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Response Surface Designs

Response Surface Designs Overview

Response surface methods are used to examine the relationship between one or more response variables and a set of quantitative experimental variables or factors. These methods are often employed after you have identified a "vital few" controllable factors and you want to find the factor settings that optimize the response. Designs of this type are usually chosen when you suspect curvature in the response surface.

Response surface methods may be employed to

- find factor settings (operating conditions) that produce the "best" response
- find factor settings that satisfies operating or process specifications
- identify new operating conditions that produce demonstrated improvement in product quality over the quality achieved by current conditions
- model a relationship between the quantitative factors and the response

Many response surface applications are sequential in nature in that they require more than one stage of experimentation and analysis. The steps shown below are typical of a response surface experiment. Depending on your experiment, you may carry out some of the steps in a different order, perform a given step more than once, or eliminate a step.

1 Choose a response surface design for the experiment. Before you begin using MINITAB, you must determine what the influencing factors are, that is, what the process conditions are that influence the values of the response variable. See *Choosing a Design* on page 20-3.

2 Use **Create Response Surface Design** to generate a central composite or Box-Behnken design—see *Creating Response Surface Designs* on page 20-4.

   Use **Define Custom Response Surface Design** to create a design from data you already have in the worksheet. Custom designs allows you to specify which columns are your factors and other design characteristics. You can then easily fit a model to the design and generate plots. See *Defining Custom Designs* on page 20-19.

3 Use **Modify Design** to rename the factors, change the factor levels, replicate the design, and randomize the design. See *Modifying Designs* on page 20-20.

4 Use **Display Design** to change the order of the runs and the units in which MINITAB expresses the factors in the worksheet. See *Displaying Designs* on page 20-24.

5 Perform the experiment and collect the response data. Then, enter the data in your MINITAB worksheet. See *Collecting and Entering Data* on page 20-25.

6 Use **Analyze Response Surface Design** to fit a model to the experimental data. See *Analyzing Response Surface Designs* on page 20-26.

7 Use **Contour/Surface (Wireframe) Plots** to visualize response surface patterns. You can display contour and surface (wireframe) plots—see page 20-34.

8 If you are trying to optimize responses, use **Response Optimizer** (page 23-2) or **Overlaid Contour Plot** (page 23-20) to obtain a numerical and graphical analysis.

Choosing a Design

Before you use MINITAB, you need to determine what design is most appropriate for your experiment. Choosing your design correctly will ensure that the response surface is fit in the most efficient manner. MINITAB provides central composite and Box-Behnken designs. When choosing a design you need to

■ identify the number of factors that are of interest.

■ determine the number of runs you can perform.

■ ensure adequate coverage of the region of interest on the response surface. You should choose a design that will adequately predict values in the region of interest.

■ determine the impact that other considerations (such as cost, time, or the availability of facilities) have on your choice of a design.

Depending on your problem, there are other considerations that make a design desirable. You need to choose a design that shows consistent performance in the criteria that you consider important, such as the ability to

■ increase the order of the design sequentially.

■ perform the experiment in orthogonal blocks. Orthogonally blocked designs allow for model terms and block effects to be estimated independently and minimize the variation in the estimated coefficients.

■ rotate the design. Rotatable designs provide the desirable property of constant prediction variance at all points that are equidistant from the design center, thus improving the quality of the prediction.

■ detect model lack of fit.

More   Our intent is to provide only a brief introduction to response surface methods. There are many resources that provide a thorough treatment of these designs. For a list of resources, see *References* on page 20-38.

Creating Response Surface Designs

MINITAB provides two response surface designs: central composite designs (page 20-4) and Box-Behnken designs (page 20-5).
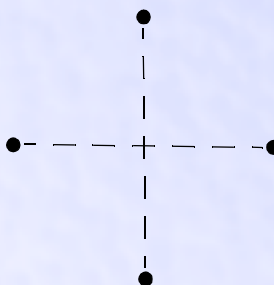
Central composite designs

You can create blocked or unblocked central composite designs. Central composite designs consist of

- 2^k factorial or "cube" points, where K is the number of factors
- axial points (also called star points)
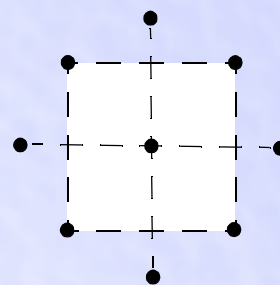- center points

A central composite design with two factors is shown below. Points on the diagrams represent the experimental runs that are performed:
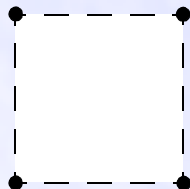
The points in the "cube" portion of the design are coded to be –1 and +1.

The points in the axial or star portion of the design are at
(+α,0) (–α,0)
(0, +α) (0, –α)

Here, the "cube" and axial portions along with the center point is shown. The design center is at (0,0).

Central composite designs are often recommended when the design plan calls for sequential experimentation because these designs can incorporate information from a properly planned factorial experiment. The factorial or "cube" portion and center points may serve as a preliminary stage where you can fit a first-order (linear) model, but still provide evidence regarding the importance of a second-order contribution or curvature.

You can then build the "cube" portion of the design up into a central composite design to fit a second-degree model by adding axial and center points. Central composite designs allow for efficient estimation of the quadratic terms in the second-order model,

and it is also easy to obtain the desirable design properties of orthogonal blocking and rotatability.

More Orthogonally blocked designs allow for model terms and block effects to be estimated independently and minimize the variation in the regression coefficients. Rotatable designs provide the desirable property of constant prediction variance at all points that are equidistant from the design center, thus improving the quality of the prediction.

Box-Behnken designs

You can create blocked or unblocked Box-Behnken designs. The illustration below shows a three-factor Box-Behnken design. Points on the diagram represent the experimental runs that are performed:
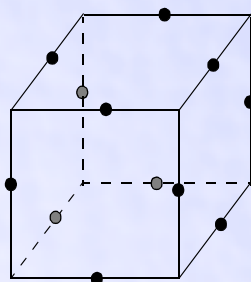
You may want to use Box-Behnken designs when performing non-sequential experiments. That is, you are only planning to perform the experiment once. These designs allow efficient estimation of the first- and second-order coefficients. Because Box-Behnken designs have fewer design points, they are less expensive to run than central composite designs with the same number of factors.

Box-Behnken designs can also prove useful if you know the safe operating zone for your process. Central composite designs usually have axial points outside the "cube" (unless you specify an α that is less than or equal to one). These points may not be in the region of interest, or may be impossible to run because they are beyond safe operating limits. Box-Behnken designs do not have axial points, thus, you can be sure that all design points fall within your safe operating zone. Box-Behnken designs also ensure that all factors are never set at their high levels simultaneously.

Chapter 20   Creating Response Surface Designs

▶ **To create a response surface design**

1 Choose **Stat ➤ DOE ➤ Response Surface ➤ Create Response Surface Design**
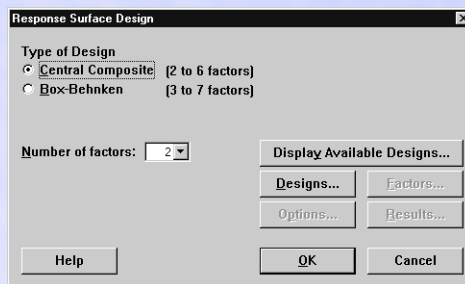
2 If you want to see a summary of the response surface designs, click **Display Available Designs**. Use this table to compare design features. Click **OK**.

