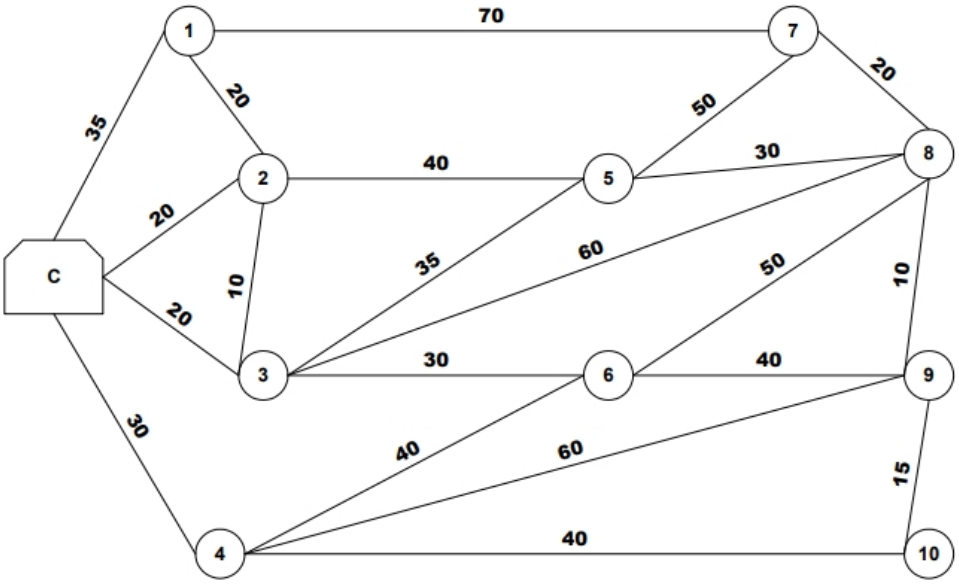
Illustrated Problem 6.1: Step 11



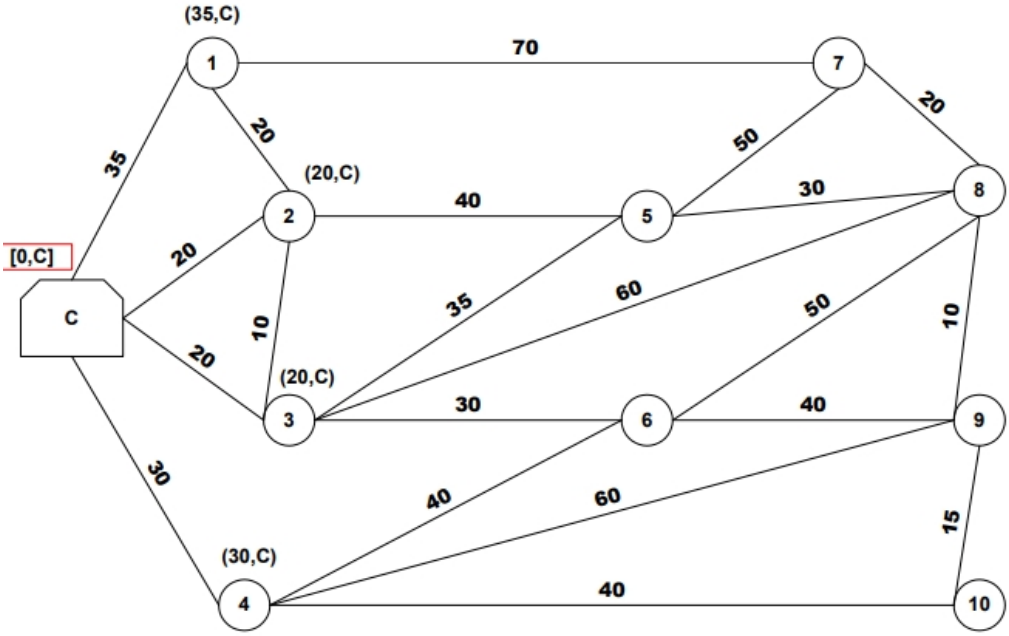
– Illustrated Problem 6.1: Step 12 (Final)

Node	Minimum Distance	Shortest Route
2	4	1-2
3	6	1-4-3
4	5	1-4
5	8	1-4-3-5
6	11	1-4-3-5-6
7	13	1-4-3-5-6-7

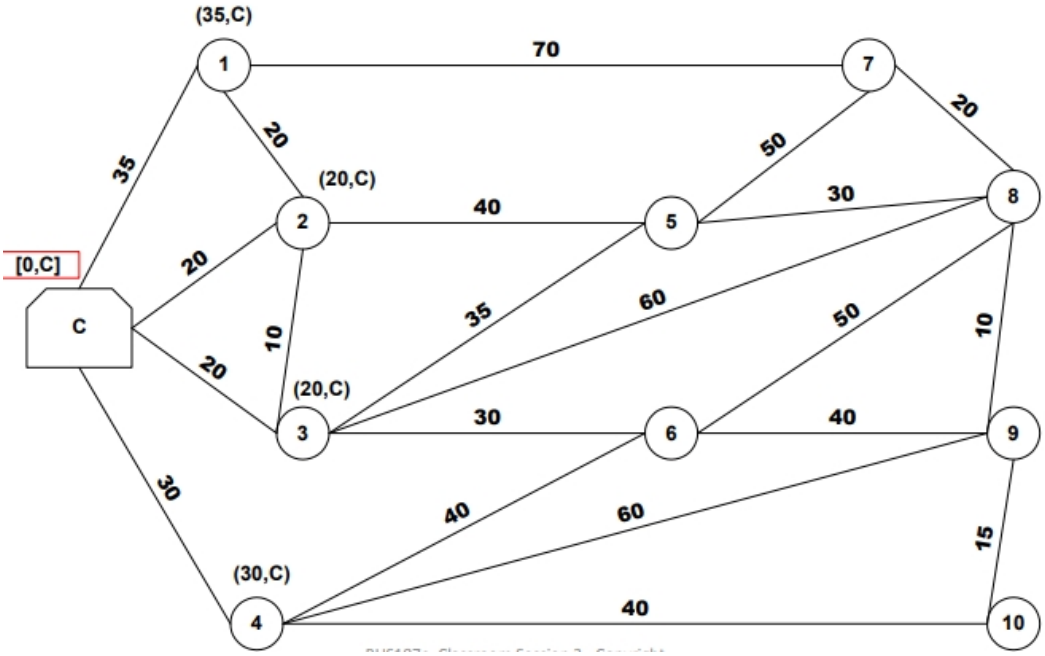
2. EXAMPLE 2



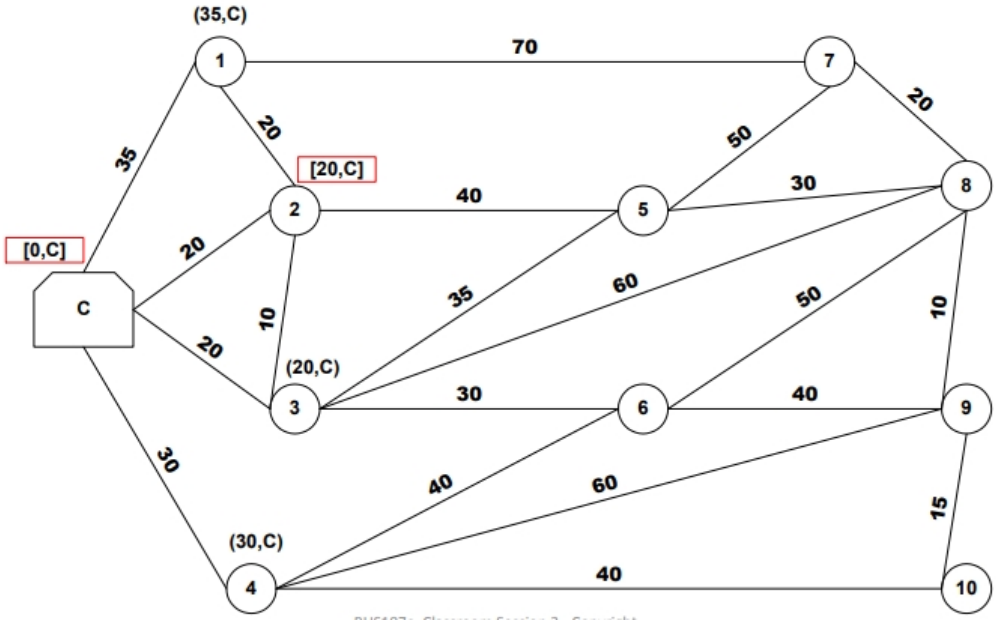
– An Old Exercise : Step 1 **Network Modeling** Shortest Route Algorithm



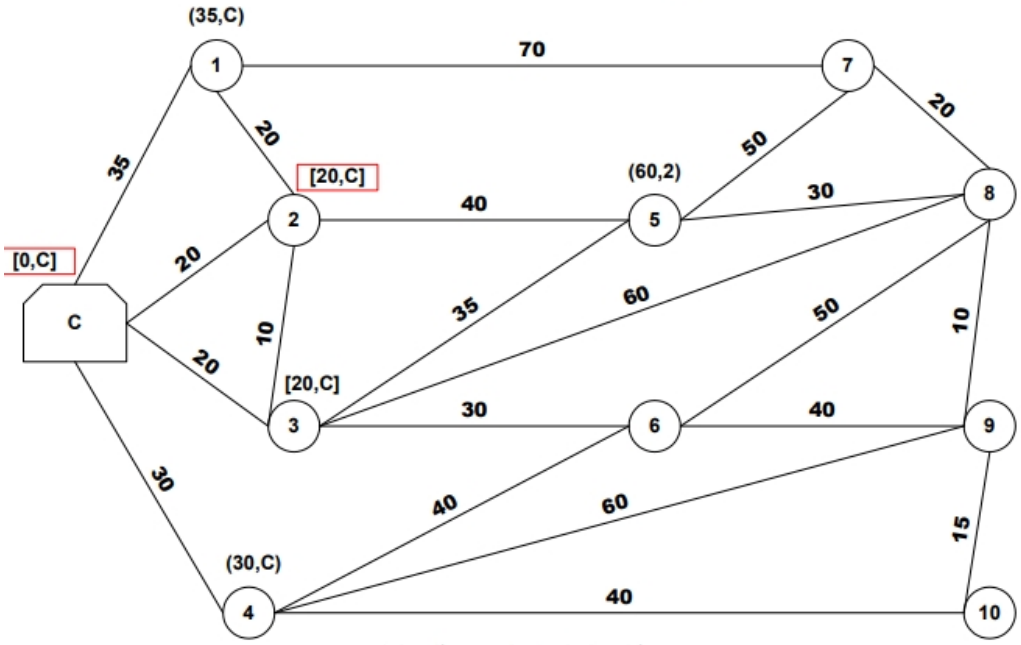
– An Old Exercise : Step 2 **Network Modeling** Shortest Route Algorithm



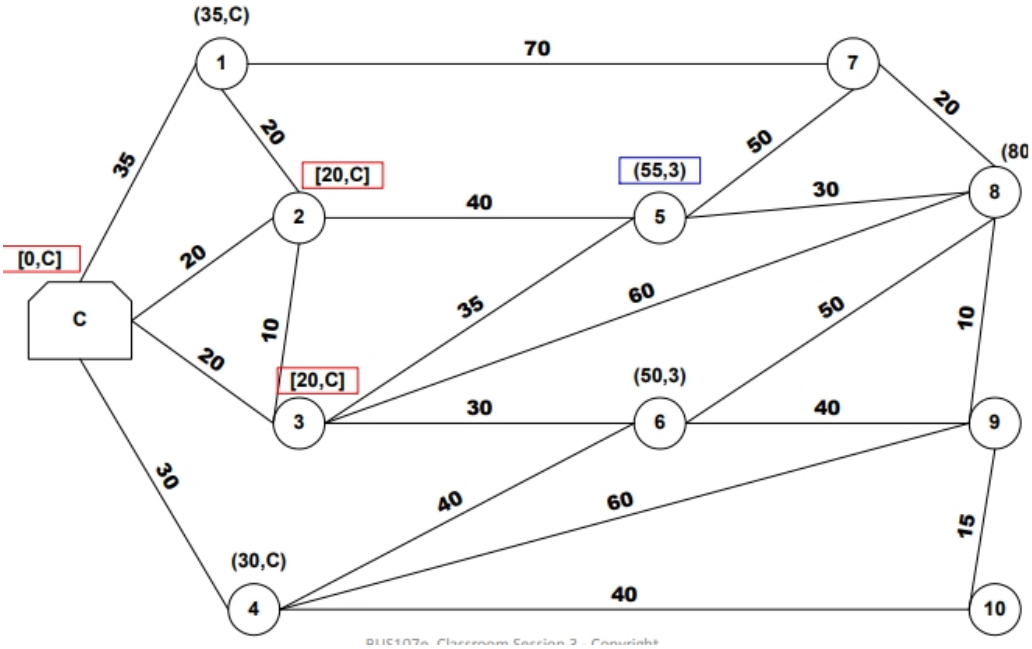
– An Old Exercise : Step 3 **Network Modeling** Shortest Route Algorithm



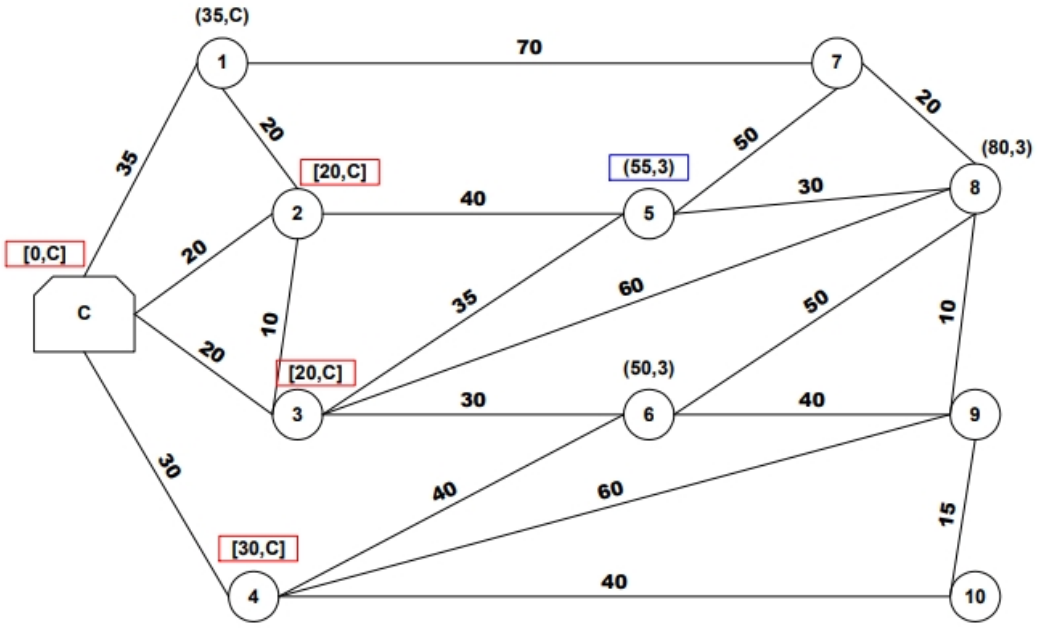
– An Old Exercise : Step 4 **Network Modeling** Shortest Route Algorithm



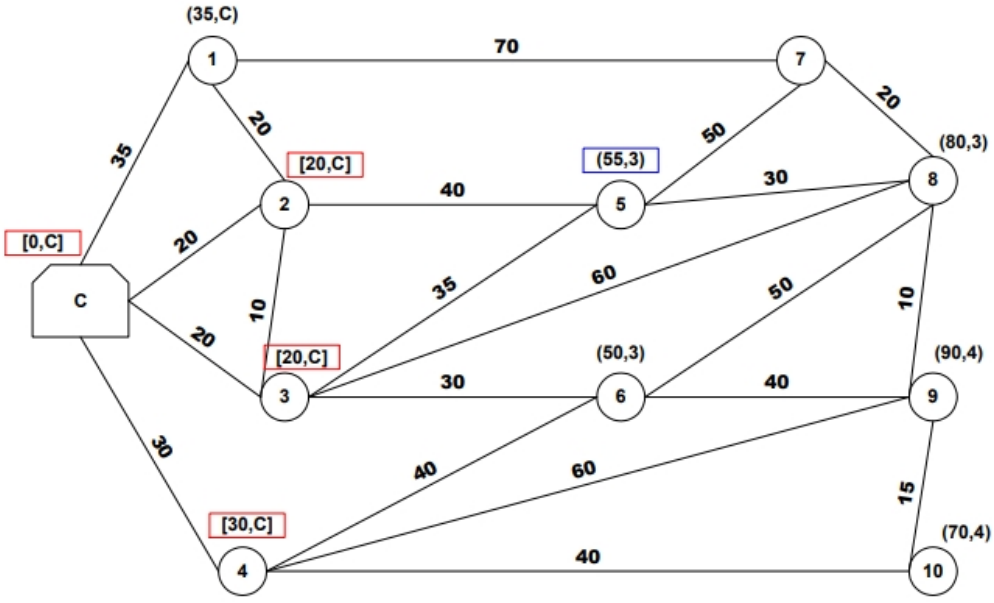
– An Old Exercise : Step 5 **Network Modeling** Shortest Route Algorithm