3 Under **Type of Design**, choose **Central composite** or **Box-Behnken**.
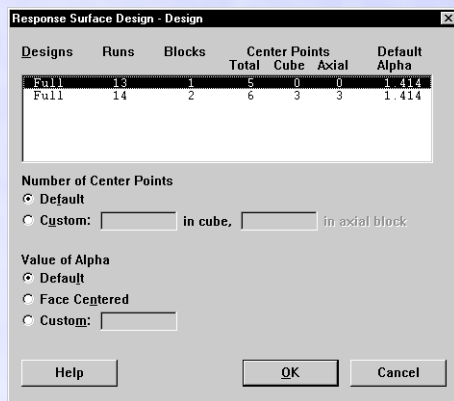
4 From **Number of factors**, choose a number:

- for a central composite design, choose a number from 2 to 6

- for a Box-Behnken design, choose a number from 3 to 7

5 Click **Designs**.

Creating Response Surface Designs   Response Surface Designs

The subdialog box that displays depends whether you choose **Central composite** or **Box-Behnken** in step 3.

Central Composite Design   **Box-Behnken Design**

6 Do one of the following:

  ■ for a central composite design, choose the design you want to create from the list shown at the top of the subdialog box

  ■ for a Box-Behnken design, you do not have to choose a design because the number of factors determines the number of runs

7 If you like, use any of the options listed under *Design subdialog box* below.

8 Click **OK** even if you do not change any of the options. This selects the design and brings you back to the main dialog box.

9 If you like, click **Options, Factors,** or **Results** to use any of the options listed below, then click **OK** to create your design.

Note   *Central composite designs* on page 20-18 shows the central composite designs you can generate with **Create Response Surface Design**. The default values of α provide orthogonal blocking and, whenever possible, rotatability. The "cube" portions of central composite designs are identical to those generated by **Create Factorial Design** with the same number of center points and blocks. Thus, a design generated by **Create Factorial Design** with the same number of runs, center points, and blocks can be built up into an orthogonally-blocked central composite design.

Any factorial design with the right number of runs and blocks can be built up into a blocked central composite design. However, to make the blocks orthogonal, **Create Factorial Design** must use the number of center points shown in *Central composite designs* on page 20-18.

Options

Design subdialog box

- block the design—see *Blocking the design* on page 20-8

- change the number of center points—see *Changing the number of center points* on page 20-9

- for a central composite design, change the position of the axial settings (α)—see *Changing the value of α for a central composite design* on page 20-10

Options subdialog box

- randomize the design—see *Randomizing the design* on page 20-10

- store the design—see *Storing the design* on page 20-11

Factors subdialog box

- name factors—see *Naming factors* on page 20-12

- set factor levels—see *Setting factor levels* on page 20-12

- for a central composite design, define the low and high values of the experiment in terms of the axial points rather than the "cube" points—see *Setting factor levels* on page 20-12

Results subdialog box

- display the summary and data tables or suppress all Session window results

Blocking the design

When the number of runs is too large to be completed under steady state conditions, you need to be concerned with the error that may be introduced into the experiment. Running an experiment in blocks allows you to separately and independently estimate the block effects (or different experimental conditions) from the factor effects. For example, blocks might be days, suppliers, batches of raw material, machine operators, or manufacturing shift.

Central composite designs

For a central composite design, the number of orthogonal blocks depends on the number of factors, the number of runs, and the design fraction you choose. A central composite design can always be separated into a factorial block and an axial point block. With three or more factors, the factorial block can also be divided into two or more blocks. When you are creating a design, MINITAB displays the appropriate choices.

Creating Response Surface Designs | Response Surface Designs

More | The value of α, in combination with the number of center points, determines whether a design exhibits the properties of rotatability and orthogonal blocking. MINITAB provides default designs that achieve rotatability and orthogonal blocking, when both properties can be achieved simultaneously. When the design is blocked and you cannot achieve both properties simultaneously, the default designs provide for orthogonal blocking.

Box-Behnken designs

For a Box-Behnken design, the number of ways to block a design depends on the number of factors. All of the blocked designs have orthogonal blocks. A design with

- three factors cannot be blocked
- four factors can be run in three blocks
- five, six, or seven factors can be run in two blocks

When you are creating a design, MINITAB displays the appropriate choices.

Changing the number of center points

The number of center points, along with α (for a central composite design), determines whether or not a design can be orthogonally blocked. By default, MINITAB chooses the number of center points to achieve orthogonal blocking.

The inclusion of center points provides an estimate of experimental error and allows you to check the adequacy of the model (lack of fit). Checking the adequacy of the fitted model is important as an incorrect or under-specified model can result in misleading conclusions.

The default number of center points is shown in the Designs subdialog box. For a table showing the default number of center points for all designs, see *Central composite designs* on page 20-18 and *Box-Behnken designs* on page 20-18.

▶ **To change the default number of center points**

1 In the Create Response Surface Design dialog box, click **Designs**.

2 Do one of the following, then click **OK**.

- For a central composite design, under **Number of center points** choose **Custom** and enter a number in **in cube**. When you have more than one block in your design, you also need to enter a number to indicate the number of center points in **in axial block**.

- For a Box-Behnken design, under **Number of center points** choose **Custom** and enter a number in the box.

Note | When a Box-Behnken design is blocked, the center points are divided equally (as much as possible) among the blocks.

Changing the value of α for a central composite design

The position of the axial points in a central composite design is denoted by α. The value of α, along with the number of center points, determines whether a design can be orthogonally blocked and is rotatable. Orthogonally blocked designs allow for model terms and block effects to be estimated independently and minimize the variation in the regression coefficients. Rotatable designs provide the desirable property of constant prediction variance at all points that are equidistant from the design center, thus improving the quality of the prediction.

MINITAB's default designs achieve rotatability and orthogonal blocking when both properties can be achieved simultaneously. When the design is blocked and you cannot achieve both properties simultaneously, the default designs use α such that the design is orthogonally blocked. When there are no blocks, the default designs use α such that the design is rotatable.

The default value for α for each central composite design is shown on page 20-18.

▶ **To change the default value of α**

1 In the Create Response Surface Design dialog box, click **Designs**.

2 Do one of the following, then click **OK**.

   ■ To set α equal to 1, choose **Face Centered**. When $\alpha = 1$, the axial points are placed on the "cube" portion of the design. This is an appropriate choice when the "cube" points of the design are at the operational limits.

   ■ Choose **Custom** and enter a positive number in the box. A value less than one places the axial points inside the "cube" portion of the design; a value greater than one places the axial points outside the "cube."

Note | A value of $\alpha = (F)^{\frac{1}{4}}$, where F is the number of factorial points in the design, guarantees rotatability.

Randomizing the design

By default, MINITAB randomizes the run order of the design. The ordered sequence of the factor combinations (experimental conditions) is called the *run order*. It is usually a good idea to randomize the run order to lessen the effects of factors that are not included in the study, particularly effects that are time-dependent.

However, there may be situations when randomization leads to an undesirable run order. For instance, in industrial applications, it may be difficult or expensive to change factor levels. Or, after factor levels have been changed, it may take a long time for the system to return to steady state. Under these conditions, you may not want to randomize the design in order to minimize the level changes.

Every time you create a design, MINITAB reserves and names C1 (StdOrder) and C2 (RunOrder) to store the standard order and run order, respectively.

- StdOrder shows what the order of the runs in the experiment would be if the experiment was done in standard order—also called Yates' order.

- RunOrder shows what the order of the runs in the experiment would be if the experiment was run in random order.

If you do not randomize, the run order and standard order are the same.

If you want to re-create a design with the same ordering of the runs (that is, the same design order), you can choose a base for the random data generator. Then, when you want to re-create the design, you just use the same base.

More | You can use **Stat ➤ DOE ➤ Display Design** (page 20-24) to switch back and forth between a random and standard order display in the worksheet.

Storing the design

If you want to analyze a design, you *must* store it in the worksheet. By default, MINITAB stores the design. If you want to see the properties of various designs before selecting the design you want to store, uncheck **Store design in worksheet** in the Options subdialog box.