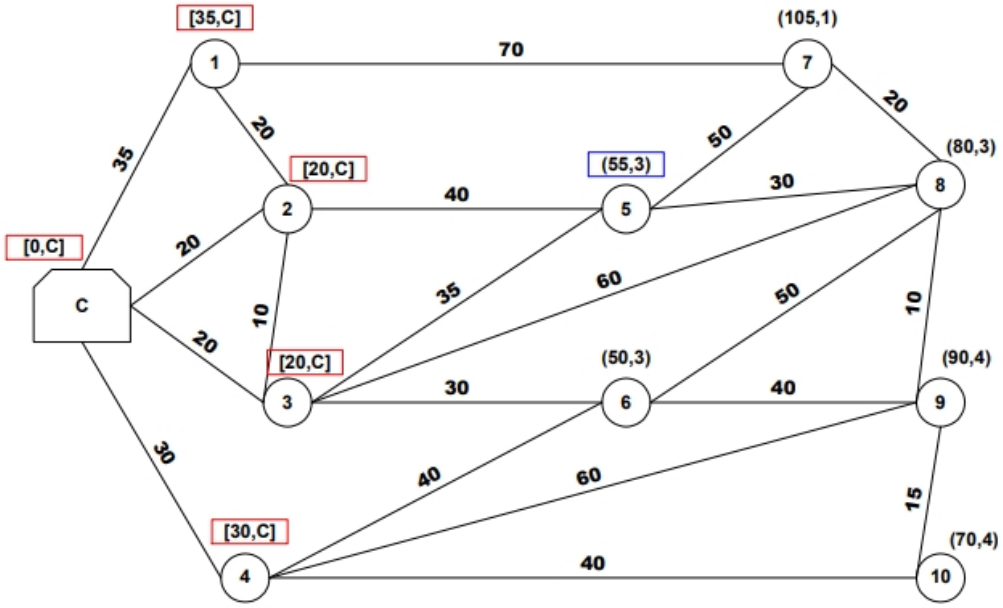
– An Old Exercise : Step 6 **Network Modeling** Shortest Route Algorithm



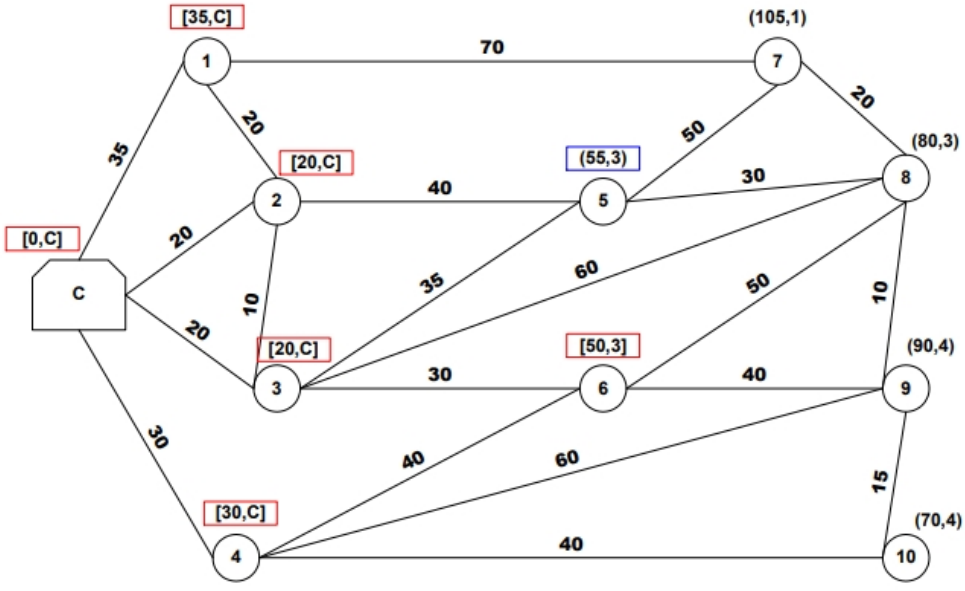
– An Old Exercise : Step 7 **Network Modeling** Shortest Route Algorithm



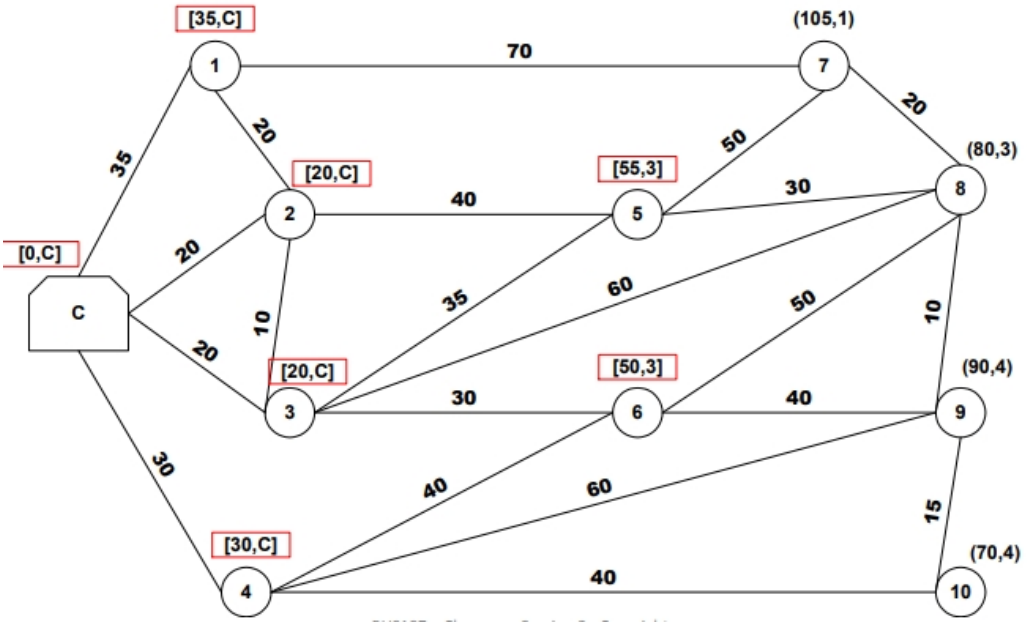
– An Old Exercise : Step 8 **Network Modeling** Shortest Route Algorithm



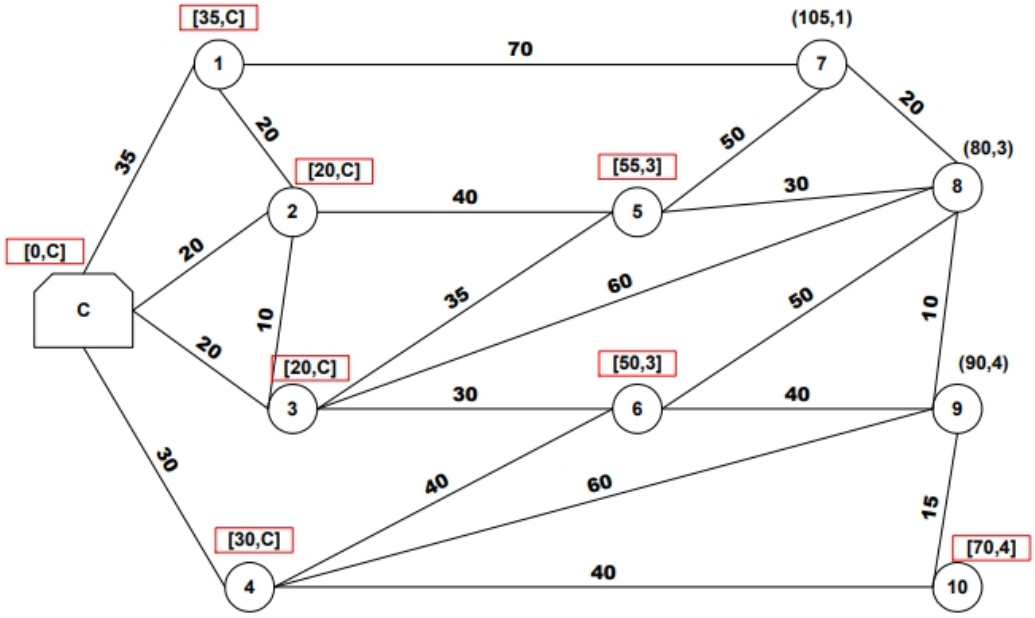
– An Old Exercise : Step 9 **Network Modeling** Shortest Route Algorithm



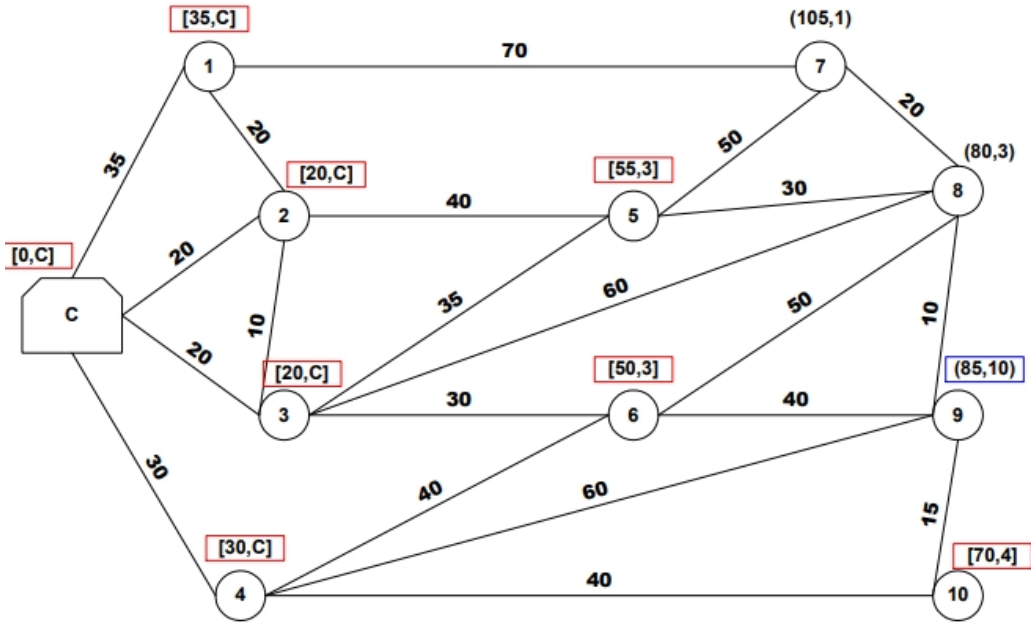
– An Old Exercise : Step [N] **Network Modeling** Shortest Route Algorithm



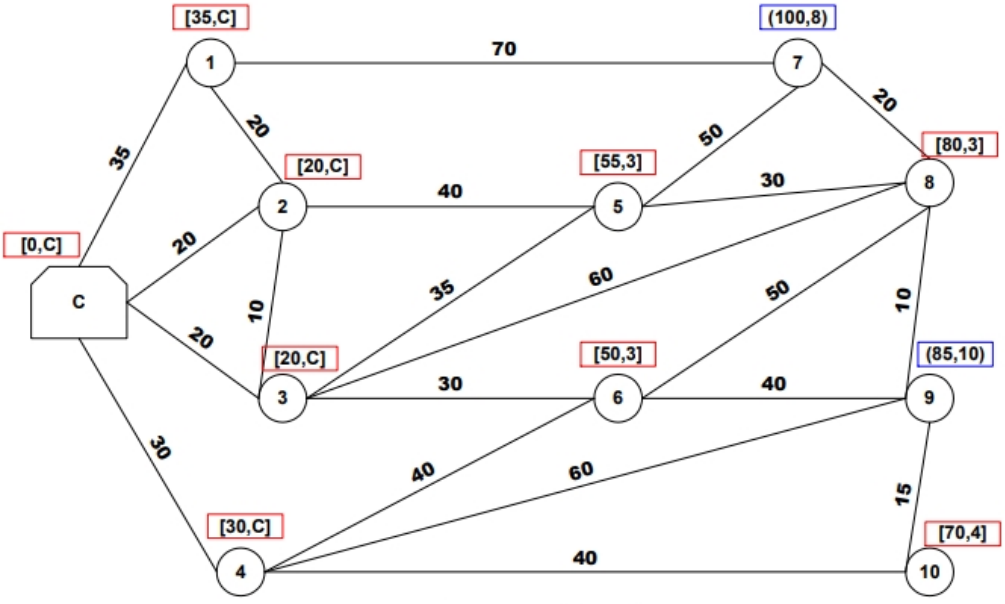
– An Old Exercise : Step [N] **Network Modeling** Shortest Route Algorithm



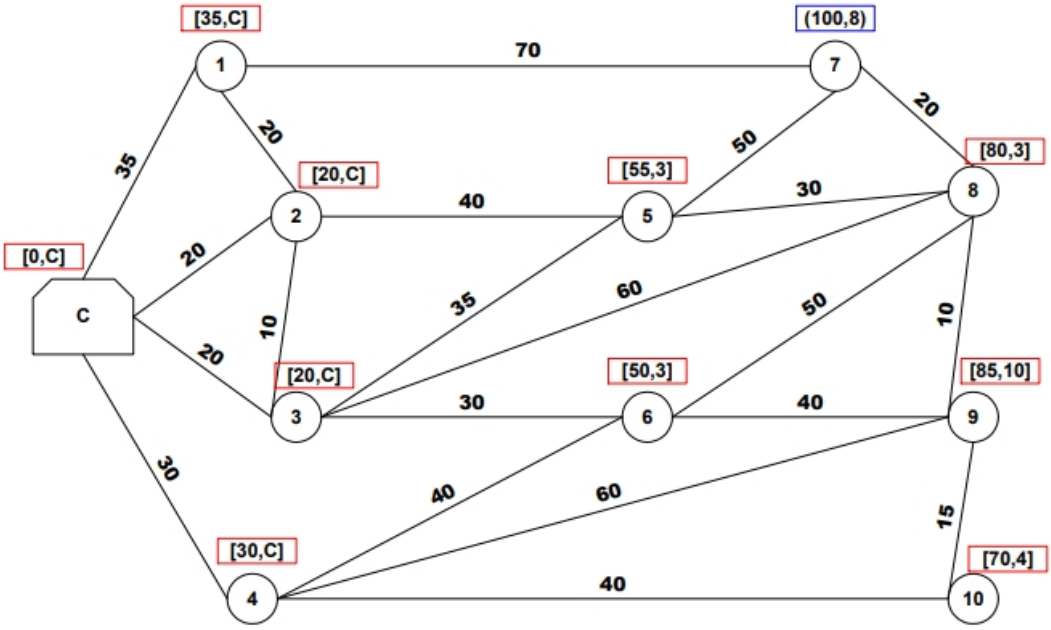
– An Old Exercise : Step [2] **Network Modeling** Shortest Route Algorithm



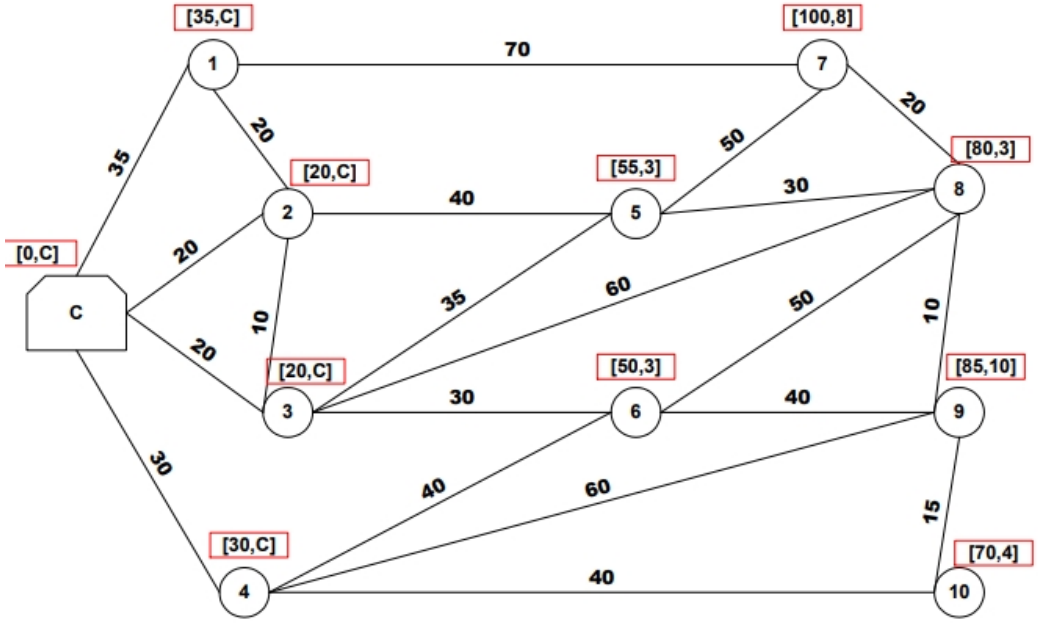
– An Old Exercise : Step [3] **Network Modeling** Shortest Route Algorithm



– An Old Exercise : Step [14] **Network Modeling** Shortest Route Algorithm



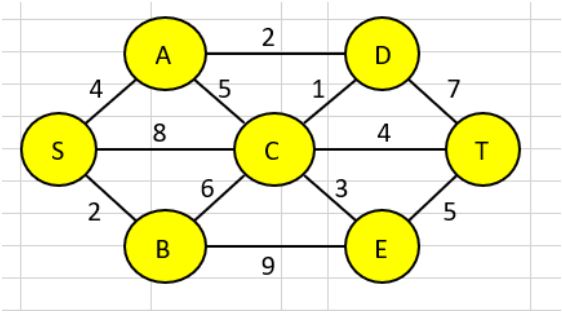
– An Old Exercise : Step [15] **Network Modeling** Shortest Route Algorithm



Node	Shortest Route from Node C	Distance
1	C - 1	35
2	C - 2	20
3	C - 3	20
4	C - 4	30
5	C - 3 - 5	55
6	C - 3 - 6	50
7	C - 3 - 8 - 7	100
8	C - 3 - 8	80
9	C - 4 - 10 - 9	85
10	C - 4 - 10	70

B. USING EXCEL

File: https://www.alvinang.sg/s/shortest-path-problem.xlsx



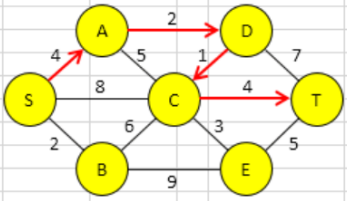
- We are trying to get from S to T
- We need to find the Shortest Path and the Shortest Distance.

The optimal solution:

Shortest Path Problem

From	To	Distance	Go
S	A	4	1
S	B	2	0
S	C	8	0
A	C	5	0
A	D	2	1
B	C	6	0
B	E	9	0
C	A	5	0
C	B	6	0
C	D	1	0
C	E	3	0
C	T	4	1
D	A	2	0
D	C	1	1
D	T	7	0
E	B	9	0
E	C	3	0
E	T	5	0
		Total Distance	11

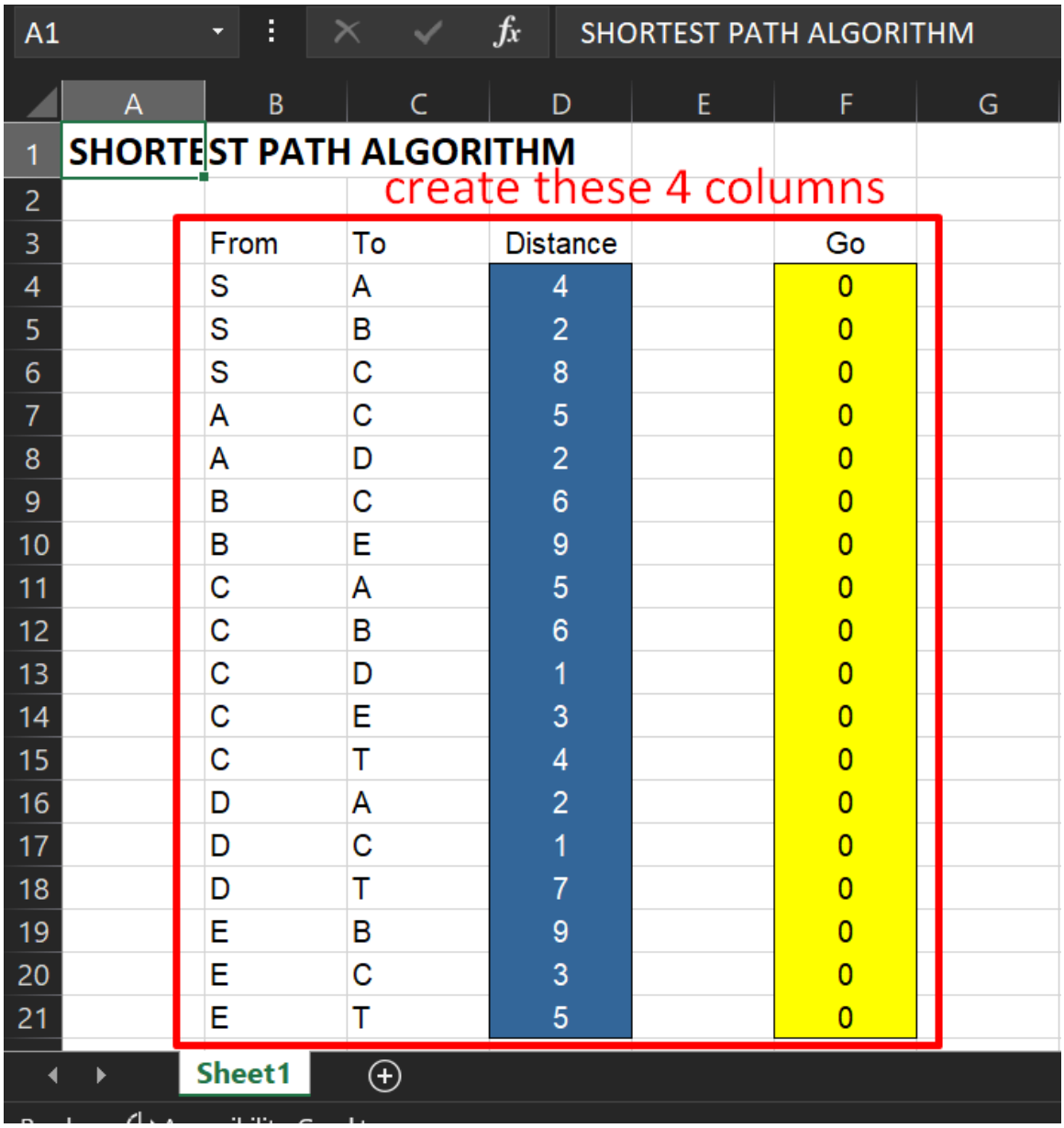
Nodes	Net Flow		Supply/Demand
S	1	=	1
A	0	=	0
B	0	=	0
C	0	=	0
D	0	=	0
E	0	=	0
T	-1	=	-1

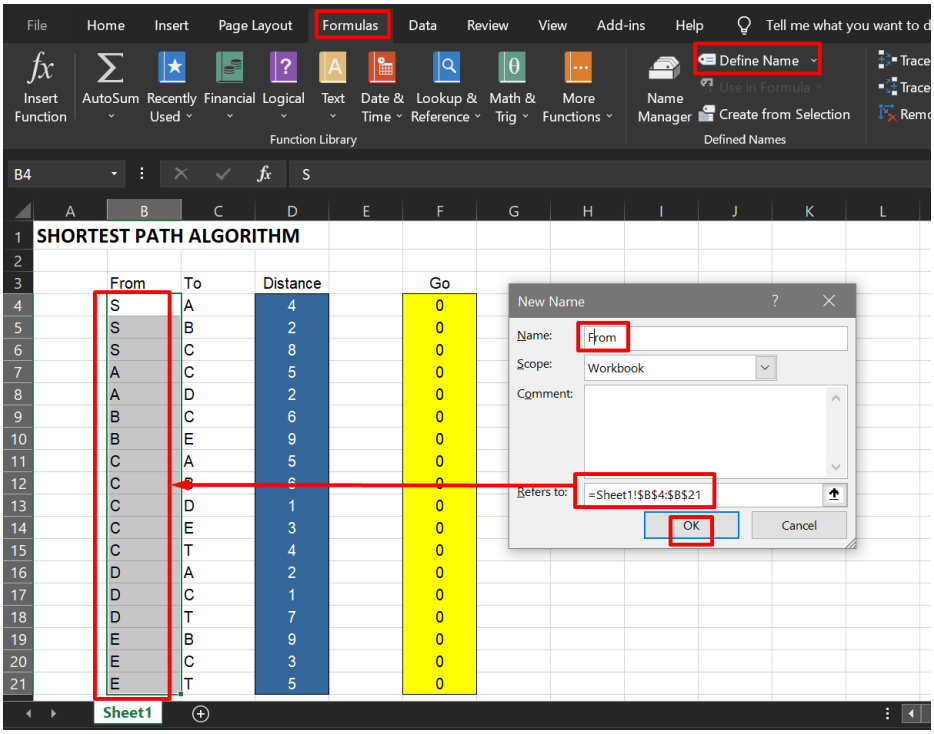


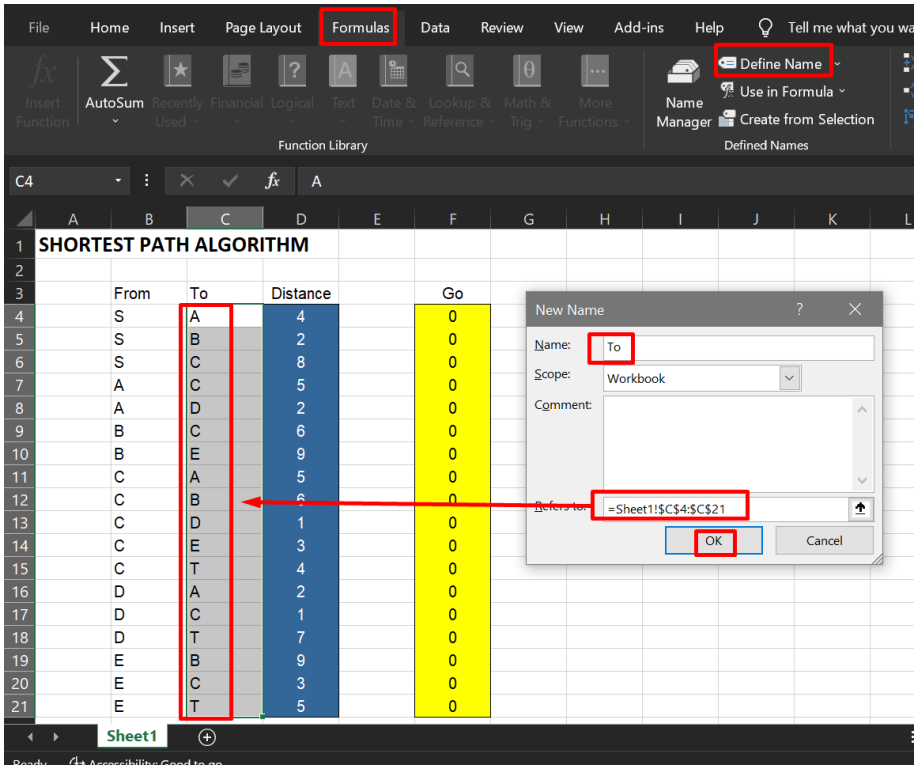
Conclusion: SADCT is the shortest path with a total distance of 11.

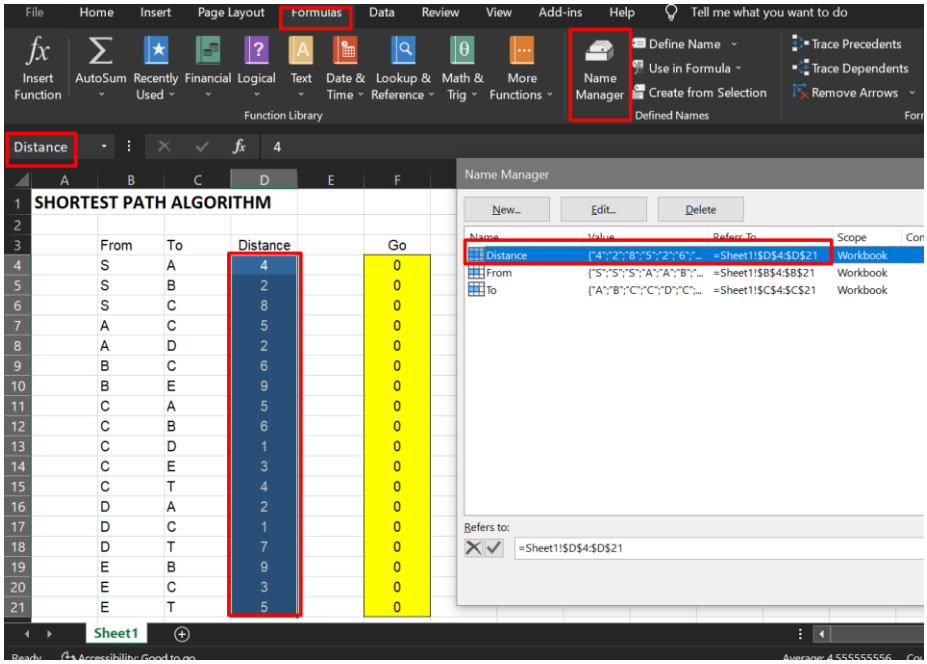
- Minimal Distance is 11
- Shortest Path: SADCT.
- We will try to replicate this in Excel, Step by Step.

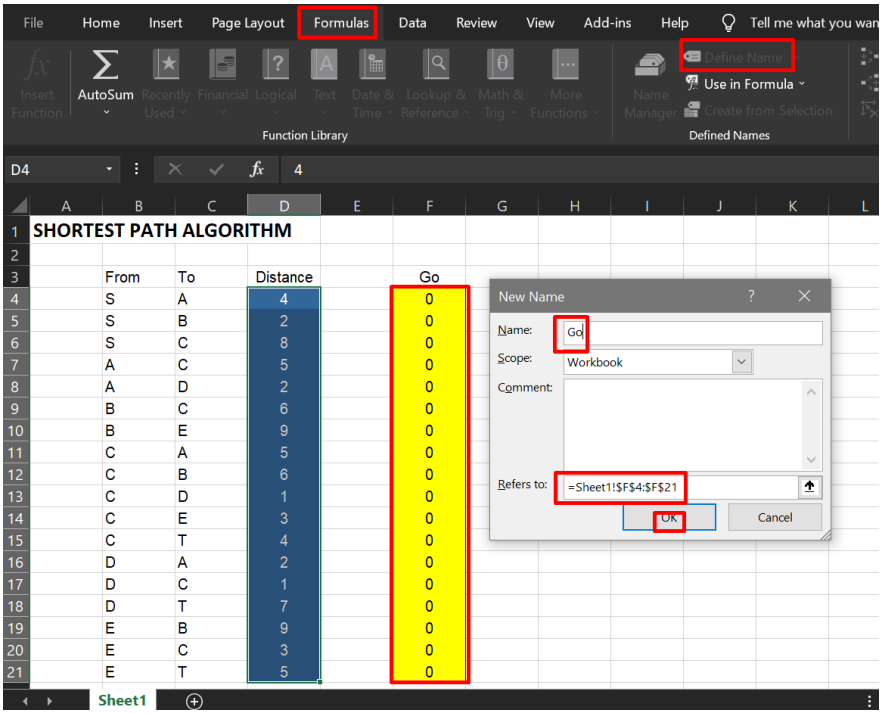
1. STEP 1: DEFINE FROM / TO / DISTANCE / GO