Every time you create a design, MINITAB reserves and names the following columns:

- C1 (StdOrder) stores the standard order.

- C2 (RunOrder) stores run order.

- C3 (Blocks) stores the blocking variable. When the design is not blocked, MINITAB sets all column values to one.

- C4 – Cn stores the factors. MINITAB stores each factor in your design in a separate column.

If you named the factors, these names display in the worksheet. If you did not provide names, MINITAB names the factors alphabetically. After you create the design, you can change the factor names directly in the Data window or with **Stat ➤ DOE ➤ Modify Design** (page 20-20).

If you did not assign factor levels in the Factors subdialog box, MINITAB stores factor levels in coded form (all factor levels are –1 or +1). If you assigned factor levels, the uncoded levels display in the worksheet. To switch back and forth between a coded and an uncoded display, use **Stat ➤ DOE ➤ Display Design** (page 20-24).

Caution | When you create a design using Create Response Surface Design, MINITAB stores the appropriate design information in the worksheet. MINITAB needs this stored information to analyze and plot data. If you want to use Analyze Response Surface Design, you must follow certain rules when modifying the worksheet data. See *Modifying and Using Worksheet Data* on page 18-4.

If you make changes that corrupt your design, you may still be able to analyze it with Analyze Response Surface Design after you use Define Custom Response Surface Design (page 20-19).

Naming factors

By default, MINITAB names the factors alphabetically.

▶ **To name factors**

1   In the Create Response Surface Design dialog box, click **Factors**.
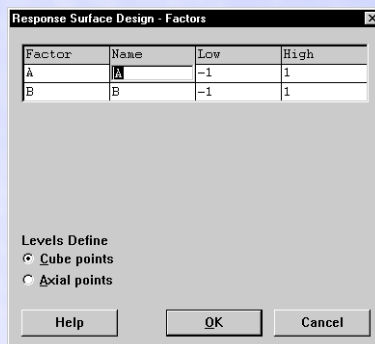
2   Under **Name**, click in the first row and type the name of the first factor. Then, use the ⬇ key to move down the column and enter the remaining factor names. Click **OK**.

More | After you have created the design, you can change the factor names by typing new names in the Data window or with Modify Design (page 20-20).

Setting factor levels

In a response surface design, you designate a low level and a high level for each factor. These factor levels define the proportions of the "cube" around which the design is built. The "cube" is often centered around the current operating conditions for the process. For a central composite design, you may have design points inside the "cube," on the "cube," or outside the "cube." For a Box-Behnken design, the factor levels are the lowest and highest points in the design. See the illustrations on pages 20-4 and 20-6.

By default, MINITAB sets the low level of all factors to –1 and high level to +1.

▶ **To assign factor levels**

1 In the Create Response Surface Design dialog box, click **Factors**.

This option is only
available for central
composite designs.

2 Under **Low**, click in the row for the factor you would like to assign values and enter
any numeric value. Use the ⇥ key to move to **High** and enter a numeric value that is
greater than the value you entered in **Low**.

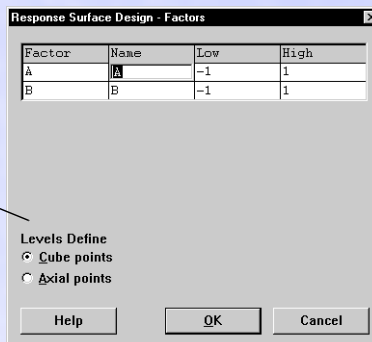3 Repeat step 2 to assign levels for other factors.

4 For a central composite design, under **Levels Define**, choose **Cube points** or **Axial
points** to specify which values you entered in **Low** and **High**. Click **OK**.

Note | In a central composite design, the values you enter for the factor levels are usually not the
minimum and maximum values in the design. They are the low and high settings for the
"cube" portion of the design. The axial points are usually outside the "cube" (unless you
specify an α that is less than or equal to 1). If you are not careful, this could lead to axial
points that are not in the region of interest or may be impossible to run.

Choosing **Axial points** in the Factors subdialog box guarantees all of the design points
will fall between the defined minimum and maximum value for the factor(s). MINITAB will
then determine the appropriate low and high settings for the "cube" as follows:

$$\text{Low Level Setting} = \frac{(\alpha - 1)\ max + (\alpha + 1)\ min}{2\alpha}$$

$$\text{High Level Setting} = \frac{(\alpha - 1)\ min + (\alpha + 1)\ max}{2\alpha}$$

More | To change the factor levels after you have created the design, use **Stat ➤ DOE ➤ Modify
Design**.

▶ **Example of a central composite design**

Suppose you want to conduct an experiment to maximize crystal growth. You have determined that three variables—time the crystals are exposed to a catalyst, temperature in the exposure chamber, and percentage of the catalyst in the air inside the chamber—explain much of the variability in the rate of crystal growth.

You generate the default central composite design for three factors and two blocks (to represent the two days you conduct the experiment). You assign the factor levels and randomize the design.

1 Choose **Stat ➤ DOE ➤ Response Surface ➤ Create Response Surface Design**.

2 Under **Type of Design**, choose **Central Composite**.

3 From **Number of factors**, choose *3*.

4 Click **Designs**. To create the design with 2 blocks, highlight the second row in the **Design** box at the top. Click **OK**.

5 Click **Factors**. Complete the **Name**, **Low**, and **High** columns of the table as shown below:

Factor	Name	Low	High
A	Time	6	9
B	Temperature	40	60
C	Catalyst	3.5	7.5

6 Click **OK**.

7 Click **Results**. Choose **Summary table and data table**. Click **OK** in each dialog box. .

Session window output

Central Composite Design

Central Composite Design

```
Factors:     3   Blocks:      2   Center points in cube:   4
Runs:       20   Alpha:   1.633   Center points in star:   2

Data Matrix (randomized)

Run Block      A      B      C
  1    1    1.000 -1.000 -1.000
  2    1    1.000  1.000 -1.000
  3    1   -1.000  1.000  1.000
  4    1    0.000  0.000  0.000
  5    1    0.000  0.000  0.000
  6    1   -1.000  1.000 -1.000
  7    1   -1.000 -1.000 -1.000
  8    1   -1.000 -1.000  1.000
  9    1    0.000  0.000  0.000
 10    1    0.000  0.000  0.000
 11    1    1.000  1.000  1.000
 12    1    1.000 -1.000  1.000
 13    2    0.000 -1.633  0.000
 14    2    0.000  0.000  0.000
 15    2   -1.633  0.000  0.000
 16    2    0.000  0.000  1.633
 17    2    0.000  0.000  0.000
 18    2    0.000  1.633  0.000
 19    2    0.000  0.000 -1.633
 20    2    1.633  0.000  0.000
```

Interpreting the results

You have created a central composite design with three factors which will be run in two blocks. This design is both rotatable and orthogonally blocked—see *Central composite designs* on page 20-18.

Because you chose to display the summary and data tables, MINITAB shows the experimental conditions or settings for each of the factors for the design points. When you perform the experiment, use the order that is shown to determine the conditions for each run. For example, in the first run of your experiment, you would set the time (A) at 9 minutes (1 = high), the temperature (B) at 40° (–1 = low), and use 3.5 grams of the catalyst (C) (–1 = low).

Note MINITAB randomizes the design by default, so if you try to replicate this example, your runs may not match the order shown.

▶ **Example of a Box-Behnken design**

Suppose you have a process for pressure treating utility poles with creosote. In the treating step of the process, you place air-dried poles inside a treatment chamber. The pressure in the chamber is increased and the chamber is flooded with hot creosote. The poles are left in the chamber until they have absorbed 12 pounds of creosote per cubic foot. You would like to experiment with different settings for the air pressure, temperature of the creosote, and time in the chamber. Your goal is to get the creosote absorption as close to 12 pounds per cubic foot as possible, with minimal variation. Previous investigation suggests that the response surface for absorption exhibits curvature.