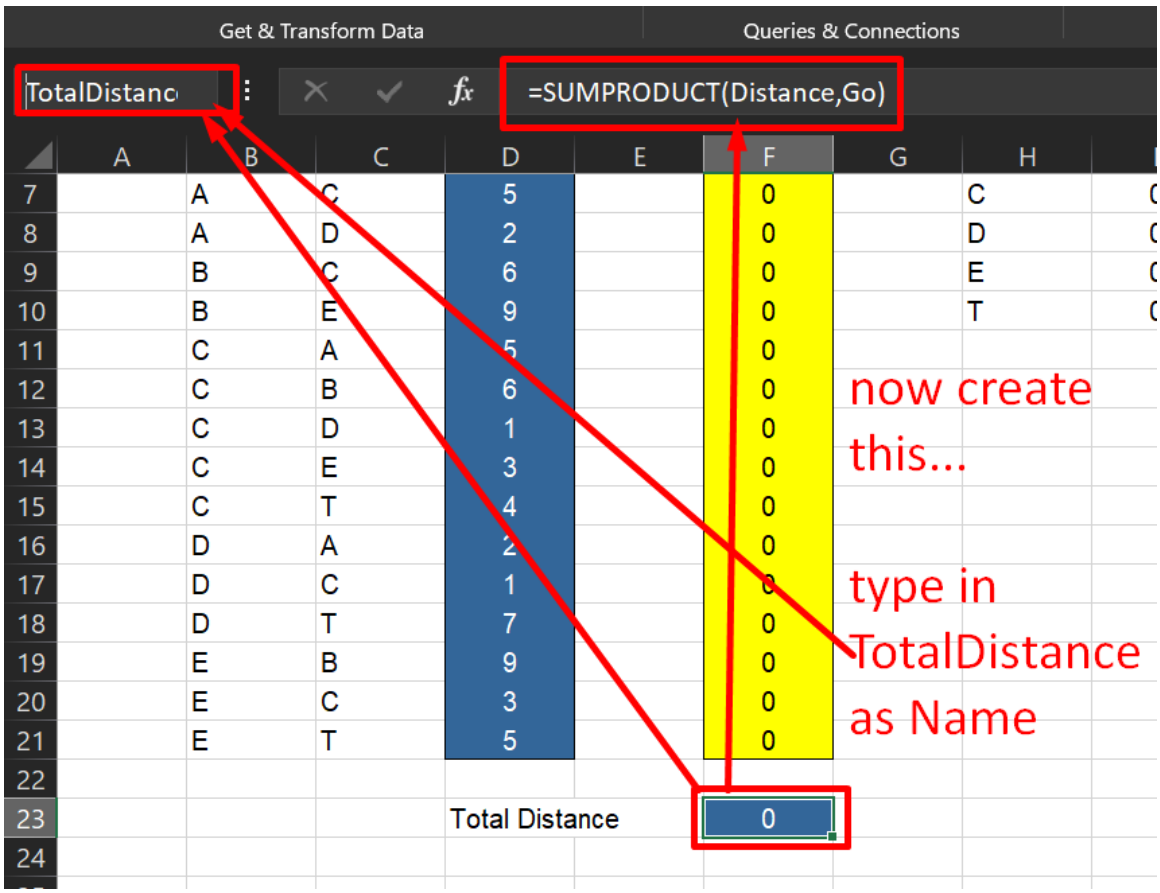




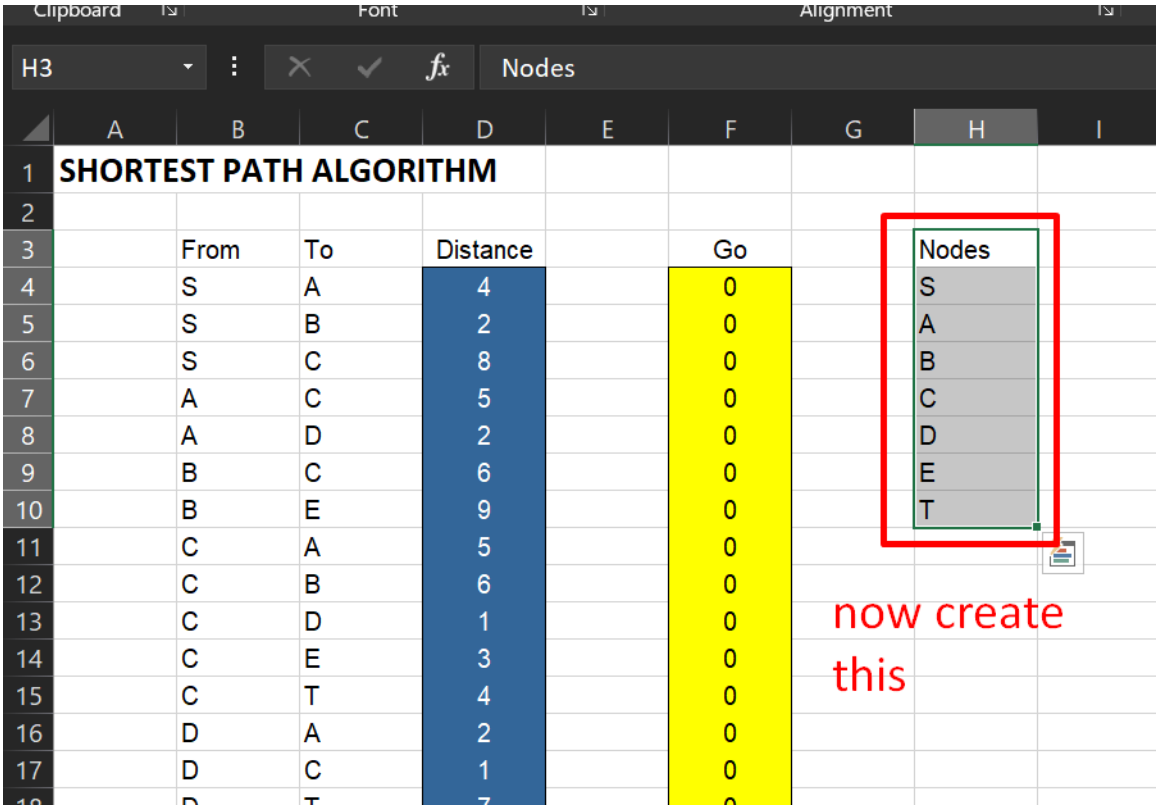


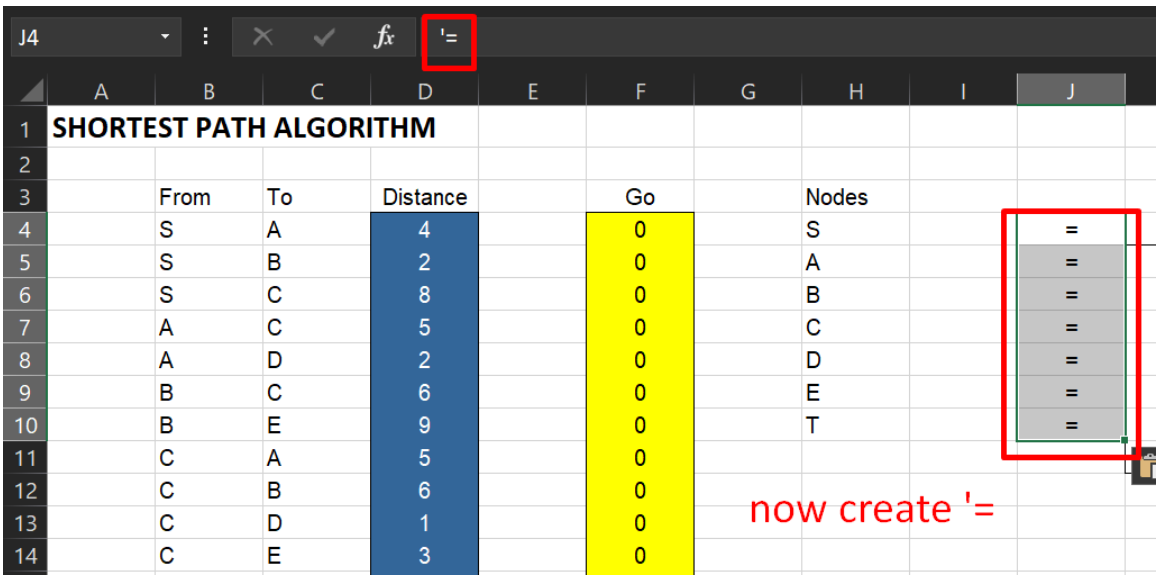


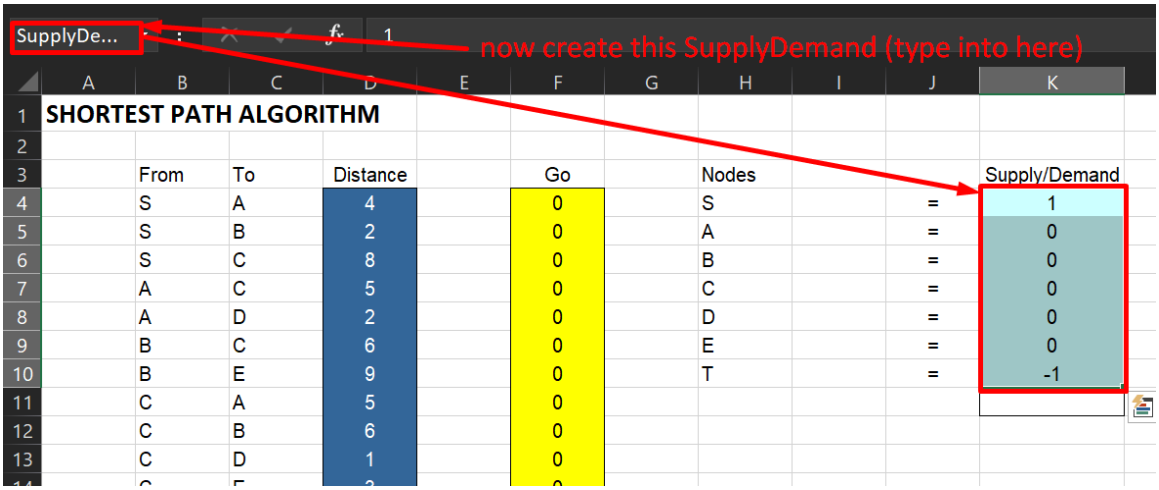
2. STEP 2: CREATE TOTAL DISTANCE



3. STEP 3: CREATE NODES / = / SUPPLY DEMAND







4. STEP 4: CREATE NET FLOW

Nodes	Net Flow
S	=SUMIF(From,H4,Go)
A	=SUMIF(From,H5,Go)-SUMIF(To,H5,Go)
B	=SUMIF(From,H6,Go)-SUMIF(To,H6,Go)
C	=SUMIF(From,H7,Go)-SUMIF(To,H7,Go)
D	=SUMIF(From,H8,Go)-SUMIF(To,H8,Go)
E	=SUMIF(From,H9,Go)-SUMIF(To,H9,Go)
T	=-SUMIF(To,H10,Go)

=SUMIF(From,H4,Go)

=SUMIF(From,H5,Go)-SUMIF(To,H5,Go)

=SUMIF(From,H6,Go)-SUMIF(To,H6,Go)

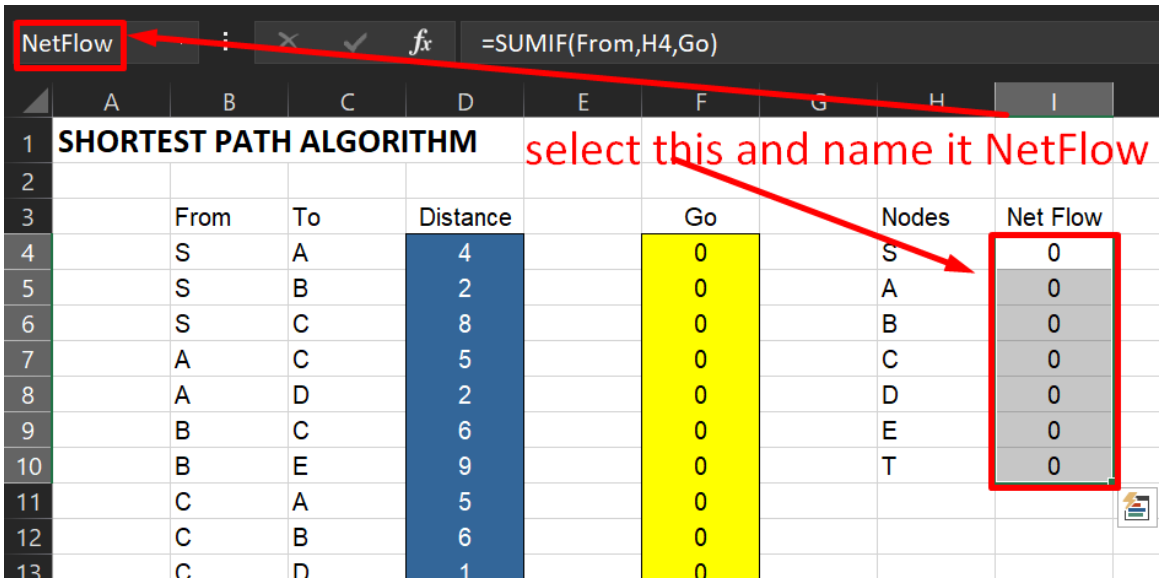
=SUMIF(From,H7,Go)-SUMIF(To,H7,Go)

=SUMIF(From,H8,Go)-SUMIF(To,H8,Go)

=SUMIF(From,H9,Go)-SUMIF(To,H9,Go)

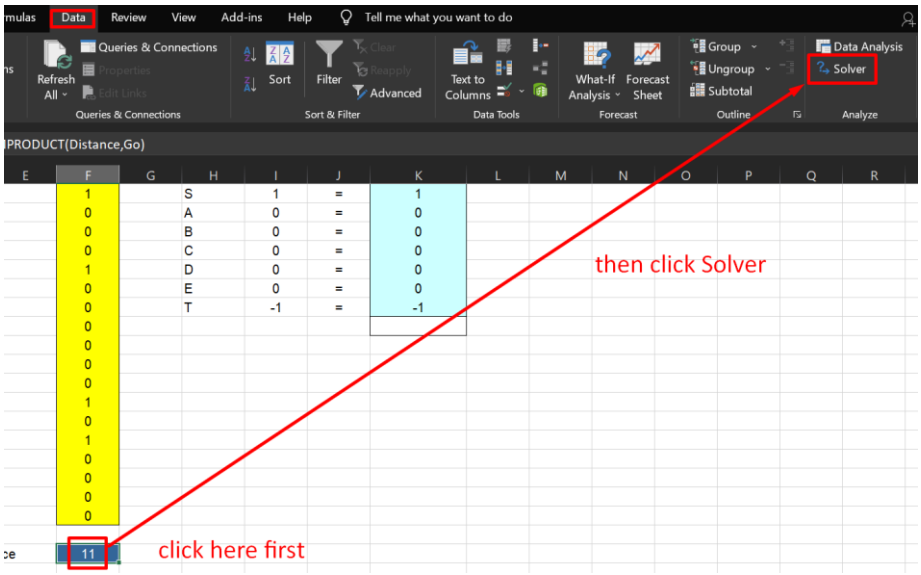
=-SUMIF(To,H10,Go)

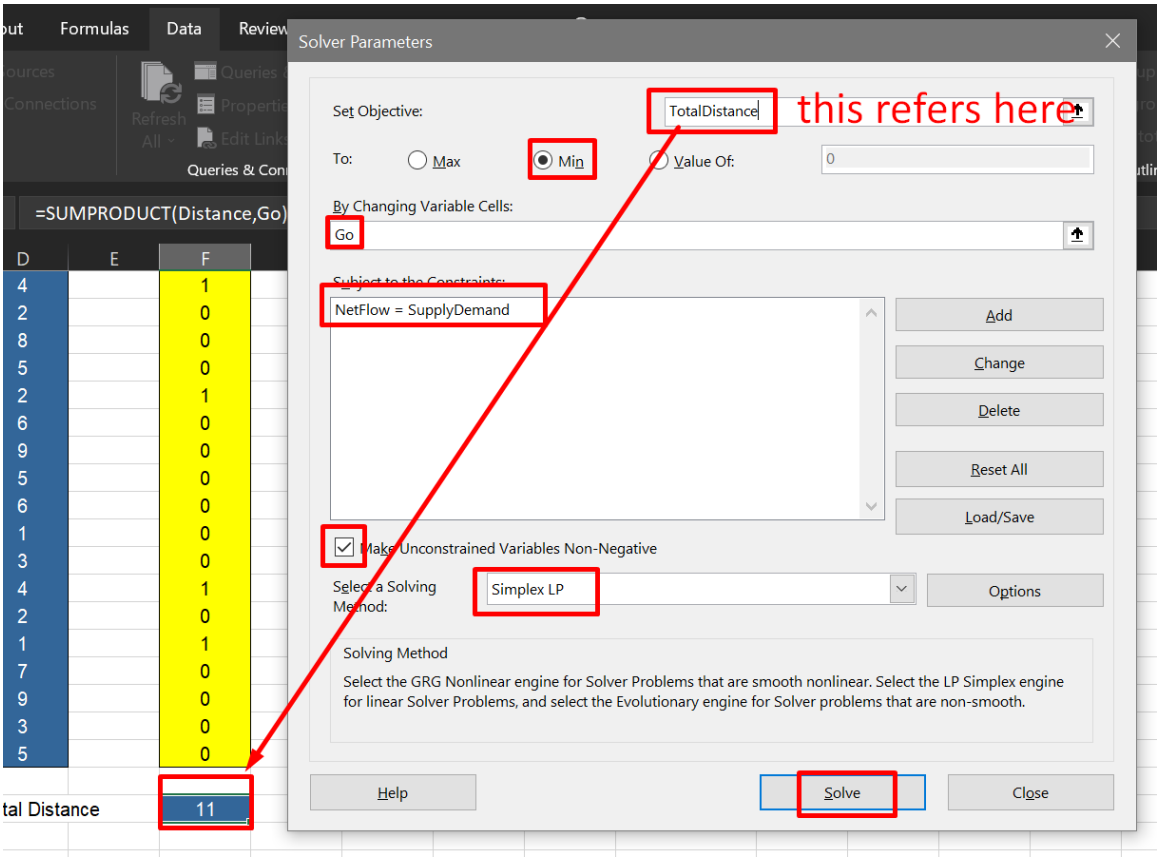
- SUMIF calculates the Net Flow of each node.
- Node S: SUMIF sums values in Go column with an "S" in the From column.
- As a result, only cell F4, F5 or F6 can be 1 (one outgoing arc).
- Node T: SUMIF sums values in Go column with a "T" in the To column.
- As a result, only cell F15, F18 or F21 can be 1 (one ingoing arc).
- For all other nodes, Excel looks in the From and To column.

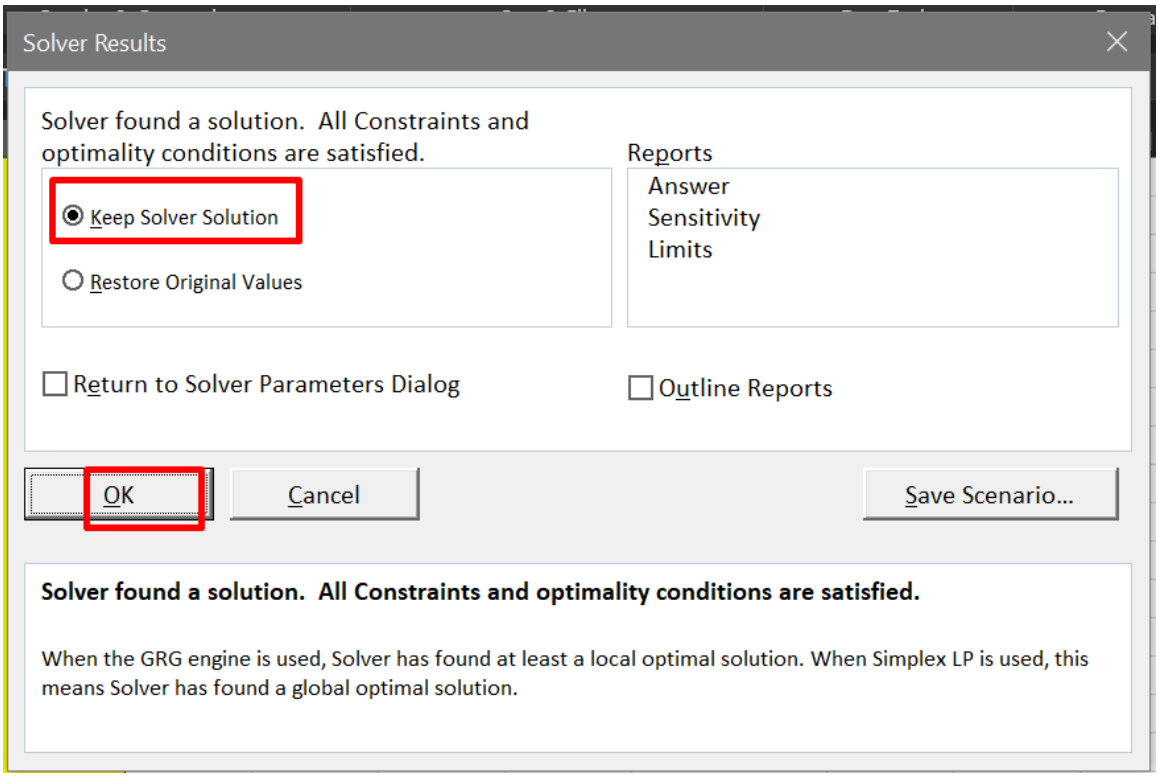


	From	To	Distance	Go	Nodes	Net Flow
4	S	A	4	0	S	0
5	S	B	2	0	A	0
6	S	C	8	0	B	0
7	A	C	5	0	C	0
8	A	D	2	0	D	0
9	B	C	6	0	E	0
10	B	E	9	0	T	0
11	C	A	5	0		
12	C	B	6	0		
13	C	D	1	0		

5. STEP 5: SOLVER



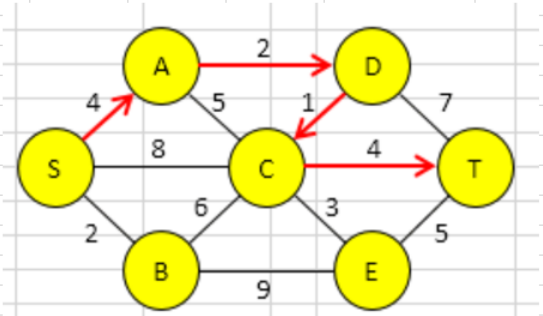




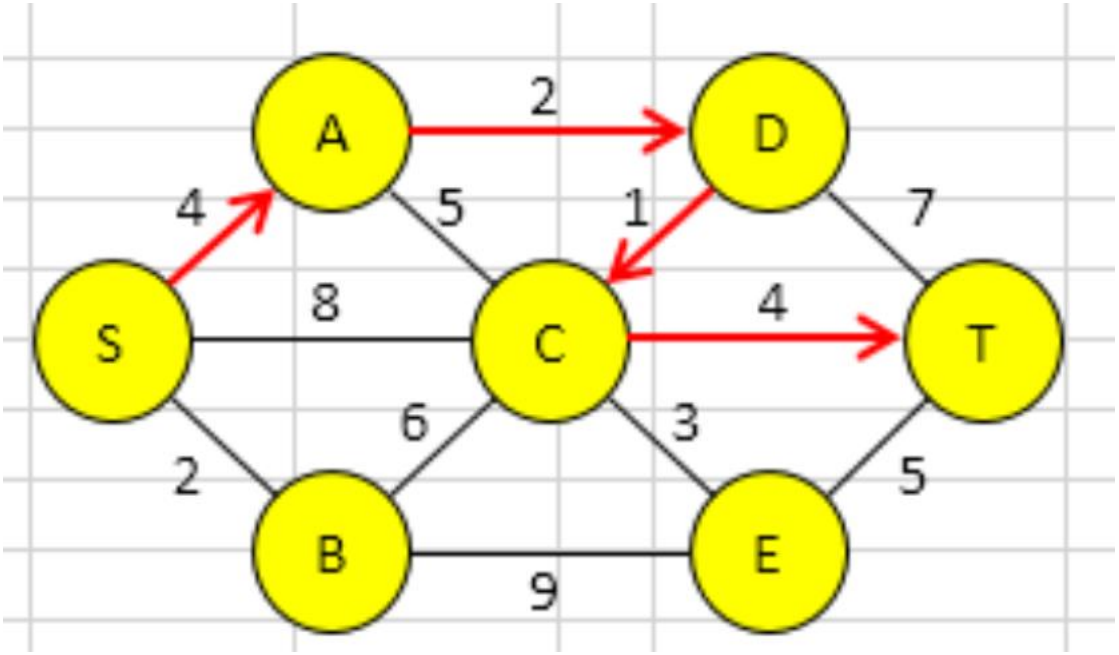
6. STEP 6: SOLUTION

From	To	Distance	Go
S	A	4	1
S	B	2	0
S	C	8	0
A	C	5	0
A	D	2	1
B	C	6	0
B	E	9	0
C	A	5	0
C	B	6	0
C	D	1	0
C	E	3	0
C	T	4	1
D	A	2	0
D	C	1	1
D	T	7	0
E	B	9	0
E	C	3	0
E	T	5	0
	Total Distance		11

Nodes	Net Flow		Supply/Demand
S	1	=	1
A	0	=	0
B	0	=	0
C	0	=	0
D	0	=	0
E	0	=	0
T	-1	=	-1



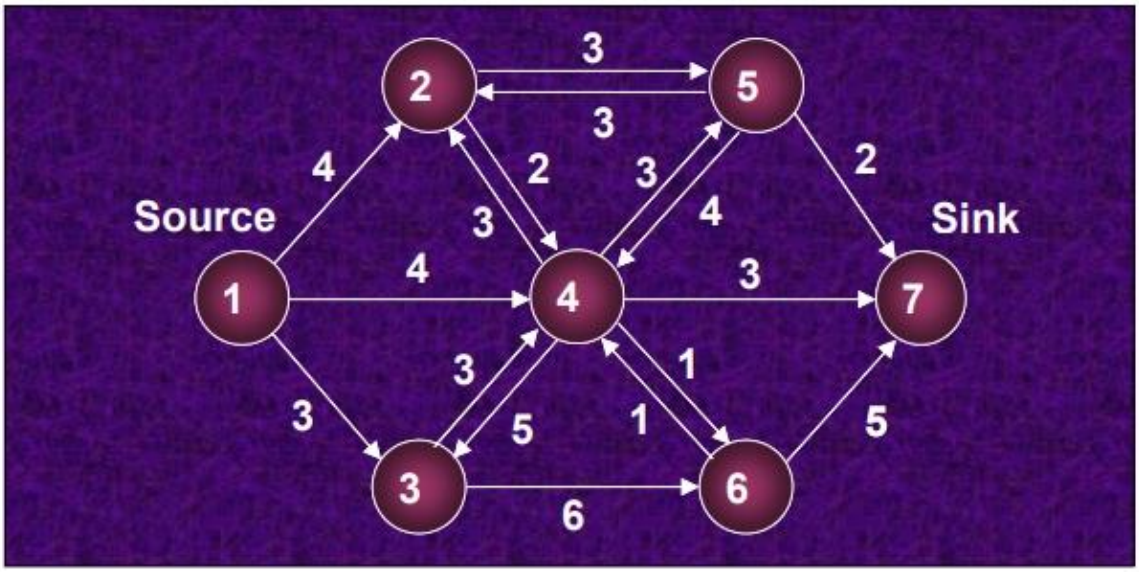
SOLUTION



- Maximal Flow Problem
  - Find Maximum Volume of flow from 1 node to another node
  - Practical Use:
    - Find the Maximum No. of People Arriving in an Airport per day (from another country)
    - Find the Max. No. of Goods delivered to a City per day (from another State)

A. BY HAND

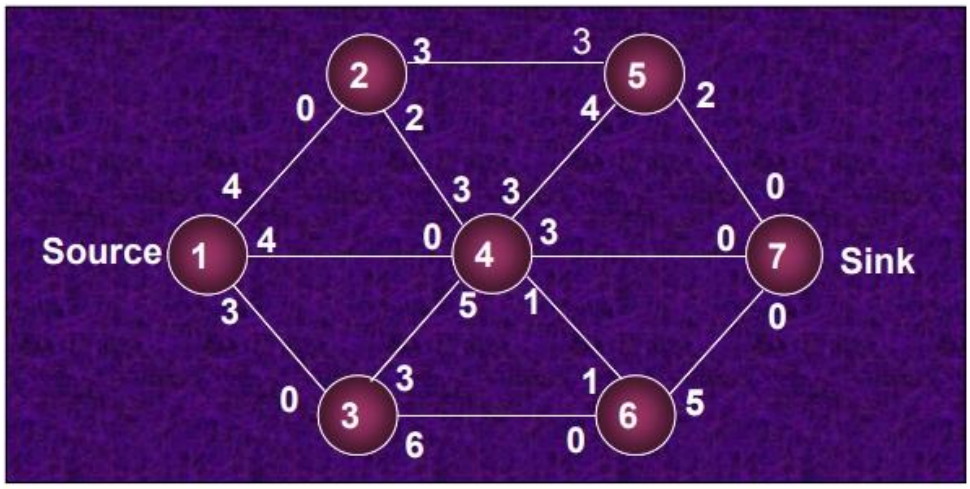
1. EXAMPLE 1



– Illustrated Problem 6.3
– Step 1

Network Modeling

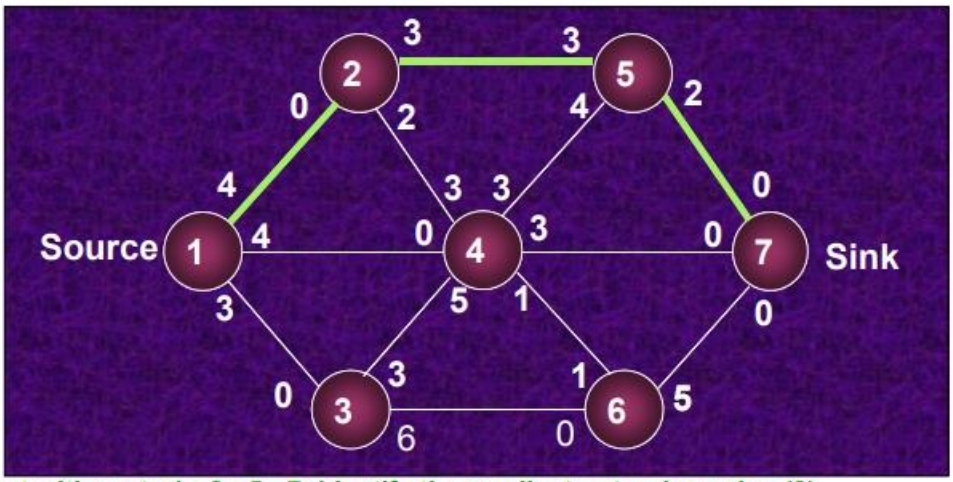
Maximal Flow Algorithm



– Illustrated Problem 6.3
– Step 2

Network Modeling

Maximal Flow Algorithm

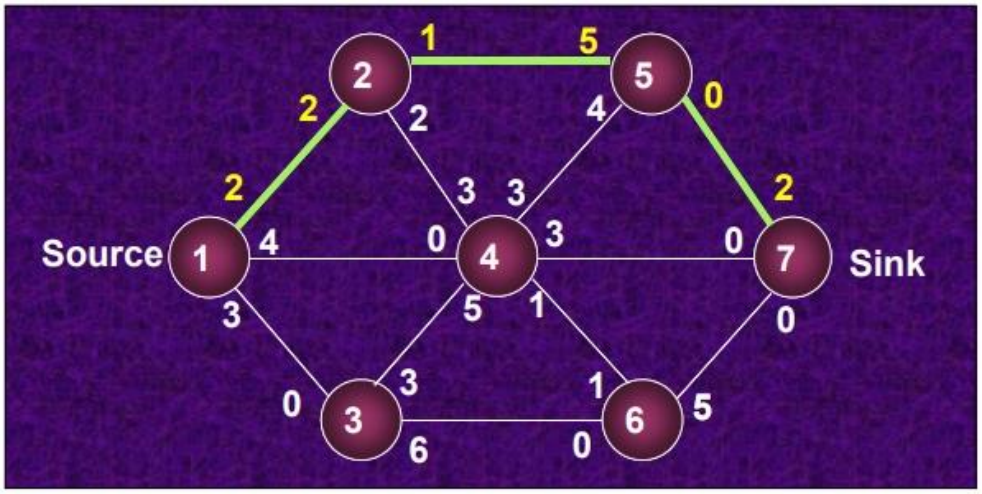


Start with route 1->2->5->7: identify the smallest out-going value (2); Subtract 2 from out-going value and add 2 to in-coming value.

– Illustrated Problem 6.3
– Step 3

Network Modeling

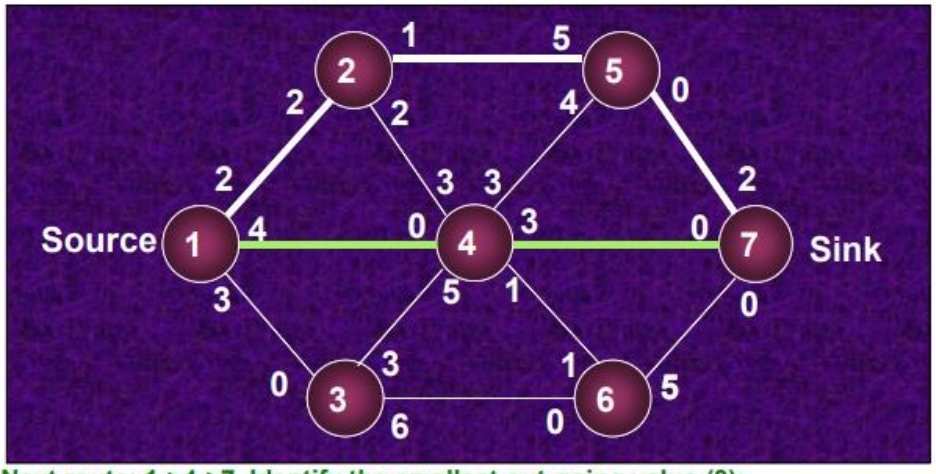
Maximal Flow Algorithm



– Illustrated Problem 6.3
– Step 4

Network Modeling

Maximal Flow Algorithm

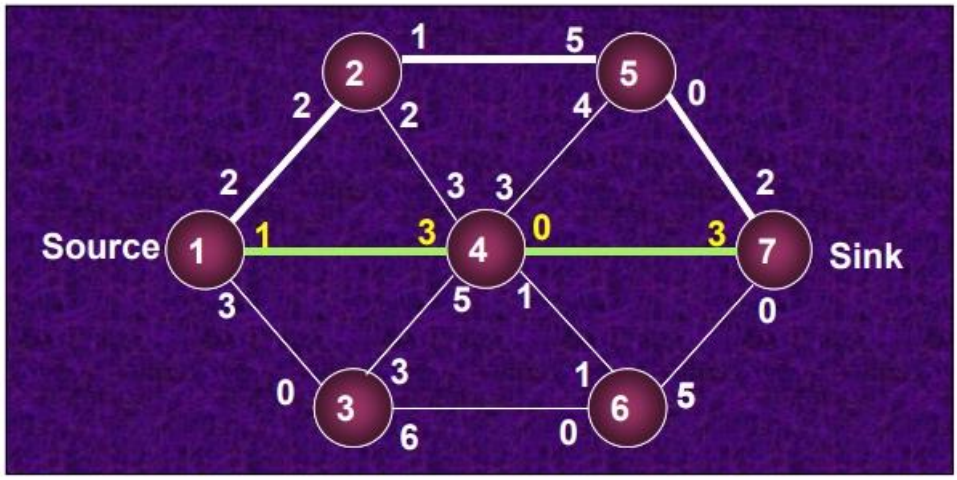


Next route: 1->4->7. Identify the smallest out-going value (3); Subtract 3 from out-going value and add 3 to in-coming value.