The chamber will withstand internal pressures up to 220 psi, although the strain on equipment is pronounced at over 200 psi. The current operating value is at 175 psi, so you feel comfortable with a range of values between 150 and 200. Current operating values for temperature and time are 210° F and 5 hours, respectively. You feel that temperature cannot vary by more than 10° from the current value. Time can be varied from 4 to 6 hours.

A Box-Behnken design is a practical choice when you cannot run all of the factors at their high (or low) levels at the same time. Here, the high level for pressure is already at the limit of what the chamber can handle. If temperature were also at its high level, this increases the effective pressure, and running at these settings for a long period of time is not recommended. The Box-Behnken design will assure that no runs require all factors to be at their high settings simultaneously.

1 Choose **Stat ➤ DOE ➤ Response Surface ➤ Create Response Surface Design**.

2 Under **Type of Design**, choose **Box-Behnken**.

3 From **Number of factors**, choose *3*.

4 Click **Designs**. Click **OK**.

5 Click **Factors**. Complete the **Name**, **Low**, and **High** columns of the table as shown below:

Factor	Name	Low	High
A	Pressure	150	200
B	Temperature	200	220
C	Time	4	6

6 Click **OK**.

7 Click **Results**. Choose **Summary table and data table**. Click **OK** in each dialog box.

Session window output

Box-Behnken Design

```
Box-Behnken Design

Factors:   3      Blocks:        none
Runs:     15      Center points:   3


Data Matrix (randomized)

    Run  A  B  C
     1   0  +  -
     2   +  -  0
     3   -  0  -
     4   0  -  -
     5   +  0  -
     6   +  +  0
     7   0  0  0
     8   0  +  +
     9   -  0  +
    10   0  0  0
    11   0  0  0
    12   -  +  0
    13   0  -  +
    14   +  0  +
    15   -  -  0
```

Interpreting the results

Because you chose to display the summary and data tables, MINITAB shows the experimental conditions or settings for each of the factors for the design points. When you perform the experiment, use the order that is shown to determine the conditions for each run. For example, in the first run of your experiment, you would set the pressure at 175 psi (0 = center), the temperature at 220°F (+ = high), and treat the utility poles for 4 hours (− = low).

Note   MINITAB randomizes the design by default, so if you try to replicate this example, your runs may not match the order shown.

Summary of Available Designs

Central composite designs

factors	total runs	total blocks	cube blocks	cube runs	total center points	cube center points	axial center points	default alpha	orthogonal blocks	rotatable
2	13	—	—	4	5	—	—	1.414	—	y
	14	2	1	4	6	3	3	1.414	y	y
3	20	—	—	8	6	—	—	1.682	—	y
	20	2	1	8	6	4	2	1.633	y	n
	20	3	2	8	6	4	2	1.633	y	n
4	31	—	—	16	7	—	—	2.000	—	y
	30	2	1	16	6	4	2	2.000	y	y
	30	3	2	16	6	4	2	2.000	y	y
5 half	32	—	—	16	6	—	—	2.000	—	y
	33	2	1	16	7	6	1	2.000	y	y
5	52	—	—	32	10	—	—	2.378	—	y
	54	2	1	32	12	8	4	2.366	y	n
	54	3	2	32	12	8	4	2.366	y	n
6 half	53	—	—	32	9	—	—	2.378	—	y
	54	2	1	32	10	8	2	2.366	y	n
	54	3	2	32	10	8	2	2.366	y	n
6	90	—	—	64	14	—	—	2.828	—	y
	90	2	1	64	14	8	6	2.828	y	y
	90	3	2	64	14	8	6	2.828	y	y
	90	5	4	64	14	8	6	2.828	y	y

Box-Behnken designs

| factors	runs	blocks	center points
3	15	1	3
4	27	3	3
5	46	2	6
6	54	2	6
7	62	2	6

Defining Custom Designs

Use **Define Custom Response Surface Design** to create a design from data you already have in the worksheet. For example, you may have a design that you created using MINITAB session commands, entered directly into the Data window, imported from a data file, or created with earlier releases of MINITAB. You can also use **Define Custom Response Surface Design** to redefine a design that you created with Create Response Surface Design and then modified directly in the worksheet.

Define Custom Response Surface Design allows you to specify which columns contain your factors and other design characteristics. After you define your design, you can use Modify Design (page 20-20), Display Design (page 20-24), and Analyze Response Surface Design (page 20-26).

▶ **To define a custom response surface design**

1 Choose **Stat ➤ DOE ➤ Response Surface ➤ Define Custom Response Surface Design**.
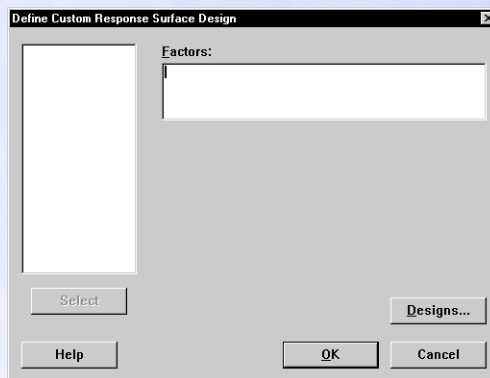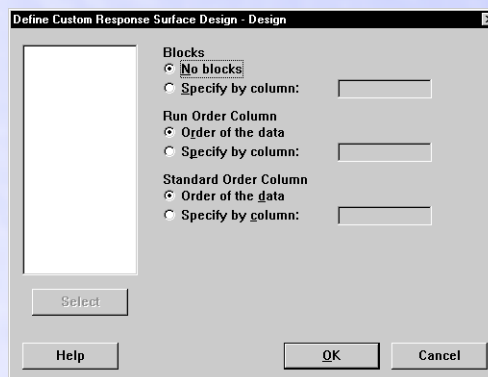

2 In **Factors**, enter the columns that contain the factor levels.

3 Do one of the following:

- If you do not have any columns containing the blocks, run order, or standard order, click **OK**.

- If you have additional columns that contain data for the blocks, run order, or standard order, click **Designs**.

1 If your design is blocked, under **Blocks**, choose **Specify by column** and enter the column containing the blocks.

2 If you have a column that contains the run order of the experiment, under **Run Order Column**, choose **Specify by column** and enter the column containing the run order.

3 If you have a column that contains the standard order of the experiment, under **Standard Order Column**, choose **Specify by column** and enter the column containing the standard order. Click **OK** in each dialog box.

Modifying Designs

After creating a design and storing it in the worksheet, you can use Modify Design to make the following modifications:

- rename the factors and change the factor levels—see *Renaming factors and changing factor levels* below

- replicate the design—see *Replicating the design* on page 20-22

- randomize the design—see *Randomizing the design* on page 20-23

By default, MINITAB will replace the current design with the modified design.

Renaming factors and changing factor levels

If you did not name the factors, MINITAB assigns letter names alphabetically. You can use Modify Design to change the default names or names that you assigned when you created the design. You can also change factor levels from the default values of −1 and +1 or change previously assigned values.