– Illustrated Problem 6.3
– Step 5

Network Modeling

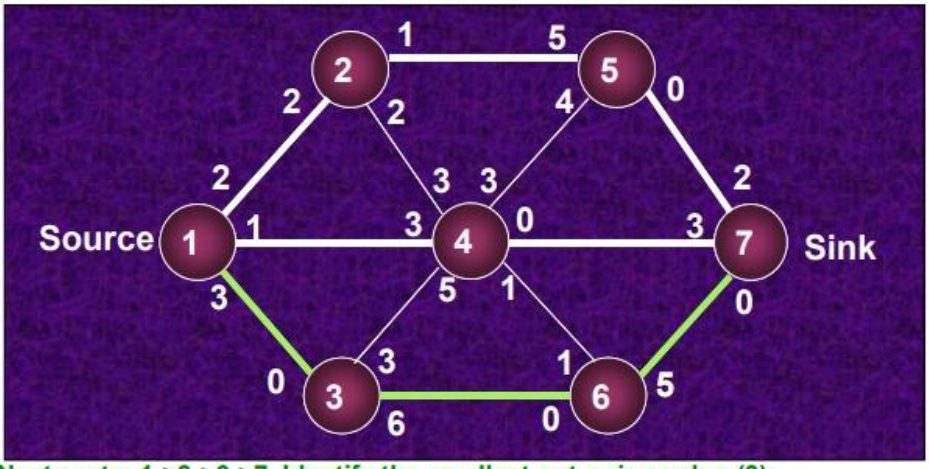
Maximal Flow Algorithm



– Illustrated Problem 6.3
– Step 6

Network Modeling

Maximal Flow Algorithm

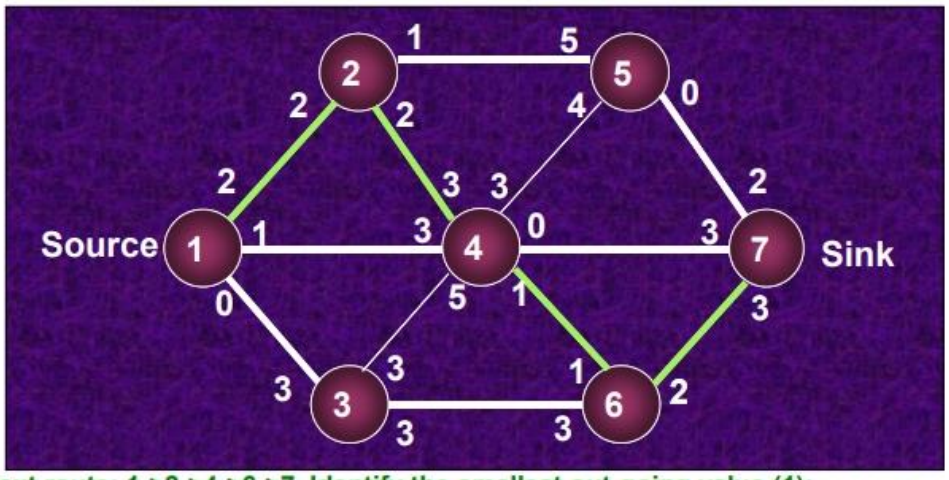


Next route: 1->3->6->7. Identify the smallest out-going value (3); Subtract 3 from out-going value and add 3 to in-coming value.

– Illustrated Problem 6.3
– Step 7

Network Modeling

Maximal Flow Algorithm

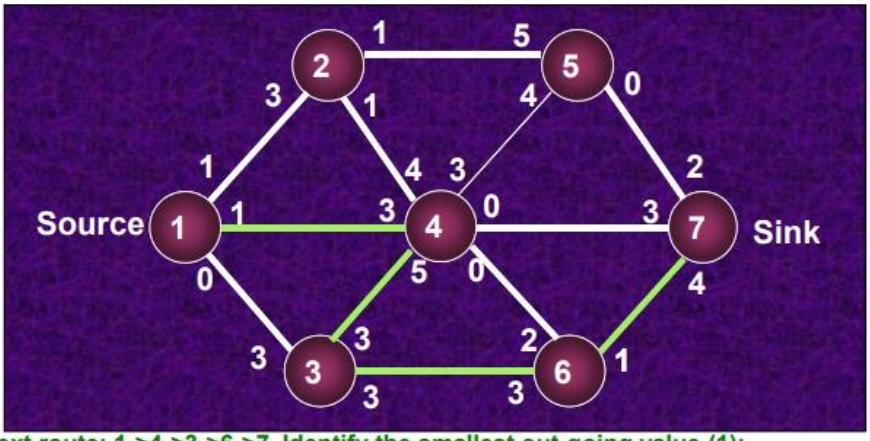


Next route: 1->2->4->6->7. Identify the smallest out-going value (1); Subtract 1 from out-going value and add 1 to in-coming value.

– Illustrated Problem 6.3
– Step 8

Network Modeling

Maximal Flow Algorithm

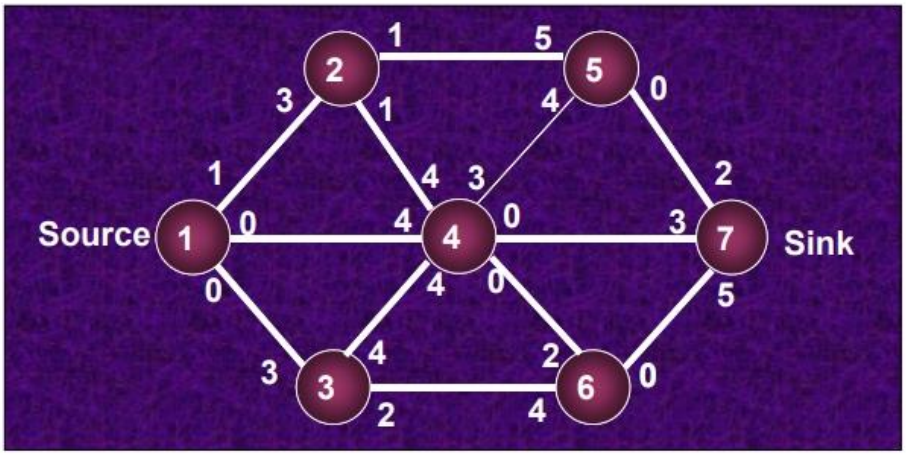


Next route: 1->4->3->6->7. Identify the smallest out-going value (1); Subtract 1 from out-going value and add 1 to in-coming value.

- Illustrated Problem 6.3
- Step 9

Network Modeling

Maximal Flow Algorithm



- Illustrated Problem 6.3
- Step 10

Network Modeling

Maximal Flow Algorithm

• Conclusion

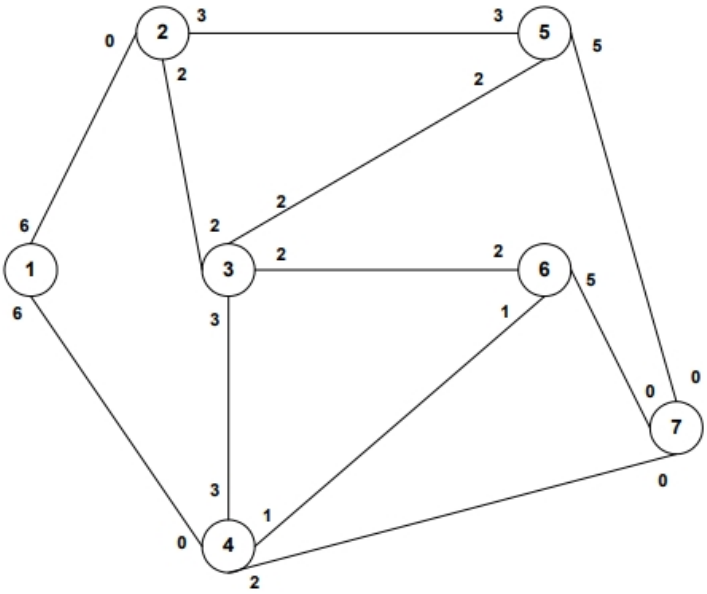
Iteration	Path	Maximum Flow
1	1->2->5->7	2
2	1->4->7	3
3	1->3->6->7	3
4	1->2->4->6->7	1
5	1->4->3->6->7	1
	Total	10

2. EXAMPLE 2

- Exercise 6.1
- Source = 1
- Sink = 7

Network Modeling

Maximal Flow Algorithm

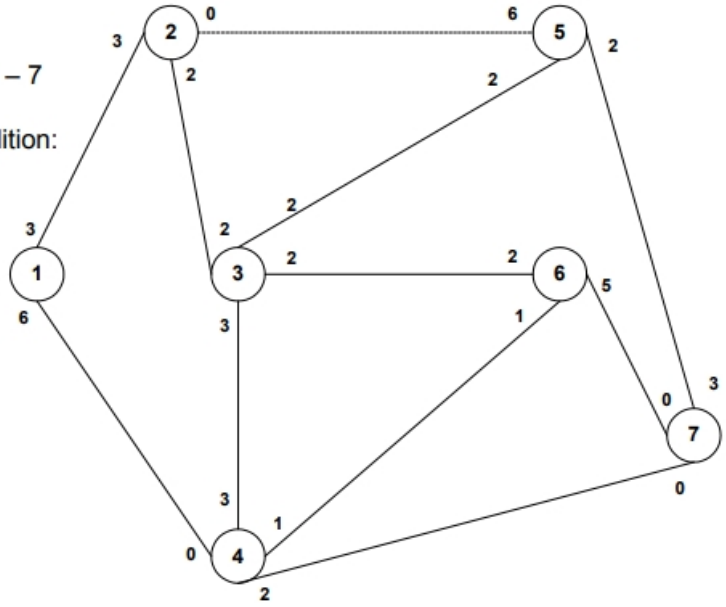


- Exercise 6.1
- Step 1

Network Modeling

Maximal Flow Algorithm

Iteration 1:
Chosen Path: 1 – 2 – 5 – 7
Smallest Arc Flow: 3
After reduction and addition:

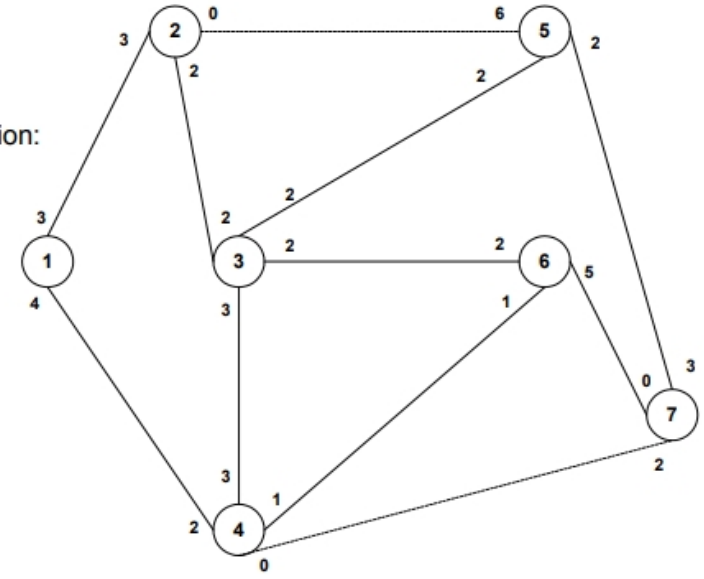


- Exercise 6.1
- Step 2

Network Modeling

Maximal Flow Algorithm

Iteration 2:
Chosen Path: 1 – 4 – 7
Smallest Arc Flow: 2
After reduction and addition:

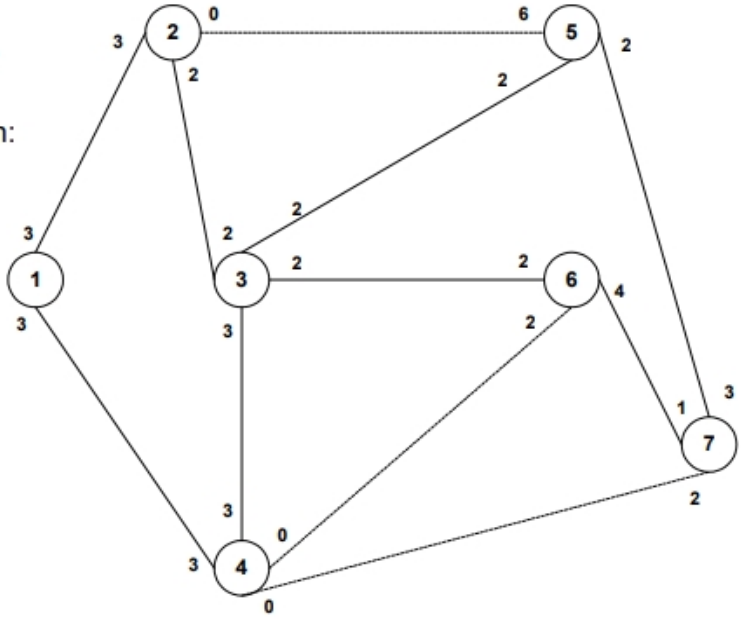


- Exercise 6.1
- Step 3

Network Modeling

Maximal Flow Algorithm

Iteration 3:
Chosen Path: 1 – 4 – 6 – 7
Smallest Arc Flow: 1
After reduction and addition:

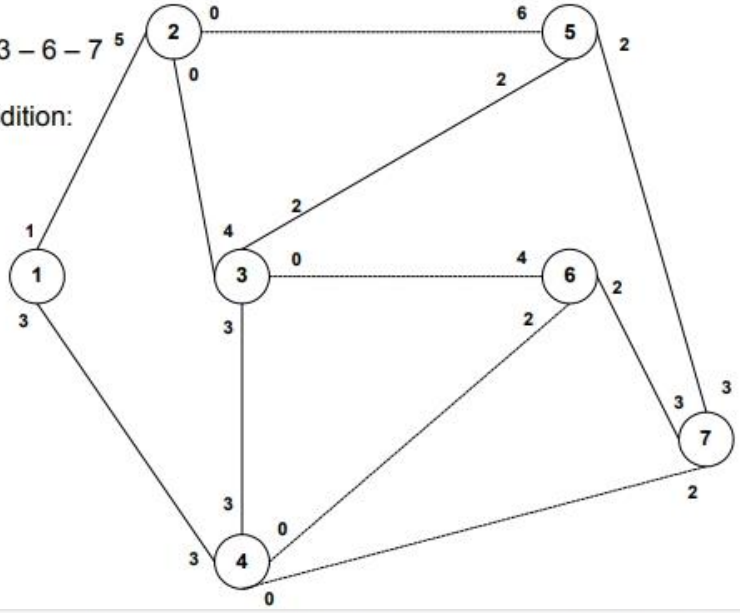


– Exercise 6.1
– Step 4

Network Modeling

Maximal Flow Algorithm

Iteration 4:
Chosen Path: 1 – 2 – 3 – 6 – 7
Smallest Arc Flow: 2
After reduction and addition:

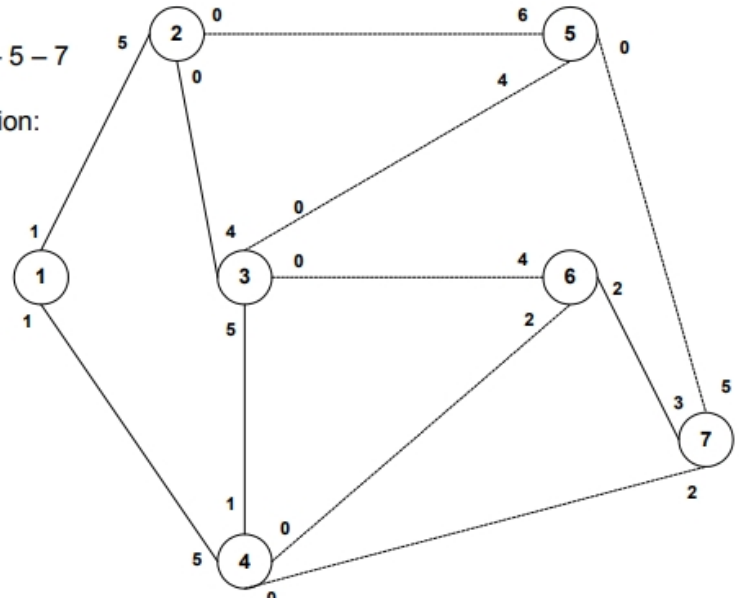


– Exercise 6.1
– Step 5

Network Modeling

Maximal Flow Algorithm

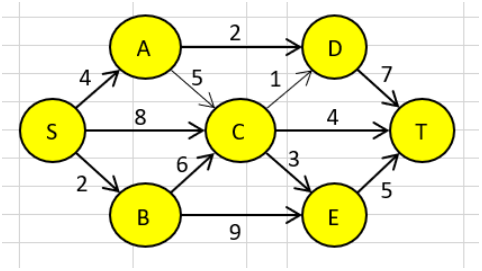
Iteration 5:
Chosen Path: 1 – 4 – 3 – 5 – 7
Smallest Arc Flow: 2
After reduction and addition:



Iteration	Path	Maximum Flow
1	1 – 2 – 5 – 7 – 3	3
2	1 – 4 – 7	2
3	1 – 4 – 6 – 7	1
4	1 – 2 – 3 – 6 – 7	2
5	1 – 4 – 3 – 5 – 7	2
		Total: 10

B. BY EXCEL

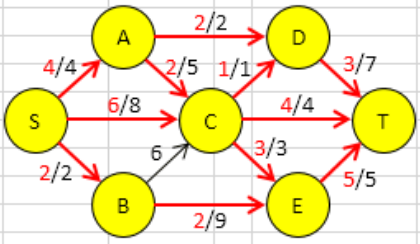
File: https://www.alvinang.sg/s/maximum-flow-problem.xlsx



- We are trying to find out the Maximum Flow of People per day coming into Airport T from Airport S.