▶ To rename factors or change factor levels

1 Choose **Stat ➤ DOE ➤ Modify Design**.
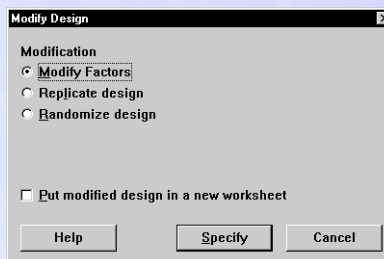
2 Choose **Modify factors** and click **Specify**.
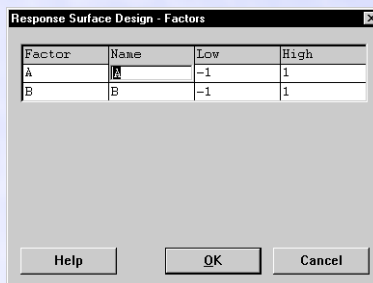
3 Enter new factor names or factor levels as shown in *Naming factors* on page 20-12 and *Setting factor levels* on page 20-12. Click **OK**.

Tip   You can also type new factor names directly into the Data window.

Replicating the design

You can add up to ten replicates of your design. When you replicate a design, you duplicate the complete set of runs from the initial design. The runs that would be added to a two-factor central composite design are as follows:

Initial design		One replicate added (total of two replicates)		Two replicates added (total of three replicates)	
A	B	A	B	A	B
-1.414	0.000	-1.414	0.000	-1.414	0.000
1.000	-1.000	1.000	-1.000	1.000	-1.000
0.000	1.414	0.000	1.414	0.000	1.414
0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000
1.414	0.000	1.414	0.000	1.414	0.000
0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000
0.000	-1.414	0.000	-1.414	0.000	-1.414
1.000	1.000	1.000	1.000	1.000	1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	1.000	-1.000	1.000	-1.000	1.000
		-1.414	**0.000**	**-1.414**	**0.000**
		1.000	**-1.000**	**1.000**	**-1.000**
		0.000	**1.414**	**0.000**	**1.414**
		0.000	**0.000**	**0.000**	**0.000**
		0.000	**0.000**	**0.000**	**0.000**
		1.414	**0.000**	**1.414**	**0.000**
		0.000	**0.000**	**0.000**	**0.000**
		0.000	**0.000**	**0.000**	**0.000**
		0.000	**0.000**	**0.000**	**0.000**
		0.000	**-1.414**	**0.000**	**-1.414**
		1.000	**1.000**	**1.000**	**1.000**
		-1.000	**-1.000**	**-1.000**	**-1.000**
		-1.000	**1.000**	**-1.000**	**1.000**
				-1.414	**0.000**
				1.000	**-1.000**
				0.000	**1.414**
				0.000	**0.000**
				0.000	**0.000**
				1.414	**0.000**
				0.000	**0.000**
				0.000	**0.000**
				0.000	**0.000**
				0.000	**-1.414**
				1.000	**1.000**
				-1.000	**-1.000**
				-1.000	**1.000**

True replication provides an estimate of the error or noise in your process and may allow for more precise estimates of effects.

Modifying Designs   Response Surface Designs

▶ **To replicate the design**

1 Choose **Stat ➤ DOE ➤ Modify Design**.

2 Choose **Replicate design** and click **Specify**.

Modify Design - Replicate Design	☒
Number of replicates to add: ▓▓1▼	
Help	OK	Cancel

3 From **Number of replicates to add**, choose a number up to ten. Click **OK**.

Randomizing the design

You can randomize the entire design or just randomize one of the blocks. For a general discussion of randomization, see page 20-10.

▶ **To randomize the design**

1 Choose **Stat ➤ DOE ➤ Modify Design**.

2 Choose **Randomize design** and click **Specify**.

Modify - Randomize Design	☒
⦿ Randomize entire design	
○ Randomize just block: [▼]	
Base for random data generator [ ]	
Note, when you randomize a design, Minitab updates the Run Order column but does not reorder the rows of the worksheet. If you want to see the rows of your design in run order, select DOE>Display Design from the menu.	
Help	OK	Cancel

3 Do one of the following:

   ■ Choose **Randomize entire design**.

   ■ Choose **Randomize just block,** and choose a block number from the list.

4 If you like, in **Base for random data generator**, enter a number. You can recreate a design by using the same base each time. Click **OK**.

More   You can use **Stat ➤ DOE ➤ Display Design** (page 20-24) to switch back and forth between a random and standard order display in the worksheet.

Displaying Designs

After you create the design, you can use Display Design to change the way the design points are stored in the worksheet. You can change the design points in two ways:

■ display the points in either run or standard order. Standard order is the order of the runs if the experiment was done in Yates' order. Run order is the order of the runs if the experiment was done in random order.

■ express the factor levels in coded or uncoded form.

▶ **To change the display order of points in the worksheet**

1 Choose **Stat ➤ DOE ➤ Display Design**.
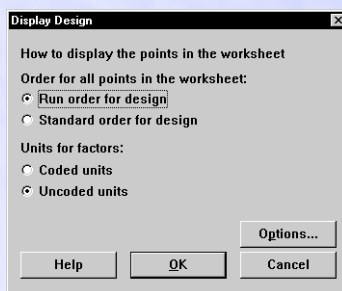
2 Choose **Run order for the design** or **Standard order for the design**. If you do not randomize a design, the columns that contain the standard order and run order are the same.

3 Do one of the following:

■ If you want to reorder all worksheet columns that are the same length as the design columns, click **OK**.

■ If you have worksheet columns that you do not want to reorder:

1 Click **Options**.

2 In **Exclude the following columns when sorting**, enter the columns. These columns *cannot* be part of the design. Click **OK** in each dialog box.

Displaying the design in coded or uncoded units

If you assigned factor levels in the Factors subdialog box, the uncoded or actual levels initially display in the worksheet. For example, if you entered 50 for the low level of temperature and 80 for the high level of temperature in the Factors subdialog box, these uncoded levels display in the worksheet. The coded levels are –1 and +1.

If you do not assign factor levels, displaying the design in coded and uncoded units is the same.

▶ **To change the display units for the factors**

1 Choose **Stat ➤ DOE ➤ Display Design**.

2 Choose **Coded units** or **Uncoded units**. Click **OK**.

Collecting and Entering Data

After you create your design, you need to perform the experiment and collect the response (measurement) data. To print a data collection form, follow the instructions below. After you collect the response data, enter the data in any worksheet column not used for the design. For a discussion of the worksheet structure, see *Storing the design* on page 20-11.

Printing a data collection form

You can generate a data collection form in two ways. You can simply print the Data window contents, or you can use a macro. A macro can generate a "nicer" data collection form—see Help for more information. Although printing the Data window will not produce the prettiest form, it is the easiest method. Just follow these steps:

1 When you create your experimental design, MINITAB stores the run order, block assignment, and factor settings in the worksheet. These columns constitute the basis of your data collection form. If you did not name factors or specify factor levels when you created the design, and you want names or levels to appear on the form, use Modify Design (page 20-20).

2 In the worksheet, name the columns in which you will enter the measurement data obtained when you perform your experiment.

3 Choose **File ➤ Print Worksheet**. Make sure **Print Grid Lines** is checked, then click **OK**.

More You can also copy the worksheet cells to the Clipboard by choosing **Edit ➤ Copy Cells**. Then paste the Clipboard contents into a word-processing application, such as Microsoft Wordpad or Microsoft Word, where you can create your own form.

Analyzing Response Surface Designs

To use Analyze Response Surface Design to fit a model, you must create and store the design using Create Response Surface Design, or create a design from data that you already have in the worksheet with **Define Custom Response Surface Design**.

You can choose to fit models with the following terms:

- all linear terms
- all linear terms and all squared terms
- all linear terms and all two-way interactions
- all linear terms, all squared terms, and all two-way interactions (the default)
- a subset of linear terms, squared terms, and two-way interactions

The model you fit will determine the nature of the effect, linear or curvilinear, that you can detect—see *Selecting model terms* on page 20-29.

Data

Enter up to 25 numeric response data columns that are equal in length to the design variables in the worksheet. Each row will contain data corresponding to one run of your experiment. You may enter the response data in any column(s) not occupied by the design data. The number of columns reserved for the design data is dependent on the number of factors in your design.