- The numbers represent Maximum Allowable Flow Rate (per day) coming out.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Maximum Flow Problem											
2												
3		From	To	Flow		Capacity		Nodes	Net Flow		Supply/Demand	
4		S	A	4	≤	4		S	12			
5		S	B	2	≤	2		A	0	=	0	
6		S	C	6	≤	8		B	0	=	0	
7		A	C	2	≤	5		C	0	=	0	
8		A	D	2	≤	2		D	0	=	0	
9		B	C	0	≤	6		E	0	=	0	
10		B	E	2	≤	9		T	-12			
11		C	D	1	≤	1						
12		C	E	3	≤	3						
13		C	T	4	≤	4						
14		D	T	3	≤	7						
15		E	T	5	≤	5						
16												
17		Maximum Flow		12								
18												
19												
20												
21												

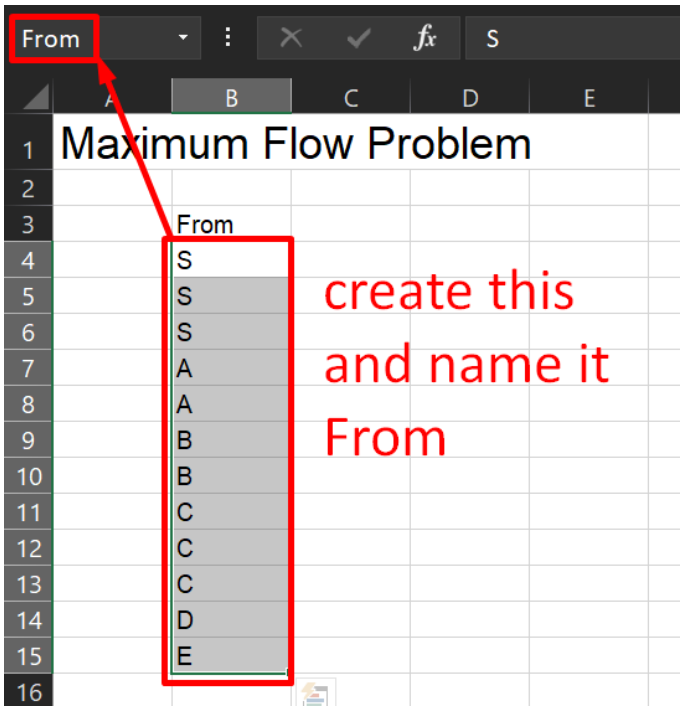


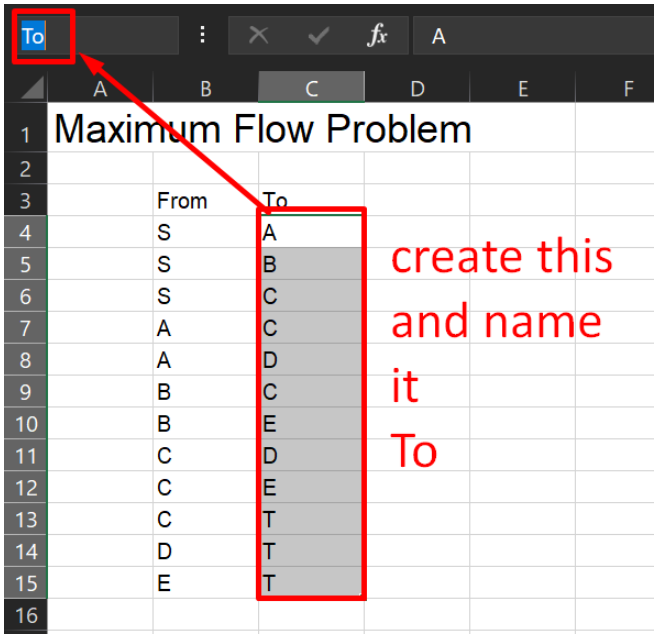
- Maximum Flow Rate = 12 people per day coming into Airport T (and flowing out from Airport S).

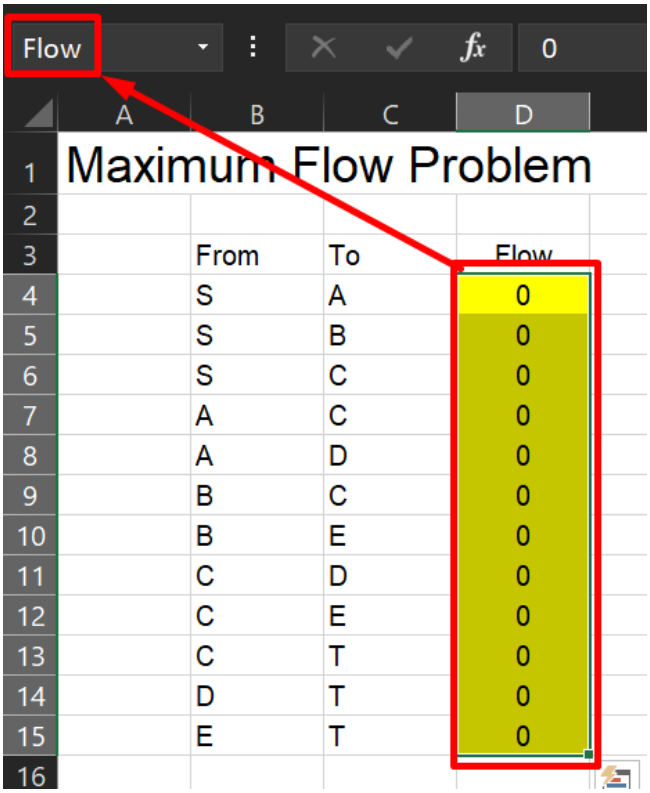
- S→A = 4/4 means max capacity is used.

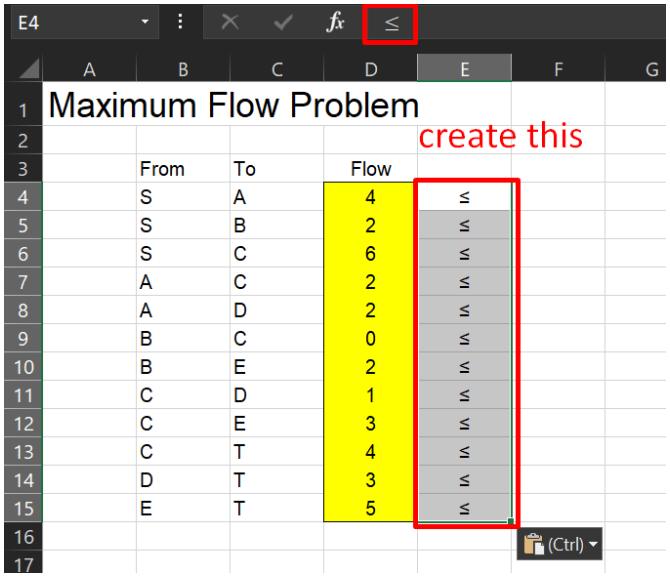
- B→E = 2/9 means only 2 out of 9 capacity is used.

1. STEP 1: CREATE FROM / TO / FLOW / CAPACITY

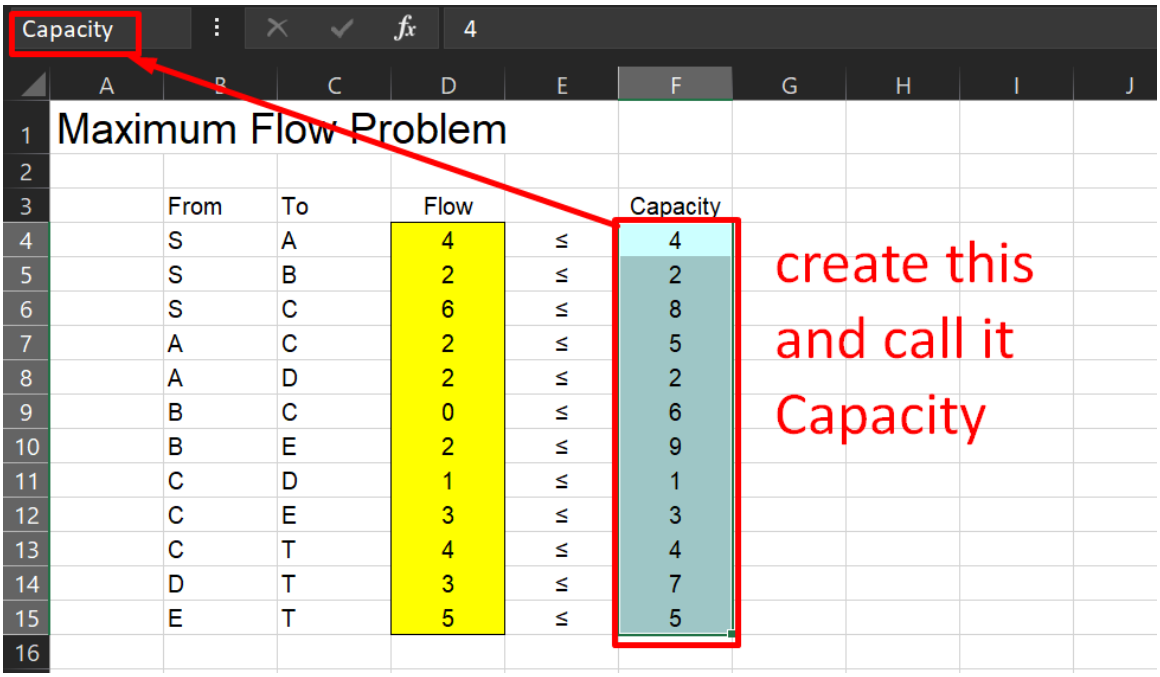




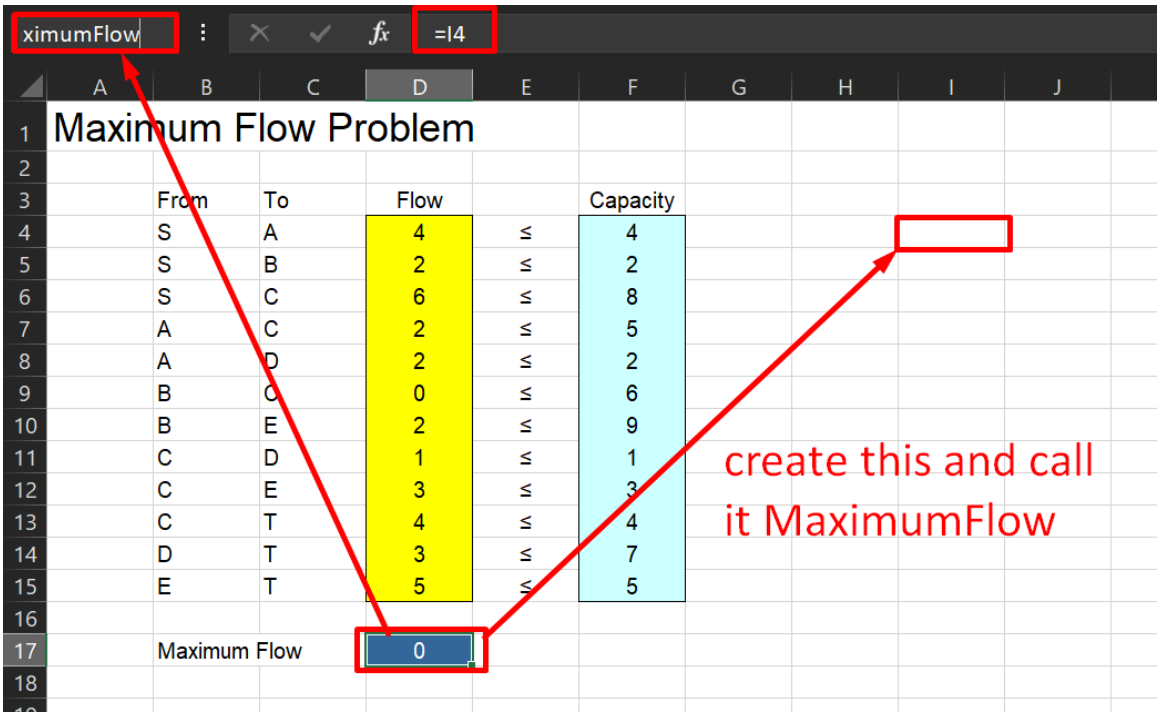




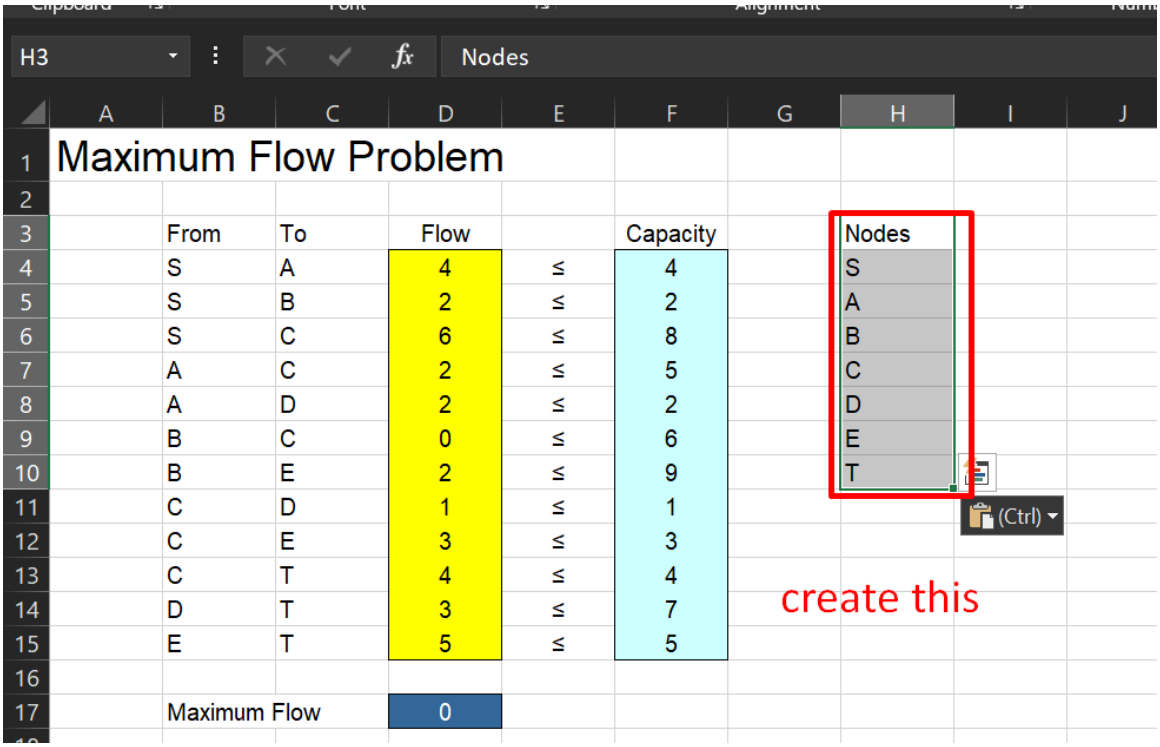
$\leqslant$

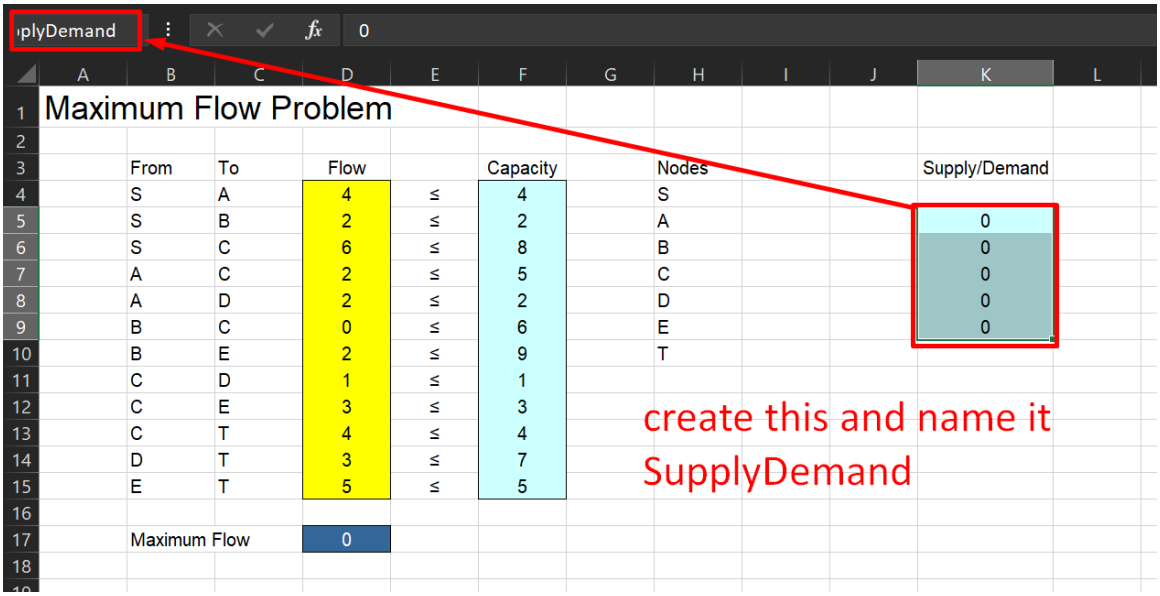


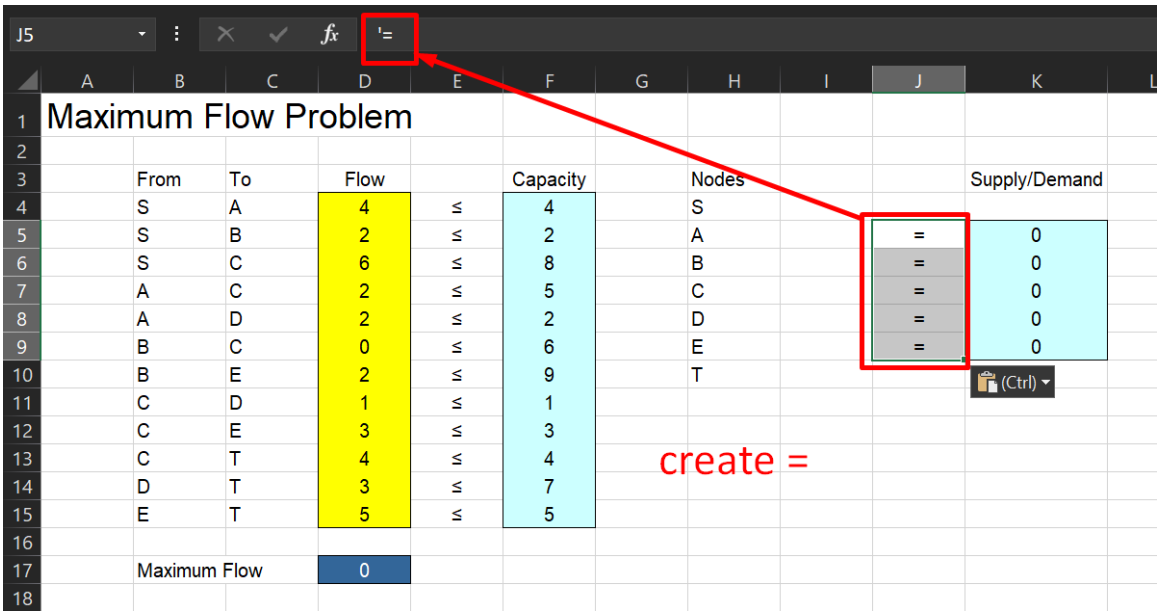
2. STEP 2: CREATE MAXIMUM FLOW



3. STEP 3: CREATE NODES / SUPPLY DEMAND / =







4. STEP 4: CREATE NETFLOW

Nodes	Net Flow
S	=SUMIF(From,H4,Flow)
A	=SUMIF(From,H5,Flow)-SUMIF(To,H5,Flow)
B	=SUMIF(From,H6,Flow)-SUMIF(To,H6,Flow)
C	=SUMIF(From,H7,Flow)-SUMIF(To,H7,Flow)
D	=SUMIF(From,H8,Flow)-SUMIF(To,H8,Flow)
E	=SUMIF(From,H9,Flow)-SUMIF(To,H9,Flow)
T	=-SUMIF(To,H10,Flow)

=SUMIF(From,H4,Flow)

=SUMIF(From,H5,Flow)-SUMIF(To,H5,Flow)

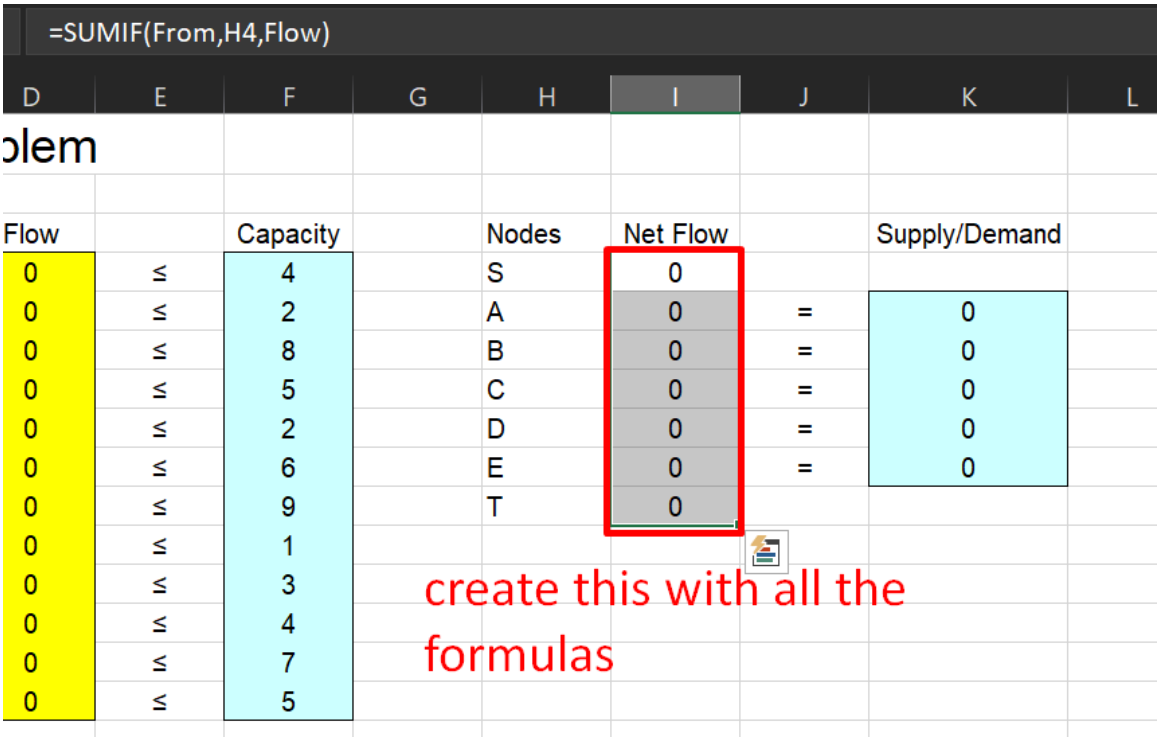
=SUMIF(From,H6,Flow)-SUMIF(To,H6,Flow)

=SUMIF(From,H7,Flow)-SUMIF(To,H7,Flow)

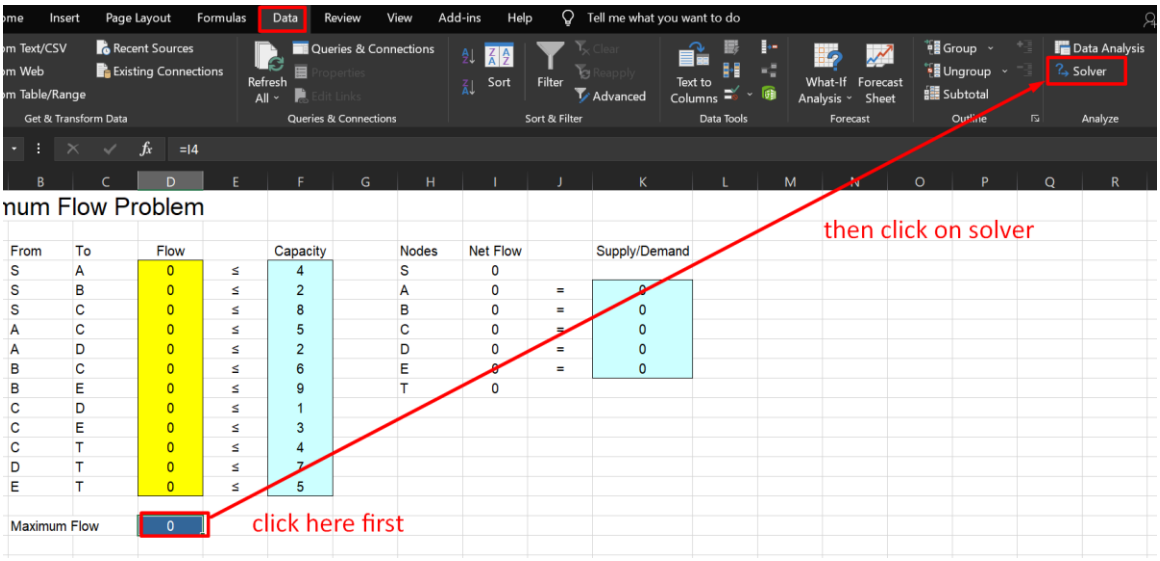
=SUMIF(From,H8,Flow)-SUMIF(To,H8,Flow)

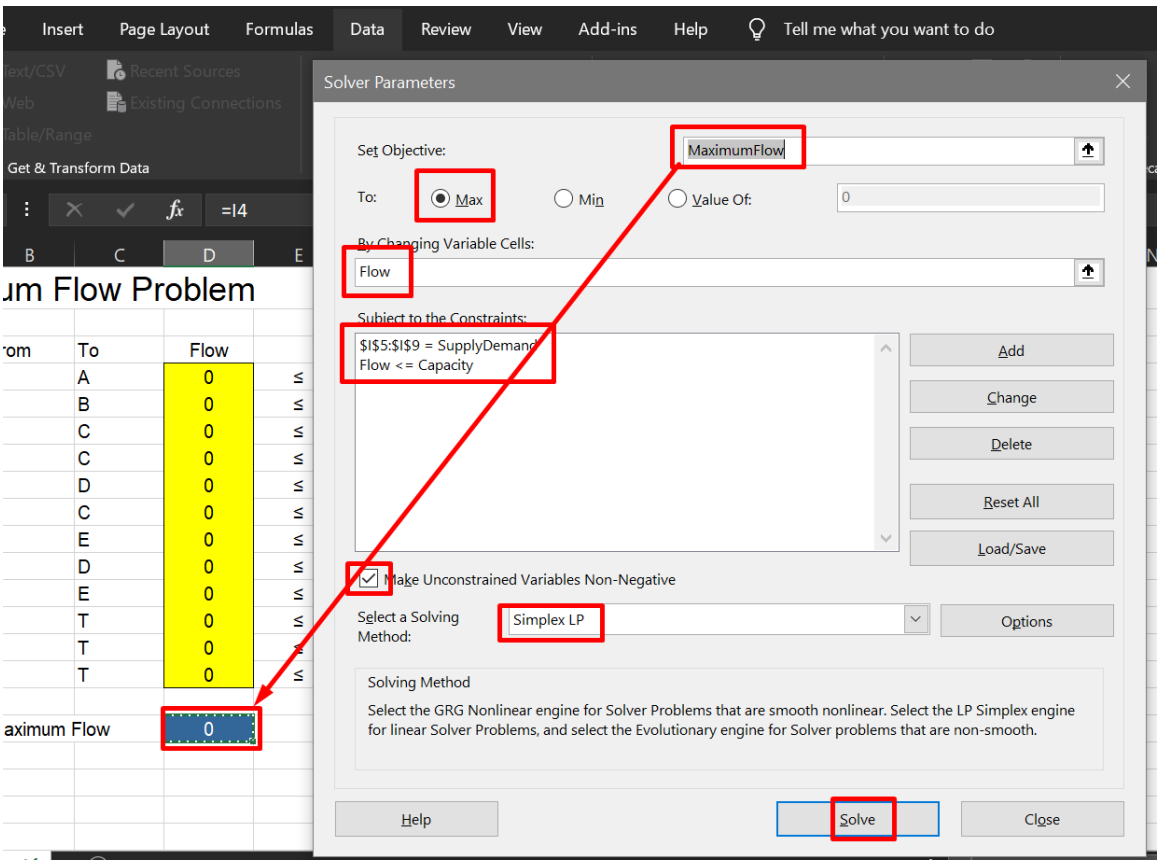
=SUMIF(From,H9,Flow)-SUMIF(To,H9,Flow)

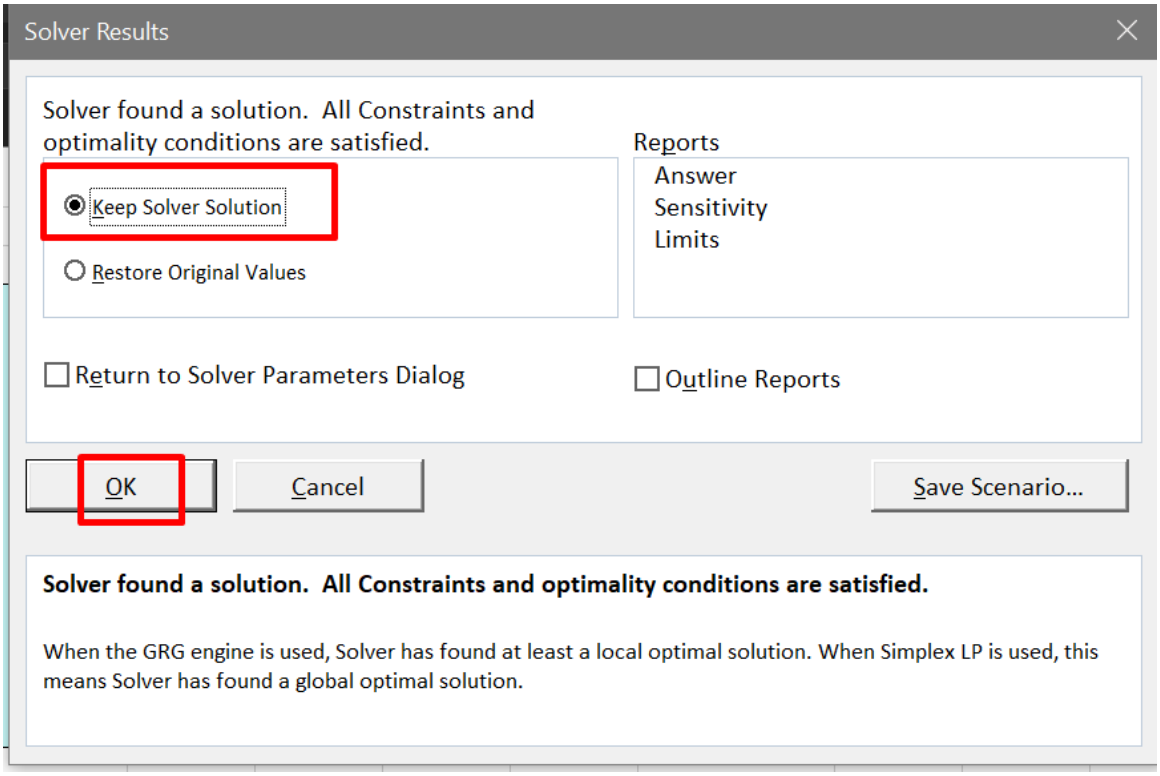
=-SUMIF(To,H10,Flow)



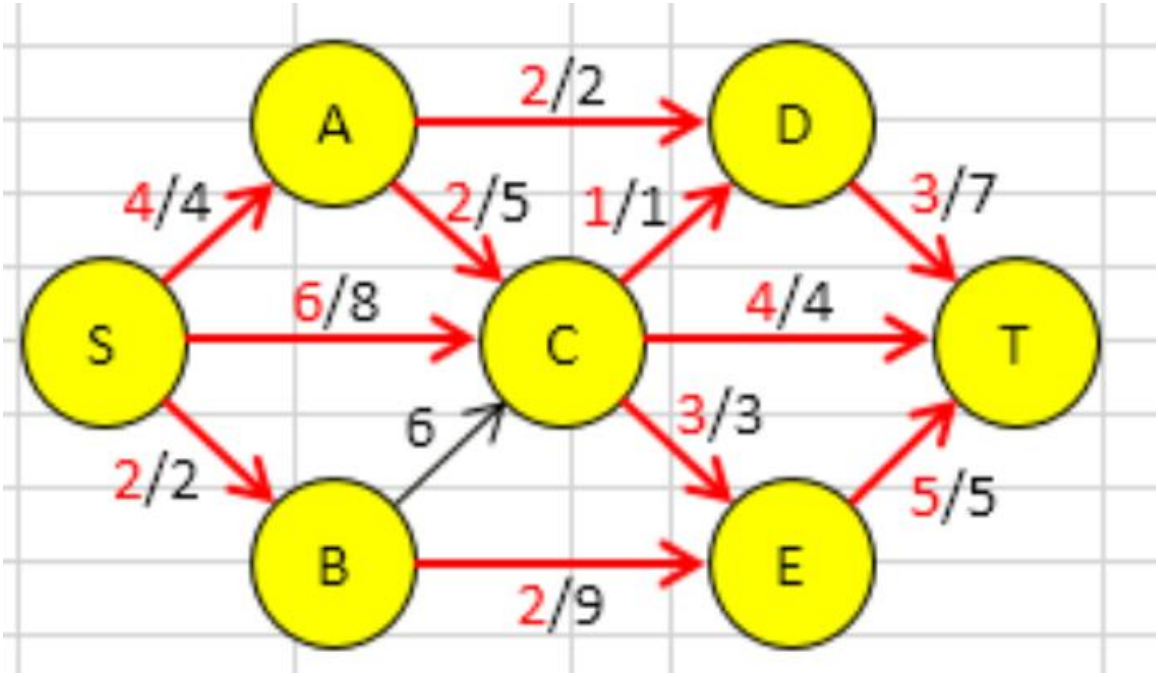
5. STEP 5: SOLVER







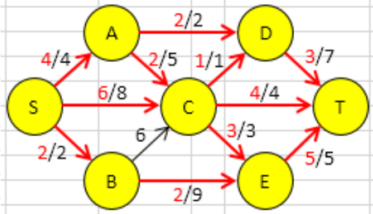
6. STEP 6: SOLUTION



Maximum Flow Problem

From	To	Flow		Capacity
S	A	4	≤	4
S	B	2	≤	2
S	C	6	≤	8
A	C	2	≤	5
A	D	2	≤	2
B	C	0	≤	6
B	E	2	≤	9
C	D	1	≤	1
C	E	3	≤	3
C	T	4	≤	4
D	T	3	≤	7
E	T	5	≤	5
Maximum Flow		12		

Nodes	Net Flow		Supply/Demand
S	12		
A	0	=	0
B	0	=	0
C	0	=	0
D	0	=	0
E	0	=	0
T	-12		



Max flow out of S
= 4+6+2
= 12

Max flow in to T
= 3+4+5
= 12

IV. ABOUT DR. ALVIN ANG

Dr. Alvin Ang earned his Ph.D., Masters and Bachelor degrees from NTU, Singapore. He is a scientist, entrepreneur, as well as a personal/business advisor. More about him at www.AlvinAng.sg.