If there is more than one response variable, MINITAB fits separate models for each response.

MINITAB omits missing data from all calculations.
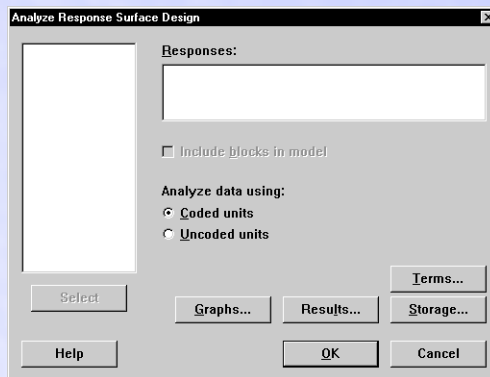
Note   When all the response variables do not have the same missing value pattern, MINITAB displays a message. Since you would get different results, you may want to repeat the analysis separately for each response variable.

Analyzing Response Surface Designs   Response Surface Designs

▶ **To fit a response surface model**

1 Choose **Stat ➤ DOE ➤ Response Surface ➤ Analyze Response Surface Design**.

Analyze Response Surface Design

Responses:

☐ Include blocks in model

Analyze data using:
○ Coded units
○ Uncoded units

Terms...

Select   Graphs...   Results...   Storage...

Help   OK   Cancel

2 In **Responses**, enter up to 25 columns that contain the response data.

3 If you like, use any of the options listed below, then click **OK**.

Options

Analyze Response Surface Design dialog box

■ include blocks in the model. This option is available when the blocks column contains more than one distinct value.

■ fit the model with coded or uncoded factor levels. See *Choosing data units* on page 20-28.

Graphs subdialog box

■ draw five different residual plots for regular, standardized, or deleted residuals—see *Choosing a residual type* on page 2-5. Available residual plots include a
  – histogram.
  – normal probability plot.
  – plot of residuals versus the fitted values ($\hat{Y}$).
  – plot of residuals versus data order. The row number for each data point is shown on the x-axis—for example, 1 2 3 4... *n.*
  – separate plot for the residuals versus each specified column.

For a discussion, see *Residual plots* on page 2-6.

Terms subdialog box

■ fit a model by specifying the maximum order of the terms, or choose which terms to include from a list of all estimable terms—see *Selecting model terms* on page 20-29.

Results subdialog box

■ display the following in the Session window:
  – no results
  – the default results, which includes a table of coefficients, s, R^2, adjusted R^2, the analysis of variance table, and the unusual values in the table of fits and residuals
  – the default results, plus a table of all the fits and residuals

Storage subdialog box

■ store the fits and regular, standardized, and deleted residuals separately for each response—see *Choosing a residual type* on page 2-5.

■ store the coefficients for the model and the design matrix, separately for each response. The design matrix multiplied by the coefficients will yield the fitted values.

■ store information about the fitted model in a column by checking **Quadratic** in the Storage subdialog box—see Help for the structure of this column.

■ store leverages, Cook's distances, and DFITS for identifying outliers—see *Identifying outliers* on page 2-9.

Choosing data units

The following results differ depending on whether you analyze the data in coded or uncoded units (the actual factor levels):

■ coefficients and their standard deviations

■ t-value for the constant term

■ p-value for the constant term

Additional results would be the same, including which terms in the model are significant.

▶ **To specify the data units for analysis**

1 In the Create Response Surface Design dialog box, under **Analyze data using**, choose **coded units** or **uncoded units**.

More   Analyze Response Surface Design uses the same method of coding as General Linear Model—see *Design matrix used by General Linear Model* on page 3-42.

Analyzing Response Surface Designs   Response Surface Designs

Selecting model terms

The model you choose determines what terms are fit and whether or not you can model linear or curvilinear aspects of the response surface. If you include any second-order terms (squares or interactions), you can model curvilinearity.

You can fit a linear, linear and squares, linear and two-way interactions, or full quadratic (default) model. Or, you can fit a model that is a subset of these terms. The table below shows what terms would be fit for a model with four factors.

This model type	fits these terms
linear	A B C D
linear and squares	A B C D A∗A B∗B C∗C D∗D
linear and two-way interactions	A B C D A∗B A∗C A∗D B∗C B∗D C∗D
full quadratic (default)	A B C D A∗A B∗B C∗C D∗D A∗B A∗C A∗D B∗C B∗D C∗D

▶ **To specify the model**

1 In the Analyze Response Surface Design dialog box, click **Terms**.
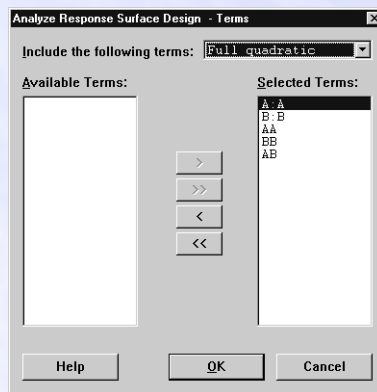
2 Do one of the following:

■ from **Include the following terms**, choose **linear**, **linear + squares**, **linear + interactions**, or **full quadratic**

■ move the terms you do not want to include in the model from **Selected Terms** to **Available Terms** using the arrow buttons
   – to move one or more factors, highlight the desired terms then click ⟨ or ⟩
   – to move all of the terms, click ⟪ or ⟫

You can also move a term by double-clicking it.

▶ Example of fitting a response surface model

The following examples use data from [3]. The experiment uses three factors—nitrogen, phosphoric acid, and potash—all ingredients in fertilizer. The effect of the fertilizer on snap bean yield was studied in a central composite design using the default (coded) factor levels.

The actual (uncoded) units for the −1 and +1 levels are 2.03 and 5.21 for nitrogen, 1.07 and 2.49 for phosphoric acid, 1.35 and 3.49 for potash. If we were to analyze the design in uncoded units, a few things would change: the coefficients and their standard deviations, and the t-value and p-value for the constant term. Additional results would be the same, including which terms in the model are significant.

Step 1: Generating the central composite design

1 Choose **Stat ➤ DOE ➤ Response Surface ➤ Create Response Surface Design**.

2 Under **Type of Design**, choose **Central composite**.

3 From **Number of factors**, choose *3*.

4 Click **Designs**. Click **OK**.

5 Click **Factors**. In the **Name** column, enter *Nitrogen PhosAcid Potash* in rows one through three, respectively. Click **OK** in each dialog box.

Step 2: Fitting a linear model

1 Open the worksheet CCD_EX1.MTW. (The design from the previous step and the response data have been saved for you.)

2 Choose **Stat ➤ DOE ➤ Response Surface ➤ Analyze Response Surface Design**.

3 In **Variables**, enter *BeanYield*.

4 Click **Terms**.

5 From **Include the following terms**, choose **Linear**. Click **OK** in each dialog box.

Session window output

Response Surface Regression

```
The analysis was done using coded units.

Estimated Regression Coefficients for BeanYiel

Term            Coef       StDev        T       P
Constant      10.1980     0.3473    29.364   0.000
Nitrogen      -0.5738     0.4203    -1.365   0.191
PhosAcid       0.1834     0.4203     0.436   0.668
Potash         0.4555     0.4203     1.084   0.295

s = 1.553       R-Sq = 16.8%     R-Sq(adj) = 1.2%
Analysis of Variance for BeanYiel

Source            DF    Seq SS    Adj SS    Adj MS      F      P
Regression         3     7.789     7.789    2.5962   1.08   0.387
  Linear           3     7.789     7.789    2.5962   1.08   0.387
Residual Error    16    38.597    38.597    2.4123
  Lack-of-Fit     11    36.057    36.057    3.2779   6.45   0.026
  Pure Error       5     2.540     2.540    0.5079
Total             19    46.385

Unusual Observations for BeanYiel

Observation    BeanYiel       Fit    StDev Fit    Residual    St Resid
        15        8.260    11.163        0.788      -2.903      -2.17R
        18       13.190    10.500        0.807       2.690       2.03R

R denotes an observation with a large standardized residual
```

Interpreting the results

It is important to check the adequacy of the fitted model, because an incorrect or under-specified model can result in misleading conclusions. By checking the fit of the linear (first-order) model you can tell if the model is under specified. The small p-value (p = 0.026) for the lack of fit test indicates the linear model does not adequately fit the response surface. The F-statistic for this test is (Adj MS for Lack of Fit) / (Adj MS for Pure Error).

Because the linear model does not adequately fit the response surface, you need to fit a quadratic (second-order) model.

► **Example of fitting a quadratic model**

In the previous example, you determined that the linear model did not adequately represent the response surface. The next step is to fit the quadratic model. The quadratic model allows detection of curvature in the response surface.

1 Open the worksheet CCD_EX1.MTW. (The design and response data have been saved for you.)

2 Choose **Stat ➤ DOE ➤ Response Surface ➤ Analyze Response Surface Design**.

3 In **Responses**, enter *BeanYield*.

4 Click **Terms**.

5 From **Include the following terms**, choose **Full quadratic**. Click **OK**.

6 Click **Graphs**.

7 Under **Residual Plots**, check **Histogram, Normal plot, Residuals versus fits,** and **Residuals versus order**. Click **OK** in each dialog box

Session window output

Response Surface Regression

The analysis was done using coded units.

Estimated Regression Coefficients for BeanYiel

Term	Coef	StDev	T	P
Constant	10.4623	0.4062	25.756	0.000
Nitrogen	-0.5738	0.2695	-2.129	0.059
Phospori	0.1834	0.2695	0.680	0.512
Potash	0.4555	0.2695	1.690	0.122
Nitrogen*Nitrogen	-0.6764	0.2624	-2.578	0.027
Phospori*Phospori	0.5628	0.2624	2.145	0.058
Potash*Potash	-0.2734	0.2624	-1.042	0.322
Nitrogen*Phospori	-0.6775	0.3521	-1.924	0.083
Nitrogen*Potash	1.1825	0.3521	3.358	0.007
Phospori*Potash	0.2325	0.3521	0.660	0.524

s = 0.9960   R-Sq = 78.6%   R-Sq(adj) = 59.4%

Analysis of Variance for BeanYiel

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	9	36.465	36.465	4.0517	4.08	0.019
Linear	3	7.789	7.789	2.5962	2.62	0.109
Square	3	13.386	13.386	4.4619	4.50	0.030
Interaction	3	15.291	15.291	5.0970	5.14	0.021
Residual Error	10	9.920	9.920	0.9920		
Lack-of-Fit	5	7.380	7.380	1.4760	2.91	0.133
Pure Error	5	2.540	2.540	0.5079		
Total	19	46.385				

```
Unusual Observations for BeanYiel

Observation    BeanYiel        Fit    StDev Fit    Residual     St Resid
        14      11.060      12.362        0.776      -1.302       -2.09R
        15       8.260       9.514        0.776      -1.254       -2.01R
        18      13.190      12.004        0.815       1.186        2.07R

R denotes an observation with a large standardized residual
```
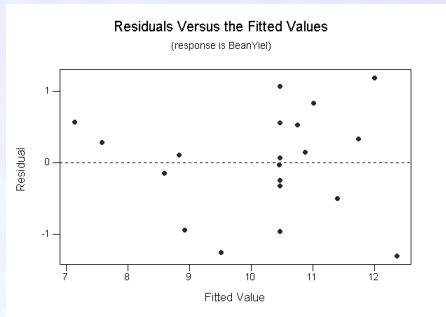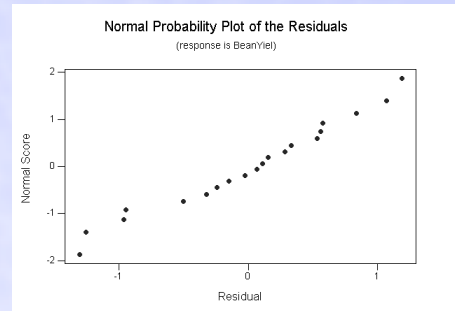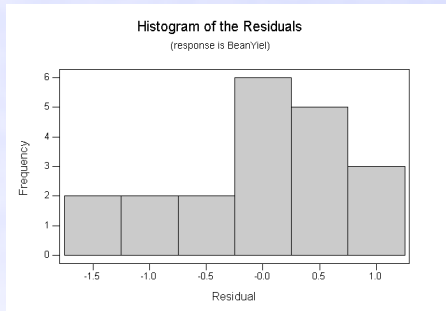
Graph window output
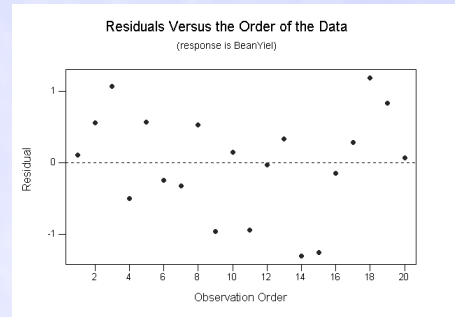
Interpreting the results

Since the linear model suggested that a higher model is needed to adequately model the response surface, you fit the full quadratic model. For the full quadratic model, the p-value for lack of fit is 0.133 suggesting that this model adequately fits the data.

The first table on the results gives the coefficients for all the terms in the model. Because you used an orthogonal design, each effect is estimated independently. Therefore, the coefficients for the linear terms are the same as when you fit just the linear model (page 20-30). The error term, s = 0.996, is smaller because you reduced the variability accounted for by error.

The Analysis of Variance table summarizes the linear terms, the squared terms, and the interactions. The small p-values for the interactions (p = 0.021) and the squared terms

(p = 0.030) suggest there is curvature in the response surface. In the table of Estimated Regression Coefficients, you will see small p-values for the Nitrogen by Potash interaction (p = 0.007), Nitrogen squared (p = 0.027), and Phosphoric acid squared (p = 0.058) suggesting these effects may be important.

In addition, MINITAB draws four residual plots. The residual plots do not indicate any problems with the model. For assistance in interpreting the residual plots, see *Residual plots* on page 2-6.

Plotting the Response Surface

You can use Contour/Surface (Wireframe) Plots to display two types of response surface plots: contour plots and surface plots (also called wireframe). These plots show how a response variable relates to two factors based on a model equation.

Contour and surface plots are useful for establishing desirable response values and operating conditions. In a contour plot, the response surface is viewed as a two-dimensional plane where all points that have the same response are connected to produce contour lines of constant responses.

A surface plot displays a three-dimensional view that may provide a clearer picture of the response surface. The illustrations below compare these two types of plots.
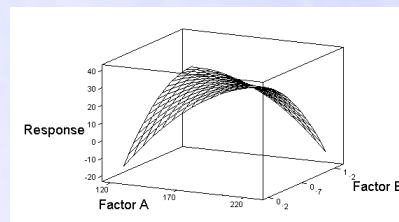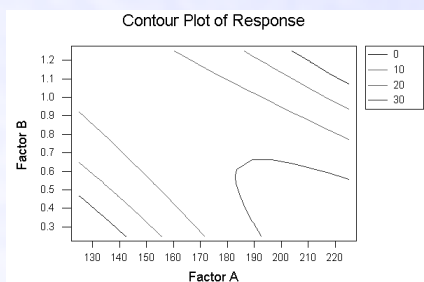
Note | When the model has more than two factors, the factor(s) that are not in the plot are held constant. You can specify the constant values at which to hold the remaining factors. See *Settings for extra factors* on page 20-36.

Data

Contour plots and surface plots are model dependent. Thus, you *must* fit a model using Analyze Response Surface Design before you can generate response surface plots with Contour/Surface (Wireframe) Plots. MINITAB looks in the worksheet for the necessary model information to generate these plots.

▶ **To plot the response surface**

1 Choose **Stat ➤ DOE ➤ Response Surface ➤ Contour/Surface (Wireframe) Plots**.

Response Surface Plots	☒
☐ Contour plot	Setup...
☐ Surface (wireframe) plot	Setup...
Help	OK	Cancel

2 Do one or both of the following:

   ■ to generate a contour plot, check **Contour plot** and click **Setup**

   ■ to generate a surface (wireframe) plot, check **Surface (wireframe) plot** and click **Setup**

3 If you like, use any of the options listed below, then click **OK** in each dialog box.

Options

Setup subdialog box

■ display a single graph for a selected factor pair

■ display separate graphs for every combination of factors in the model

■ display the data in coded or uncoded units

Settings subdialog box

■ specify values for factors that are not included in the response surface plot, instead of using the default of median (middle) values—see *Settings for extra factors* on page 20-36

Contours subdialog box

■ for contour plots, specify the number or location of the contour levels, and the contour line color and style—see *Controlling the number, type, and color of the contour lines* on page 20-36

Wireframe subdialog box

■ for surface (wireframe) plots, specify the color of the wireframe (mesh) and the surface

Options subdialog box

■ define minimum and maximum values for the x-axis and y-axis

■ replace the default title with your own title

Plotting the Response Surface

Settings for extra factors

You can set the holding level for factors that are not in the plot at their highest, lowest, or middle (calculated median) settings, or you can set specific levels at which to hold each factor.

▶ **To set the holding level for factors not in the plot**

1 In the Contour/Surface (Wireframe) dialog box, click **Setup**.

2 Click **Settings**.
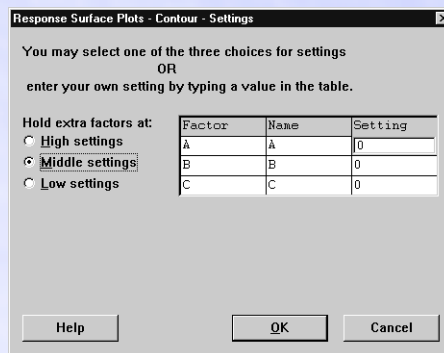
3 Do one of the following:

■ To use the preset values, choose **High settings**, **Middle settings**, or **Low settings** under **Hold extra factors at**. When you use a preset value, *all* factors not in the plot will be held at their high, middle (calculated median), or low settings. (Not available for custom designs.)

■ To specify the value(s) at which to hold the factor(s), enter a number in **Setting** for each factor you want control. This option allows you to set different hold settings for different factors.

4 Click **OK**.

Controlling the number, type, and color of the contour lines
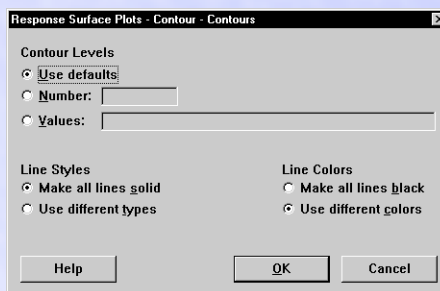
MINITAB displays from four to seven contour levels—depending on the data—by default. However, you can specify from 2 to 15 contour lines.

You can also change the line type and color of the lines.

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▶ **To control plotting of contour lines**

1 In the Contour/Surface (Wireframe) dialog box, check **Contour plot** and click **Setup**.

2 Click **Contours**.

```
Response Surface Plots - Contour - Contours                    ☒

  Contour Levels
   ◉ Use defaults
   ○ Number: [        ]
   ○ Values:  [                              ]

  Line Styles                  Line Colors
   ◉ Make all lines solid        ○ Make all lines black
   ○ Use different types         ◉ Use different colors

        Help                  OK        Cancel
```

3 To change the number of contour lines, do one of the following:

   ■ Choose **Number** and enter a number from 2 to 15.

   ■ Choose **Values** and enter from 2 to 15 contour level values in the units of your data. You must enter the values in increasing order.

4 To define the line style, choose **Make all lines solid** or **Use different types** under **Line Styles**.

5 To define the line color, choose **Make all lines black** or **Use different colors** under **Line Colors**. Click **OK**.

▶ **Example of a contour plot and a surface plot**

In the fertilizer example on page 20-30, you generated a design, supplied the response data, and fit a linear model. Since this linear model suggested that a higher model is needed to adequately model the response surface, you fit the full quadratic model. The full quadratic provides a better fit, with the squared terms for nitrogen and phosphoric acid and the nitrogen by potash interaction being important. The example below is a continuation of this analysis. Now you want to try an understand these effects by looking at a contour plot and a surface plot of snap bean yield versus the significant factors—nitrogen and phosphoric acid. By default, MINITAB selects the first factor, in this case nitrogen, for the vertical axis, and the second factor, phosphoric acid, for the horizontal axis.

1 Open the worksheet CCD_EX1.MTW. (The design, response data, and model information have been saved for you.)

2 Choose **Stat ➤ DOE ➤ Response Surface ➤ Contour/Surface (Wireframe) Plot**s.

3 Choose **Contour plot** and click **Setup**. Click **OK**.

4 Choose **Surface (wireframe) plot** and click **Setup**. Click **OK** in each dialog box.

Graph window output

Contour Plot of BeanYiel

Hold values: Potash: 0.0
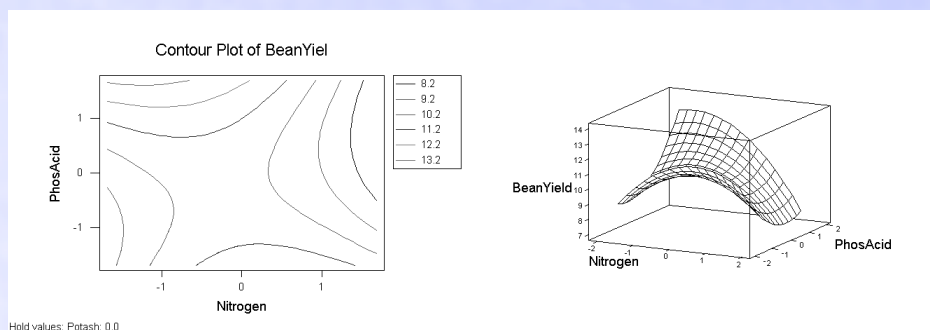
Interpreting the results

The contour plots indicates that the highest yield is obtained when nitrogen levels are low and phosphoric acid levels are high. This area appears at the upper left corner of the plot.

The surface plot also shows that the highest yield is obtained when nitrogen levels are low and phosphoric acid levels are high. In addition, you can see the shape of the response surface and get a general idea of yield at various settings of nitrogen and phosphoric acid.

Keep in mind that these plots are based on a model equation. You should be sure that your model is adequate before interpreting the plots.

References

[1] G.E.P. Box and D.W. Behnken (1960). "Some New Three Level Designs for the Study of Quantitative Variables," *Technometrics 2,* pp.455–475.

[2] G.E.P. Box and N.R. Draper (1987). *Empirical Model-Building and Response Surfaces*, John Wiley & Sons. p.249.

[3] A.I. Khuri and J.A. Cornell (1987). *Response Surfaces: Designs and Analyses,* Marcel Dekker, Inc.

[4] D.C. Montgomery (1991). *Design and Analysis of Experiments*, Third Edition, John Wiley & Sons.